INTRODUCTION TO GOLDEN SECTION

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Golden Section’s synonyms

- Golden section
- Golden ratio
- Golden proportion
- *Sectio aurea* (Latin)
- Divine proportion
- Divine section
- Phi

φ

Self-Similarity

- *Self-Similarity* is the over-arching principle within which Golden Section proportion belongs.
- Self-similarity involves inter-semiotic translation, whereby certain features or patterns recur across multiple scales or parameters creating a web of inter-relationships. The presence of self-similarity may be indicated by the presence of the following:

Indicators of Self-Similarity

- Scaling
  Truly self-similar relationships and patterns require that scaling is present without appreciable change (Madden 1). Scaling is change of size accompanied by constancy of shape or form, and is the principal criterion of self-similarity (Mandelbrot 37).
- Recursion
  Recursion involves iteration of a given object, rule or idea in a self-referential manner. Recursive patterns of numbers potentially expand infinitely, often from the simplest of generating rules.
- Geometric sequence
  We may observe scaling and recursion manifested as geometric (rather than arithmetic) sequences.

Recursion

An example of a recursive pattern is a *summation series*, whereby each successive term is created by adding together previous terms.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

Each term is calculated by adding up the two before it.

2 is found by adding the two prior numbers (1+1)

3 is found by adding the two prior numbers (1+2)

Etc…

This creates a geometric progression.

Fibonacci Series

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

- This particular summation series is known as the *Fibonacci series*, named after the thirteenth century Italian mathematician Leonardo of Pisa. ("Fibonacci" was his nickname.)
- This series expands not only infinitely, but also exponentially. The presence of scaling in this regard indicates the repeated application of a particular ratio. The fractional results of the ratios between each successive term in the Fibonacci series are listed below.

1/1 =1
1/2 =0.5
2/3 = 0.666
3/5 = 0.6
5/8 = 0.625
8/13 = 0.615

Fibonacci Series and Golden Section

- There is an intrinsic connection between the Fibonacci Series and Golden Section proportion.
- The Fibonacci Series *tends towards* GS proportion as one evaluates the ratio between successive members of the series.
- 144/233 or 0.618 is commonly taken as a reasonably close approximation of what is actually an irrational number.
- Curiously, any summation series will converge on GS regardless of value of the two the initial terms, if the same generating rule is applied.

Fibonacci Series and GS in Overtones

In musical harmony, and the development of temperament (tuning systems), we discovered early on that it was pleasurable to listen to strings tuned in relations of small integers – i.e. using the numbers 1, 2, 3, 4...
The following lists ratios and their interval equivalents:

- 1:1 unison
- 2:1 octave (*diapason*)
- 3:2 perfect fifth (*diapente*)
- 4:3 perfect fourth (*diatessaron*)
- 5:4 major third (*ditone*)

The perfect fifth is 2/3 = 0.666... which approximates GS. It is the next most consonant interval after the unison and octave, and is such a common place to modulate.

Fibonacci Series in Pitch and the Keyboard

- On the piano keyboard there are two groups of black notes – a group of 2 and a group of 3, totalling 5.
- In one octave, C to C, we have 8 white notes, giving the grand total of 13 notes.
- 2, 3, 5, 8, and 13 are all from the Fibonacci series.

Fibonacci Series and Melodic Application

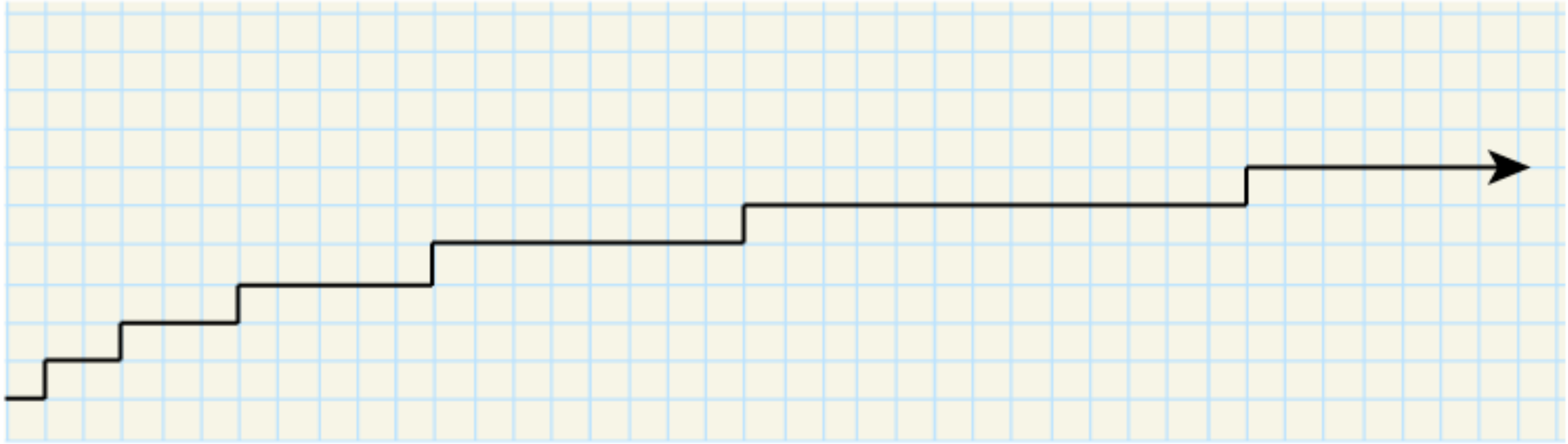
Schillinger proposes various applications of the summation series to pitch, using the values to represent intervals measured in semitones:

- Alternation of direction of interval (340)
- Readjustment of range, via octave transposition (334)
- Progressive substitution of the initial terms of the Fibonacci series by replacing

the second term with the third (e.g. 1, 3, 4, 7, ... and 1, 4, 5, 9, ...) (334)

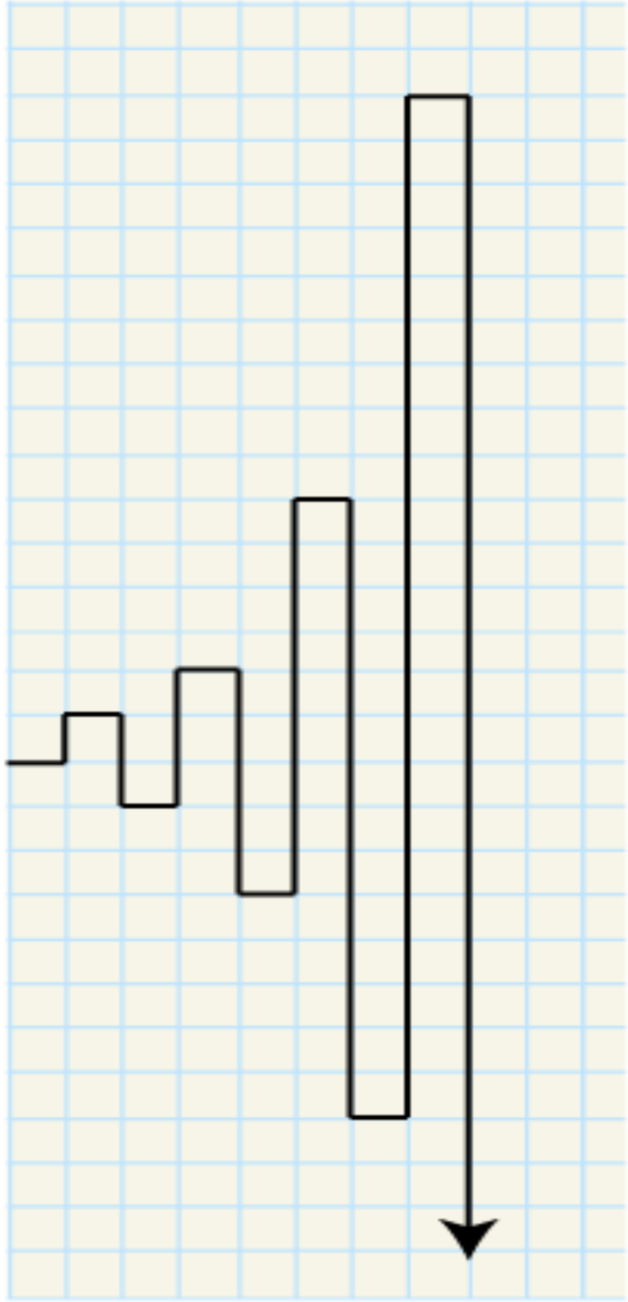
- Redefining the generating rule of the series (e.g. every term is the summation of the prior three) (335)
- Tabulation of summation series against term number on two axes that could be applied isomorphically to duration and pitch (335)
- Tabulation of summation series against term number on two axes with alternation of direction of interval to create spiral formations with bilateral symmetry (337)
- Interpolation of the original term as the pitch axes, applied to the prior symmetrical approach (339)
- Compression of the range of the series through omission of the third term, and use of balancing interval directions such as terms 1 and 2 ascend, followed by term 4 descends (340, 347).

Fibonacci Series and Pitch



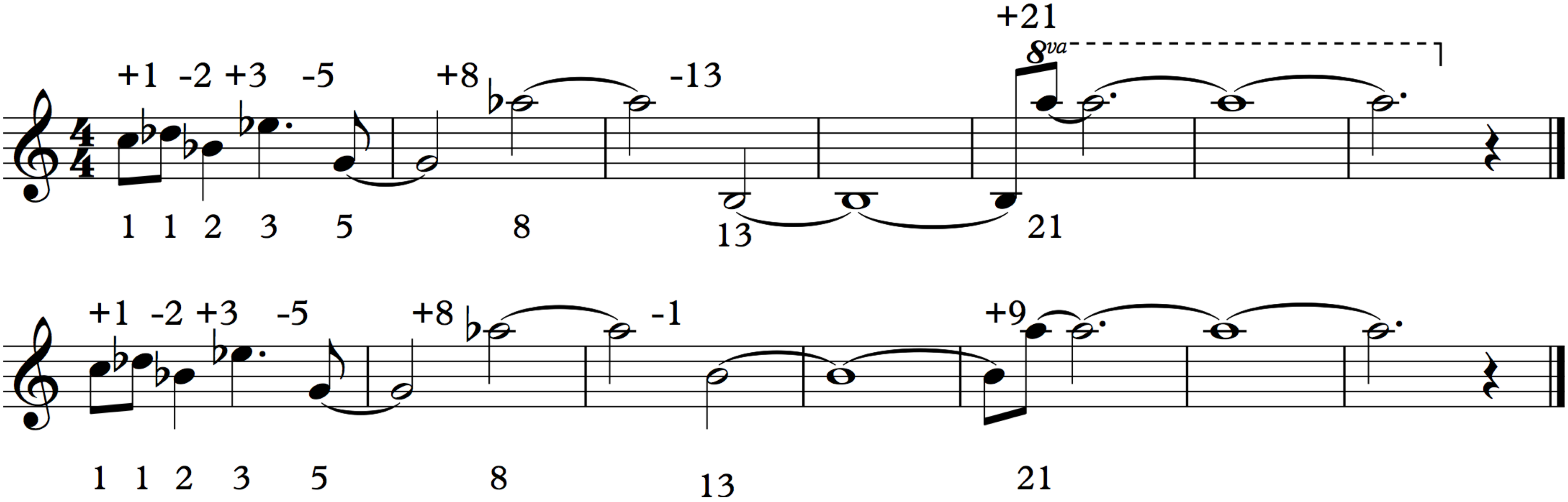
Tabulation of summation series against term number on two axes that could be applied isomorphically to duration and pitch (per Schillinger 333)

Fibonacci Series and Pitch



Tabulation of summation series against term number on two axes with alternation of direction of interval (per Schillinger 337)

Fibonacci Series and Pitch



Spiral melody as isomorphic interpretation of summation series.

Fibonacci Series and Scales

- Bartok favoured the use of the octatonic and pentatonic scales.
- The octatonic scale has a 2+1 interval generator, thus forming intervals of 1, 2, 3, 5, 8, 13 etc.
- The pentatonic scale also contains Fibonacci numbers:

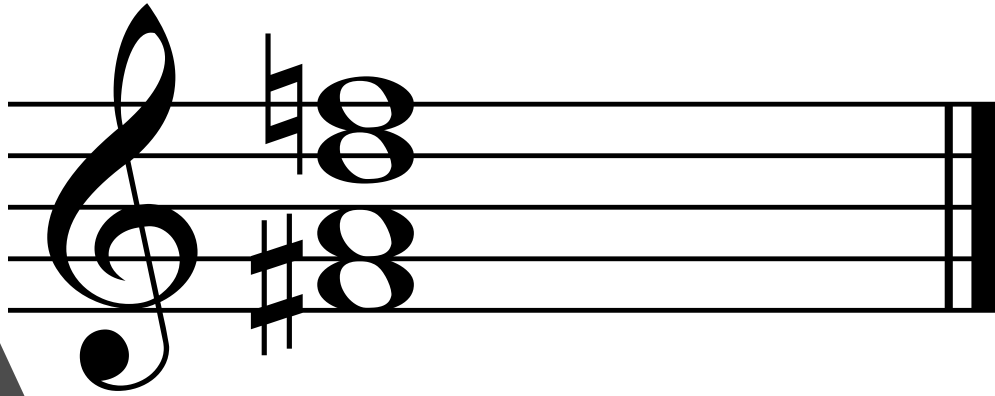
E G A C D

3 2 3 2

5 5

8

Fibonacci Series and the Major-Minor Chord



- Bartok favoured the use of a 4-note chord with both major and minor 3rds. Its intervallic construction
- The chord illustrated is in D
- It is constructed (in ascending order) with 3, 5, 3, with 8's spanning the outside to inner pitches

Fibonacci Series and Acoustic Scale

- Bartok’s use of the lydian dominant scale can be explained in many ways.
- It has a relationship to the harmonic series (the first 11 partials).
- It is also the complement of the chromatic aggregate once the Fibonacci series notes (number 1, 3, 5 etc) are removed.

Bibliography & Further Reading

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