Lecturer: Jonathan Dimond
BOM326 - Music Composition 4
Melbourne Polytechnic
August 2018

Introduction to Polyrhythm

Background

The stratification of music into constituent temporal layers has been approached by composers in a myriad of ways. Conlon Nancarrow's *temporal dissonance*, Elliott Carter's *rhythmic succession and simultaneity*, Steve Coleman's *polyrhythmic signatures*, and Olivier Messiaen's *rhythmic pedals* represent some of the diversity of approach in the music of the West during the past century, not to mention the richness of rhythm stemming from intercultural music.

Polyrhythms are inherently symmetrical. Polyrhythm allows the composer to manipulate the sense of how time flows, and provides an opportunity to be illusionary as well as strongly directional when it comes to temporality. Depending on application of pitch, instrumentation, dynamic and other musical choices a performer or composer makes, polyrhythm may not sound like two discrete pulse trains at all, but rather as a combination of their constituent lines to make a complex whole. Polyrhythm allows for "perceptual rivalry and multiplicity" (Pressing, "Black Atlantic Rhythm" 285).

Definition and terminology

Polyrhythm is a term whose definition very much in dispute, just as other temporal terms such as *pulse* and *timeline*. Rather than entering such a debate, I feel it is more appropriate to maintain a consistent and rational approach within ones own practice and teaching, and so offer my definition of the following essential terms after substantial consideration of practice and research in the field. The literal translation of the two parts to this term implies Agawu's usage, which befits the West African rhythmic textures that he researches (see Agawu, "Structural Analysis or Cultural Analysis?"). However my definition draws from contemporary drum set practice as presented by such author/teachers as Gary Chaffee and Peter Magadini (see Bibliography), and is consistent with its application in contemporary music in general.

Polyrhythm: The superimposition of two or more layers of regular groupings of pulse.

Given that the essential feature of *beat* is the periodicity of pulse grouping, and *metre* by definition comprises recurring groups of beats, it can be seen that polyrhythm could alternatively be defined as *the superimposition of two or more metres,* or termed *polymetre*.

Two-part polyrhythms are expressed as a ratio, being two numbers separated by a colon, which is read "is to" or "over" (e.g. "three is to two"). Polyrhythms are usually written in terms of their lowest common denominator. For instance, 4:6 is expressed as 2:3. Note also that the two figures in the ratio cannot be integer multiples of each other (as in 3:6).

Figure 1 shows how the ratio 3:2 corresponds to metre and its constituent layers. The numerator (3) represents the number of beats in the upper layer and the denominator (2) represents the number of beats in the lower layer - usually identical to the numerator (top figure) of the metre or time signature.

Figure 1. Correspondence of polyrhythm ratio to metre and constituent layers in western notation.

Durations are chosen in the knowledge that it is the attack point - not the duration - which essentially defines the rhythm. However, musical interpretation sees a myriad of choices as to the real length of the attacks, and how they may be realised with further subordinate attacks. The 3:2 polyrhythm in Figure 1 remains the essential structure of Figure 2.

Figure 2. Example of a rhythmic subgrouping of 3:2 polyrhythm.

Any polyrhythm is essentially a *relationship*, and consequently can be viewed or heard from the perspective of any of the constituent layers. Fundamentally 2:3 represents the same relationship as 3:2, however they tend to be notated differently, and the composer’s choices (including harmony and pitch considerations) have a bearing on the sense of beat primacy (lower layer) versus overlay.

The eleven polyrhythms which combine integers from 2 through 7 are listed in Figure 3, expressed in terms of their lowest common denominator, and profiled with their cumulative rhythms and adjacent interval vectors (to be discussed subsequently).

Polyrhythm	Cumulative rhythm	Adjacent interval vector
2:3	<2-1-1-2>	<<2,2>>
3:4	<3-1-2-2-1-3>	<<2,2,2>>
2:5	<2-2-1-1-2-2>	<<2,4>>
3:5	<3-2-1-3-1-2-3>	<<2,2,3>>
4:5	<4-1-3-2-2-3-1-4>	<<2,2,2,2>>
5:6	<5-1-4-2-3-3-2-4-1-5>	<<2,2,2,2,2>>
2:7	<2-2-2-1-1-2-2-2>	<<2,6>>
3:7	<3-3-1-2-3-2-1-3-3>	<<2,2,5>>
4:7	<4-3-1-4-2-2-4-1-3-4>	<<2,2,2,4>>
5:7	<5-2-3-4-1-5-1-4-3-2-5>	<<2,2,2,2,3>>
6:7	<6-1-5-2-4-3-3-4-2-5-1-6>	<<2,2,2,2,2,2>>

Figure 3. Eleven polyrhythms that combine integers from 2 through 7.

When two or more rhythmic layers are superimposed, their merged combination can be thought of (and heard) as a resultant rhythm, which I call a *cumulative rhythm*. In polyrhythm, this periodic rhythm is unique and characteristic of the particular ratio (in either of its inversions), and can provide essential orientation and identification information for the performer and listener. (Many musicians use mnemonic phrases to conjure the sound of specific cumulative rhythms, such as “cold cup of tea” for 2:3.) As such, cumulative rhythm acts as a kind of “hook” or “fingerprint” that can be used to orientate ones sense of beat within polyrhythm. The cumulative rhythm is the source of the *rhythmic contour* of a polyrhythm.

Cumulative rhythm: The resultant rhythmic phrase which represents the combination of two or more regular rhythmic layers in a polyrhythm.

Figure 4 notates Figure 1 as a cumulative rhythm, in its two inversions as 3:2 and 2:3 (respectively). Notice that the rhythm takes the polyphony of the original layers and creates a monophonic version, notating each onset chronologically. Notice also that cumulative rhythms are palindromic, and therefore inherently symmetrical. Both rhythms in Figure 4 have an inter-onset interval series of <2-1-1-2>, in their respective unit of pulse.

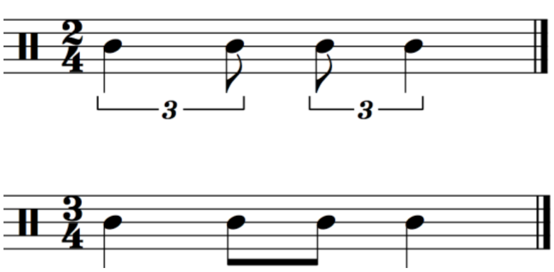

Figure 4. Cumulative rhythm for 3:2 and 2:3 polyrhythm.

There are lots of examples of this basic *hemiola* polyrhythm in genres from around the world. (*Hemiola* derived from the Greek *hemiolios*, meaning 'the whole and a half', and has featured extensively in European art music since the Baroque.)

Here are four contemporary examples:

Artist	**Album**	**Composition**
Sting	The Soul Cages	Island of Souls
Richard Bona	Munia	Sona Mama
Chiquinho Timoteo	Brazil Guitar Masters	Bahia Sarava
Nguyên Lê	Purple - Celebrating Jimi Hendrix	1983...(A Merman I Should Turn to Be)

Figure 5. Four examples of the 2:3 polyrhythm in contemporary music.

This last example actually contains 2:3 and 3:4 (or 2:3:4) concurrently at one point.

Schillinger's theory of *interferences of periodicities* resembles some of the polyrhythm theory presented in this chapter, though my theory is not derived from his. Schillinger's application *interference* theory through such methods as his *melodic attack groups* shows the nature of his unique and complementary approach. (Refer to Schillinger Book I Chapter 2 and Book VI Chapter 2).

With any measurement of rhythmic structure, it is the *onset* (attack point) that forms the point of measurement. Though the mid-point of duration would correspond to precise mathematical symmetry (Csapó 184), the primacy of rhythmic onset corresponds with practical experience, for in reality the same sense of proportion is obtained from the rhythms in Figure 4 regardless of note duration. Refer also to Dimond 2007 (chapter 5) for more information on polyrhythm theory and its practical application.

Rhythmic Contour

In the domain of pitch, melodic contour is said to describe the rise and fall of a particular phrase. Melodic contour conjures generic shape, which involves an abstraction away from the particularity of specific intervals towards the generalisation of intervals as steps, leaps or constant unisons. As such, melodic contour could be graphed (or represented gesturally) as mountains, valleys and plains. (See Straus p.126 for a discussion of melodic contour in post-tonal music.) Likewise, the rhythmic contour of a polyrhythm, and indeed any rhythm, arises from the directionality of successive changes in duration as one traverses the series of onsets.

Toussaint defines *rhythmic contour* as a qualitative measure of change in the durations of each successive pair of inter-onset intervals ("The Geometry of Musical Rhythm" 47). Toussaint's coding system however is pseudo-quantitative, as changes are represented as +1, -1, or 0 for an increase, decrease, or repetition of the prior inter-onset interval, respectively.

Due to my interest in rhythmic contour as a compositional concern, I have chosen to nominate two approaches to measuring it. *Generic rhythmic contour* adopts Toussaint's definition, and uses symbols rather than integers to maintain the qualitative measure intended. The arrows ↑ ↓ → represent a relative lengthening, shortening and repetition (respectively) of inter-onset intervals.

Specific rhythmic contour takes the series of inter-onset intervals and codes the change in duration with a specific measure of difference. In other words, this approach is one step less abstract than the *adjacent interval vector* representation of a rhythm, as it retains the chronology of the onsets. This approach also resembles melodic contour measurements, in the sense that steps (adjacencies) are distinguished from leaps. For measures of s*pecific rhythmic contour,* +x, -y, or 0 represent quantifiable changes through an increase, decrease, or repetition of the prior inter-onset interval, respectively. Refer to Figures 6 and 7.

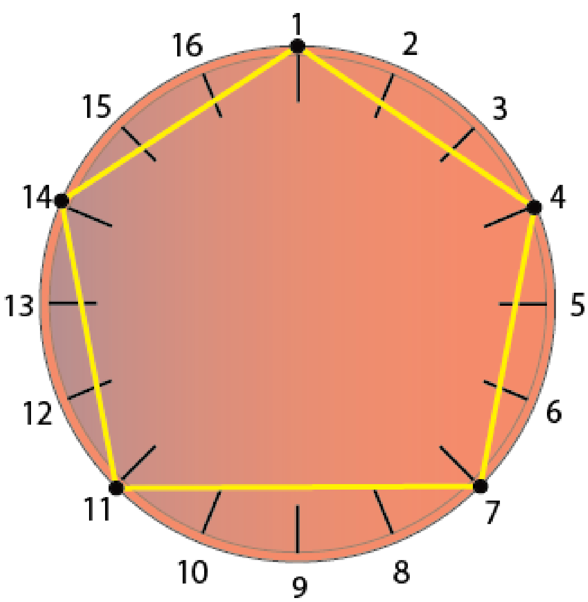

Figure 6. *Bossa-nova* timeline

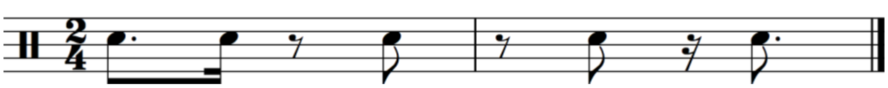

Figure 7. *Bossa-nova* notation.

This *bossa-nova* clave consists of:

- Timeline of 16 pulses
- Inter-onset intervals <3-3-4-3-3>
- The g*eneric rhythmic contour* → ↑ ↓ → →
- The s*pecific rhythmic contour* 0, +1, -1, 0, 0.

Bilateral symmetry of polyrhythm

The timeline in Figure 8 reveals a symmetrical arrangement around a vertical axis of reflection. If the timeline illustration was folded along the axis indicated by the dotted green line, all the onsets on one half would line up with the other half. The cumulative rhythm also reveals the inherent *bilateral symmetry* of any polyrhythm. (The terms *bilateral symmetry*, *plane symmetry, mirror symmetry* and *reflection symmetry* tend to be used interchangeably with identical meaning.) Timelines with bilateral symmetry create mathematical *palindromes* from their inter-onset intervals, and therefore by definition are *inversionally symmetrical*.[1] Effectively you can invert the sequence of inter-onset intervals by reversing their order around the axis of reflection. For example, the 4:5 polyrhythm's inter-onset interval series:

<4-1-3-2-2-3-1-4>

becomes

<2-3-1-4-4-1-3-2>

when the series is reversed around the axis of reflection (in this case, between the fourth and fifth onset). The inversional symmetry can be seen in the timeline of Figure 9, which appears as a mirror reflection of Figure 8. The two polygons now intersect at pulse 11 - diametrically opposite pulse 1.

---

[1] Rahn states that "an inversionally-symmetrical set always has a canonical ordering whose interval series is its own retrograde…" ("Basic Atonal Theory" 91). By "canonical ordering", Rahn means "an ascending ordering such that the adjacency-interval series of both sets related by TnI are mutually retrograde" (88). Inversion symmetry is therefore also called retrograde symmetry. Inversionally-symmetric sets have consecutive interval patterns that are palindromic. The major and minor triad is a common example of an inversionally-symmetric pitch-class set, whose structure is represented by the prime form [037].

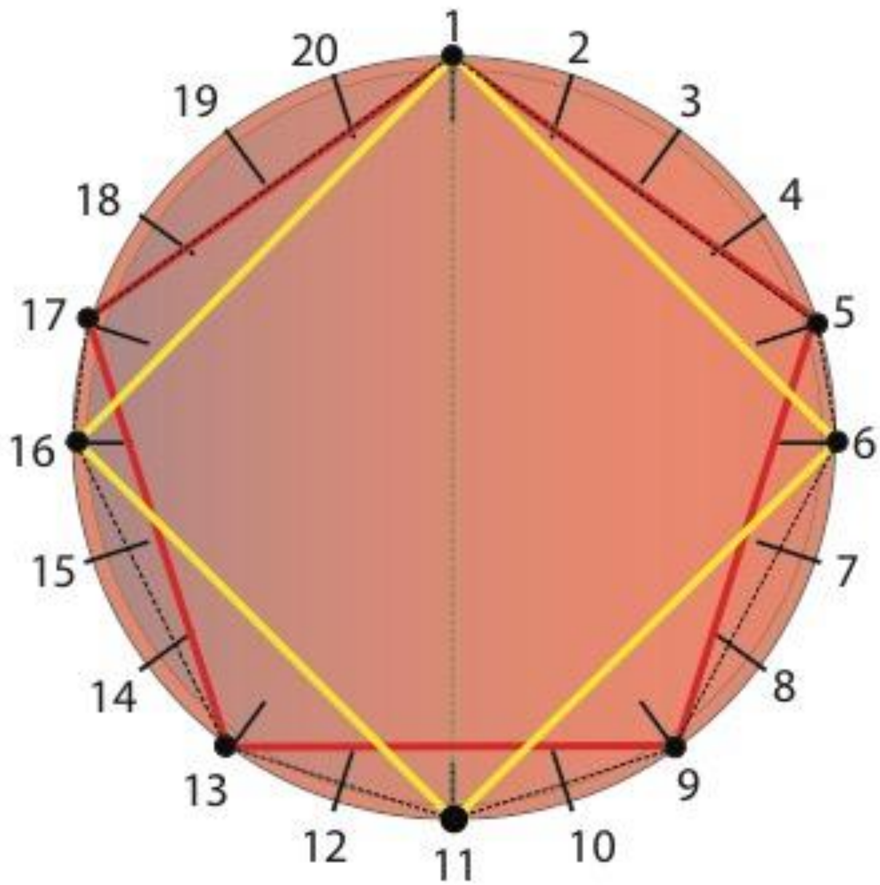

Figure 8. 4:5 polyrhythm timeline.

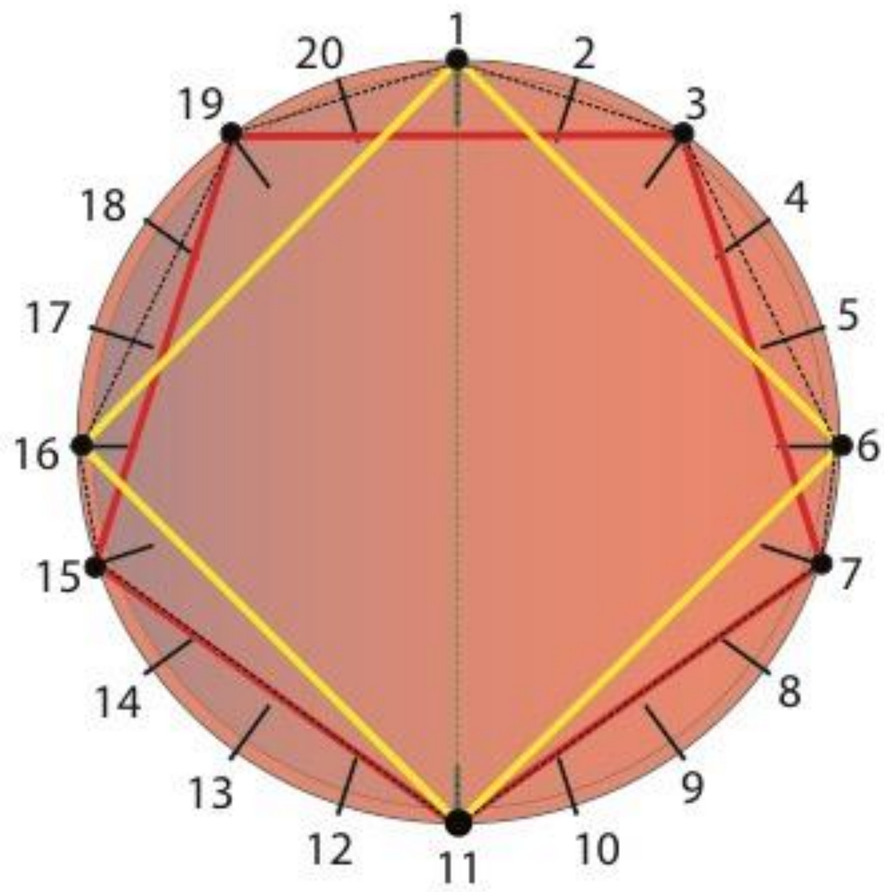

Figure 9. 4:5 polyrhythm timeline bracelet (inverted form of Figure 8).

The bilateral symmetry of polyrhythm can also be inspected via the internal design of cumulative rhythm, not just the palindromic nature of its overall inter-onset interval series or folding of the timeline about its axis of symmetry. Logically, any polyrhythm of ratio *x:y* requires that any member or pair of adjacent members in its inter-onset interval series <*a-b-c-d-…*> must equal either *x* or *y*. For example, the 4:5 polyrhythm's inter-onset interval series <4-1-3-2-2-3-1-4> contains 5 groups of 4 pulses and 4 groups of 5 pulses. Figure 10 illustrates the four binomial pairings required to render 4 groups of 5 pulses, superimposed on the monomial and overlapping binomial pairings required to render 5 groups of 4.

Figure 10. 4:5 polyrhythm inter-onset interval series partitioned into 4 groups of 5 pulses (yellow circles) and 5 groups of 4 pulses (red pentagons).

This process can also be visualised using box diagrams. The following figures illustrate the 4:5 and 5:7 polyrhythms using this alternative method to reveal bilateral symmetry in cumulative rhythm.

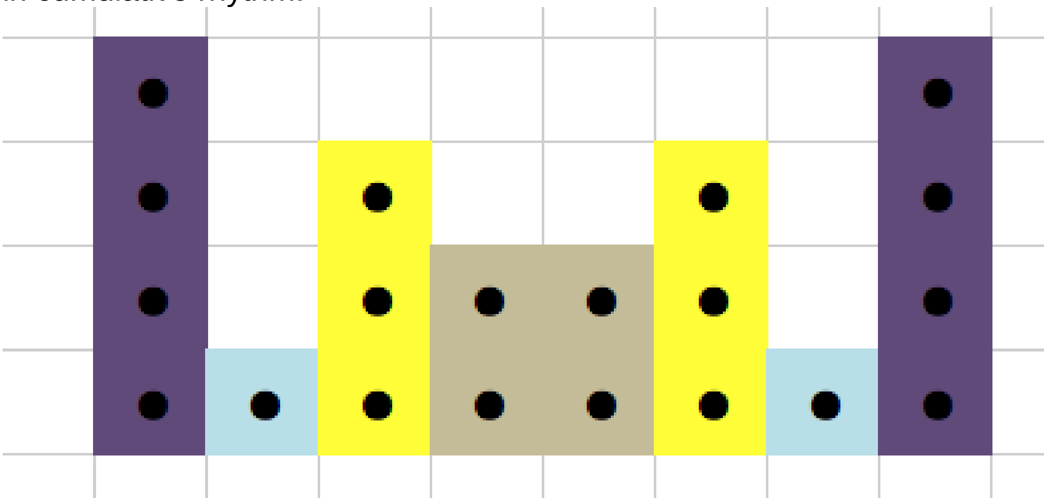

Figure 11. 4:5 polyrhythm inter-onset interval series as a box diagram.

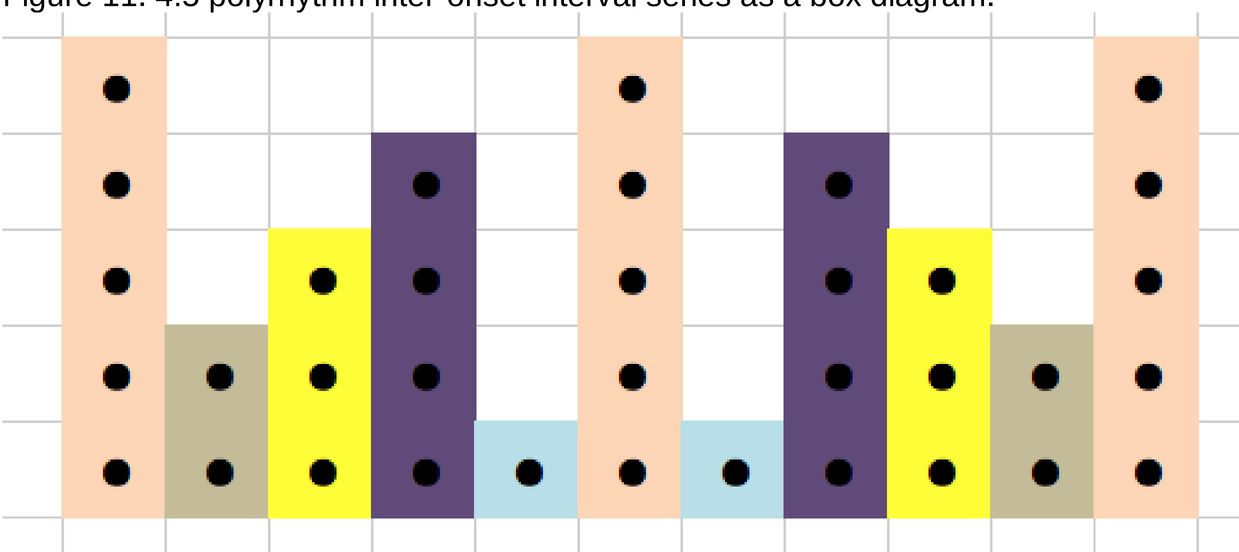

Figure 12. 5:7 polyrhythm inter-onset interval series as a box diagram.

Coincident hit point and Apsis
All the polyrhythms in this paper have a pulse on the timeline where the two constituent layers coincide, or “hit" simultaneously. (For information on non-hitting polyrhythms and polyrhythm mathematics, refer to Dimond 2007.)

In orbital mechanics this is called a *conjunction*. In this paper I refer to this special pulse as the *coincident hit point.*

Coincident hit point: The pulse in a polyrhythm where the constituent layers share an onset.

All polyrhythms also have a point where their constituent layers are most distant from each other on their periodic trajectory. I call this point the *apsis* (plural *apsides*). This point will always be diametrically opposed to (or 180° from) the coincident hit point.

Apsis: The pulse or place in the timeline diametrically opposed to the coincident hit point.

Bracelets and necklaces
Figures 6 and 7 are considered the same *bracelet,* which is a mathematical term which considers two number sequences of the same bracelet if they are mirror image reflections of each other. (In post-tonal theory, you will remember that the sets [037] and (047) bear this relationship, for example.) The two polyrhythm timeline examples contain the same sequence of onsets, but in exact inversion.

If two rhythms are mirror image reflections of each other, they are considered of the same bracelet.

Musicians habitually associate beat 1 (the 'down beat') with the coincident hit point in every ostinato or repeating pattern. You can create compositions that explore other possibilities for rhythmic alignment in time cycles to create interesting effects.[2] In the process of phase-shifting the coincident hit point of polyrhythms around timelines I create a variable experience of what I call "temporal gravity". The mathematical term for such cyclic permutations is *necklace*.

If two rhythms are rotations of each other, they are considered of the same necklace.

[2] In South Indian classical music the *eduppu* or starting point of a composition or line may differ from beat 1 (*samam*), creating a certain tension especially when it is in close proximity, such as 1 or 1.5 beats away from beat 1. This tension is not dissimilar to the necklace effect.

There are as many necklaces for a polyrhythm as there are pulses in the timeline.[3] In the case of the 4:5 polyrhythm, values for *n* from 1 to 19 (mod 20) can be added to each of the members of the set {1, 5, 6, 9, 11, 13, 16, 17} to create a unique set of onsets. There are therefore 20 (or 4x5) possible necklaces (rotations) of this polyrhythm.

Figure 13 illustrates the 4:5 polyrhythm timeline from Figure 8 in another of its 20 necklace arrangements.

[3] For any transpositionally-symmetrical rhythm that is perfectly even such as an isochronous 'beat' timeline, the number of necklaces is limited by the degree of symmetry the timeline possesses. A set is considered transpositionally symmetrical if some value of n in the transposition Tn maps the set into itself. The set [0, 3, 6, 9] for example is a tetrachord that maps into itself when n is 3, 6, or 9 semitones. (The set could also represent the beat in 12/8 metre.) It is thus transpositionally symmetrical, and is considered to have 4 degrees of symmetry in total (including n = 0).
Inversionally-symmetric timelines posses a special property which limits the number of times an individual layer in the polyrhythm can be rotated until another necklace of the timeline is recreated. As the cumulative rhythm of polyrhythms is palindromic, necklaces will always create bracelets, and bracelets will create a necklace as long as there an even number of overall pulses on the timeline.

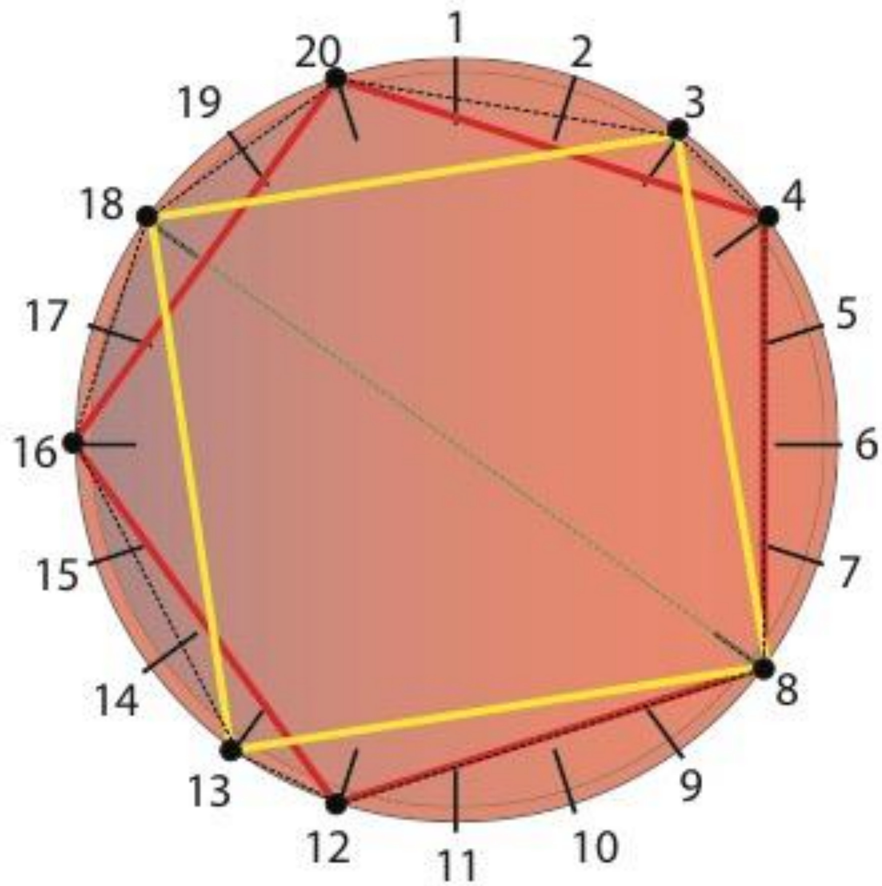

Figure 13. 4:5 polyrhythm timeline necklace with coincident hit point on pulse 8.

Inversionally-symmetric timelines with an odd number of pulses cannot be flipped 180° along the axis of reflection to create a bracelet as there are no diametrically-opposed pulse positions for the onsets. Figure 14 shows the 3:5 polyrhythm timeline with 15 pulses. Only one end of the green bilateral symmetry line for such odd-pulse timelines falls on an onset, and the apsis end of the line has no corresponding pulse. In such cases, there are only *necklaces*. (The only mathematical solution is to double the number of pulses. This is relates to the procedure for creating non-hitting polyrhythms. Refer to Dimond 2007.)

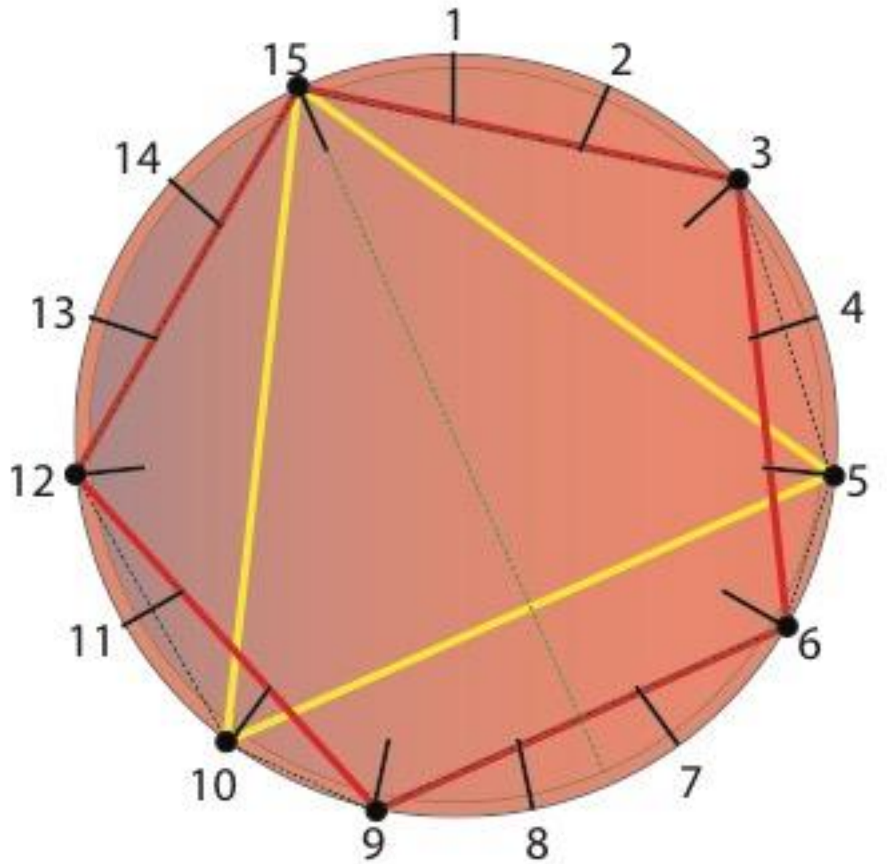

Figure 14. 3:5 polyrhythm timeline necklace showing no option for bracelet (mirror image reflection).

Metric Modulation

Metric modulation is defined by the Oxford Dictionary as a "term and technique introduced by American composer Elliott Carter for changing the rhythm (not necessarily the metre) from one section to another."[4] Tempo is primarily a function of the periodicity of beat, upon which metre depends, which is one reason this definition is so unsatisfactory. It lacks a distinction between *rhythm* and *beat*. The Oxford Companion to Music defines *metric modulation* as "a technique introduced by Elliott Carter, by which changing time signatures effect a transition from one metre to another, just as a series of chords can effect a harmonic modulation from one key to another."[5] The latter definition is more consistent with my temporal approach, and the naming of Carter as an exemplar is justified, since his linear approach is in such stark contrast to the *moment form* of Stockhausen in the 1960s (Kramer, "The Time of Music" 204). I

[4] Kennedy, Michael. "Metric Modulation." The Oxford Dictionary of Music Online. 2nd ed. 1 Mar. 2017 <http://www.oxfordmusiconline.com.ezproxy.library.uq.edu.au/subscriber/article/opr/t237/e6759>.

[5] Latham, Alison. "Metric Modulation." The Oxford Companion to Music Online. Accessed 1 Mar. 2017 <http://www.oxfordmusiconline.com.ezproxy.library.uq.edu.au/subscriber/article/opr/t114/e4389>.

advance my own definition, consistent with other terminology in this paper.

Metric Modulation: The redefinition of beat resulting in a change of perceived tempo and possibly metric structure.

A *l'istesso* change from 12/8 to 4/4 (with the dotted crotchet equalling the new crotchet) is excluded from my definition of metric modulation, as the result is merely a change of subdivision. If the crotchet was held steady in this example however, the beat would be scaled by 2/3 of the rate, resulting in metric modulation and a requisite change in tempo. See Figure 15.

Figure 15. Metric modulation is present in the bottom line but not in the top line, as it does not involve a scaling of beat/perceived tempo.

My point is that metric modulation is a natural outcome of polyrhythm, and it may be encouraged (through presentation of multiple beat-like layers with corresponding harmonic or melodic material)[6], implied (as in traditional *hemiola*) or enforced by the composer through establishment of specific modulatory formulae in notation. The establishment of recurring polyrhythmic layers in stable time cycles may result in a perception of *ad hoc* metric modulation by the listener even when it is not intended by the composer. In establishing the ambiguity and paradox that polyrhythm entails, the listener is free to flit between opposing layers in terms of beat autonomy, hierarchy and frame of reference, promoting the "perceptual rivalry and multiplicity" quoted from Pressing at the start of this paper.

[6] Check out Andy Gander's YouTube demonstrations of *time shifting* and other techniques. Also refer to DrumScene Issue 92 (April-June 2018). https://www.youtube.com/watch?v=FJBQ6sfTqbo

Bibliography / Further Reading

Agawu, K 2006, ‘Structural Analysis or Cultural Analysis? Competing Perspectives on the “Standard Pattern” of West African Rhythm’, Journal of the American Musicological Society, vol. 59, no. 1, pp. 1–46.

Csapó, G 2010, ‘Towards the Establishment of Criteria for Discussing Music in Terms of Symmetry and Asymmetry (an Outline)’, Proceedings of The First Interdisciplinary Chess Interactions Conference. Edited by HE Stanley. Published by World Scientific Publishing Co.

Chaffee, G 1994, Rhythm & Meter Patterns, PAP/COM edition., Alfred Music, Miami, FL; CPP Belwin.

Dimond, J 2007, Bass Riyaz: The practice workbook for the mastery of the 4, 5, and 6-string electric bass guitar, 3rd ed, Lulu, Los Angeles.

Magadini, P & Sykes, W 2001, Polyrhythms - The Musician’s Guide, Hal Leonard Publishing Corporation, Wisconsin.

Rahn, J 1980, *Basic Atonal Theory*, Schirmer Books, New York.

Schillinger, J & Arden, J 1946, The Schillinger System of Musical Composition, 4th ed, Clock & Rose Press.

Sethares, W 2007, *Rhythm and Transforms*, Springer, London.

Straus, JN 2016, *Introduction to Post-Tonal Theory*, 4 ed., W. W. Norton & Company, New York.

Toussaint, GT 2013, *The Geometry of Musical Rhythm*, Chapman & Hall/CRC, Boca Raton, Fla.; London.