PATHWAYS THROUGH THE OCTATONIC SCALE JONATHAN DIMOND, PHD



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INTRODUCTION

The octatonic scale is an 8-note symmetrical scale widely in use in jazz and contemporary music, and has also featured in the theoretical writings and compositions of such authors as Messiaen, Slonimsky, and Schillinger.¹ It is based upon the over-arching concept of symmetry – also a widely-researched trait found in all the arts. John Coltrane is a famous example of a jazz artist who exploited the inherent ambiguities of symmetrical divisions of the octave through his three-tonic system featured in the composition *Giant Steps*.

This paper aims not to duplicate the rich number of resources already available, but rather to provide an original perspective on the organisation and execution of the octatonic scale, in such a way that can develop its practical application by musicians. Its theoretical basis takes inspiration from Peter Schat's *Tone Clock Theory* as investigated and applied in the work of Jenny McLeod. Its artistic motivation is drawn largely from the harmony and melody of Messiaen's compositions.

The connection between Tone Clock Theory and this paper is through the concept of *steering*. The concept of steering originates in Boulez's term *frequency multiplication*,² and refers to the sequence of transpositions applied to a set of notes or a chord such that a certain group of pitches is sounded. Usually, steerings aim to saturate 12-tone pitch-space by sounding every chromatic pitch via transpositions of a pitch-class set such as a particular trichord (McLeod 23).

Steering presents as an attractive concept to adapt into the non-chromatic world of octatonic scale harmony, and offers a logical approach to organise sequences of recognisable triads through a pattern of transpositions in such a way that all eight notes of the octatonic scale can be sounded. As a result, the tonal ambiguities inherent in the octatonic scale are maintained whilst tonal regions are simultaneously emphasised. Through such a systematic methodology, it is hoped that pan-tonal patterns may be discovered by the composer and improviser, creating heretofore unfamiliar relationships within the octatonic scale.

¹ Refer to Messiaen's modes of limited transposition, and the writings of Slonimsky and Schillinger in the Bibliography.

² Boulez's term is *multiplication sonore* (Hall and Sallis 151).

TERMINOLOGY AND NOMENCLATURE



Figure 1. The octatonic scale from B, shown as four replications of a <1-2> interval generator, outlining the successive minor thirds of the diminished seventh chord.

Figure 1 illustrates the octatonic scale also known as the dominant 8-note scale, from B.³ This scale arises from four repetitions of the semitone/tone dyad, referred to as the *interval generator* of this symmetrical scale. The integers <1-2> represent the interval generator, measured in semitones.

Scale and chordal symmetry relies upon the fact that the 12-semitone octave can be equally divided.⁴ The factors of 12 are 12, 1, 6, 2, 4 and 3, and so it is partitions of 12 with an interval generator totalling the size of these factors that give rise to symmetrical scales such as Messiaen's *modes of limited transposition*. The octatonic scale's interval generator spans three semitones (1+2), and corresponds to the extent of the scale's limited transposition.⁵ In the given key, the B half-step/whole-step scale can be transposed to C and C#, but transpositions to D and further onwards create redundant scales.⁶

The number of replications of the interval generator required to create each scale is a simple division of the octave (12) by the size of the interval generator. In this case, $12 \div 3=4$. These four equally-spaced positions outline the diminished seventh chord, indicated by lower voice in figure 1. When the octatonic scale is plotted on a circle as in figure 2, its geometry reveals that the number of replications of the interval generator is congruent with the number of degrees of symmetry.

³ This is the scale used throughout this paper for illustrative purposes. It is of course intended that this model be thereafter transposed to all keys and played through multiple registers of the instrument. Standard practice sees C ascribed pitch class zero (0). Using B in this paper further encourages practice and agility with transposition.

⁴ See Slonimsky for a thorough investigation of symmetrical patterns arising from equal division across one and more octaves.

 $^{^{5}}$ This is why there is one chromatic scale <1> and two whole tone scales <2>.

⁶ D half-step/whole-step could be considered a degenerate scale with reference to B half-step/whole-step, as could C whole-step/half-step.





An alternative geometric perspective of the octatonic scale's symmetry is to consider it as two superimpositions of a four-note diminished seventh chord, instead of a series of four instances of a three-semitone interval generator. Figure 3 illustrates this perspective. This creates what is known in geometry as a *maximal-area polygon* – the eight onsets (points) of the combined octagon cover the maximum surface area able to be enclosed by eight points in twelve. The octatonic scale is the only maximal area 8-note scale with this characteristic.⁷

⁷ Toussaint also points out that the complement of this scale (formed by the pitches remaining from the chromatic collection) is also a maximal area polygon – another diminished 7th chord (Toussaint 54).





Figure 4 defines the octatonic axes illustrated in figure 3 with notation.



Figure 4. The octatonic scale from B, with labels defining the axial membership of each pitch per the superimposed diminished seventh chords of figure 3.

DEFINITION OF SYMBOLS

Table 1 summarises the symbols used in this paper.

Symbol	Meaning / Usage	Example
<x,y></x,y>	Interval generator for symmetrical scales.	<2,1,1> represents Messiaen's Third Mode of limited transposition (Messiaen 60).
[x,y,z]	Pitch-class set, such as a trichord. (See Rahn 140 for a table of all pitch- class sets.)	[0,1,3] represents C,Db,Eb and all the trichord's transpositions and inversions. In Tone Clock Theory, this set corresponds to the series 1+2, positioned at two o'clock, or the Interval Prime Form 1-2. (See McLeod 3 for an explanation of IPFs.)
⇒(x,y,z)	Steering. The pitch-class set that directs the transposition of a particular pitch-class set in order to fulfil a tiling or sounding of all available pitch classes.	$[0,1,4] \implies (0,2,5)$ In this example, PC set $[0,1,4]$ is steered by two overlapping trichords (0,2,5) and $(0,3,5)$ in order to tile 12- tone pitch-space, per figure 5. In Tone Clock Theory, McLeod prefers to refer to the clock "hour" of the respective IPF (McLeod 23).
⇒(w,x,y,z)	Steering. Identical to the above, only with a tetrachord. The tetrachord steering can still be construed to be two overlapping inversionally-related trichords.	[0,3,6] ⇒ (0,3,4,7) In this example, PC set [0,3,6] is steered by a tetrachord, which can be also construed as two overlapping inversionally-related trichords (0,3,4) and (0,1,4).

Table 1. Legend of symbols and terminology.



Figure 5. Example of steering in 12-tone pitch-space. $[0,1,4] \implies (0,2,5)$. The upper voice is steered by the lower voice, which can be interpreted as two overlapping inversionally-related trichords, or alternatively as tetrachord (0,2,5,7).⁸

⁸ The two trichords are (0,2,5) and (0,3,5). Diagrammatic examples of such trichord steerings can be easily generated online. Refer to Dijkgraaf's web site in the bibliography.



TRIADIC FORMATIONS

Figure 6 illustrates twelve common triadic formations possible in the octatonic scale. Only conventional triads are chosen (rather than trichords) for their strong sense of tonality – something that gives rise to a pervasive identity when transposed through the otherwise-ambiguous octatonic scale. This combination of traits is necessary to educe the desired pantonal effect.



Figure 6. Binomial partitions of the octatonic scale showing all possible triads drawn from the qualities of major, minor, diminished, and major b5.

Figure 6 is colour-coded to reveal intervallic structure.⁹ Augmented and suspended 4 triads are not available in the octatonic scale. However four common triads, each with their respective inversions, makes for a reasonable variety of intervallic combinations. This diagram reveals a richness of triadic possibilities not immediately apparent through superficial consideration.

We now turn to the options of these triads' pathways through the octatonic scale.

⁹ Intervals of 2 are purple, 3 yellow, 4 light blue, 5 green, and 6 red.



PATHWAYS THROUGH OCTATONIC

The next task is to identify steerings of each triad identified in figure 6, such that all pitches of the octatonic scale are sounded. The goal is to abide by the degrees of symmetry inherent in the octatonic scale revealed in figure 2, so as to transpose each given triad through no more than three additional keys in order to sound every note in the octatonic scale.¹⁰ Figure 5 confirms that any transposition must align with the specific intervals available in the octatonic scale – free 12-tone transposition is not possible.

Shat and McLeod showed that 12-tone pitch space is tiled by steerings of a trichord quality that are the result of two overlapping trichords, or alternatively viewed as a symmetric tetrachord (McLeod 23). As such, steering is a kind of *compound transposition* (116). These coexisting viewpoints can be observed in figure 5.

In tiling the octatonic scale with triads, steerings apply similarly.¹¹ Note that the identified steerings are applicable regardless of the particular triad's inversion. This means that [0,3,6] represents the sets in root position {0,3,6}, first inversion {0,3,9}, second inversion {0,6,9} and all their transpositions as well as non-cyclical combinations such as {0,6,3}.¹²

The following eight figures illustrate each triad's steering geometrically and with notation. Four triadic keys necessarily produce a duplication of four pitches in the octatonic scale. For example figure 7 illustrates the diminished triad with pitch-classes 3, 6, 7, and 10 duplicated.

DIMINISHED TRIAD STEERING

[0,3,6] ⇒ **(0,3,4,7)**

(0,3,4,7) tetrachord = (0,3,4) and (0,1,4) overlapping trichords.

¹⁰ Three "additional keys" means the initial triad's tonic is transposed to three other keys available within the scale – producing a total of four keys. As a result, these steerings do not strictly tesselate the octatonic scale, as (3x4) 12 notes are sounded in order to cover the 8 pitches, necessarily involving a duplication of 4 pitches. The word "tiling" is hereafter used in this impure mathematical sense.

¹¹ Guitarists are recommended to refer to Okazaki's approach to explaining tiling 12-tone pitch space with triads, which he calls "triadic chromaticism" (Okazaki 86).

¹² Schillinger names these latter types of operations *general permutations*, distinguishing them from *circular permutations* which retain order, as in the *necklaces* of combinatorial mathematics.



Figure 7. Diminished triad steering via (0,3,4,7) tetrachord. Colour-coding identifies the root-position triads: red (0,3,6); purple (3,6,9); green (4,7,10); and pink (7,10,1). The root of each is colour-coded to match.



Figure 8. Diminished triad steering via (0,3,4,7) tetrachord. The four root-position triads match those in figure 7.

MINOR TRIAD STEERING

[0,3,7] \Longrightarrow (0,3,6,9)

(0,3,6,9) tetrachord = two (0,3,6) overlapping trichords.



Figure 9. Minor triad steering via (0,3,6,9) tetrachord. Colour-coding identifies the root-position triads: red (0,3,7); purple (3,6,10); green (6,9,1); and pink (9,0,4). The root of each is colour-coded to match.



Figure 10. Minor triad steering via (0,3,6,9) tetrachord. The four root-position triads match those in figure 9.

MAJOR TRIAD STEERING

$(0,4,7) \Longrightarrow (0,3,6,9)$

(0,3,6,9) tetrachord = two (0,3,6) overlapping trichords.

Due to the fact that the major triad's representative set in normal form is [0,3,7], it follows that the major and minor triads share the same steering.



Figure 11. Major triad steering via (0,3,6,9) tetrachord. Colour-coding identifies the root-position triads: red (0,4,7); purple (3,7,10); green (6,10,1); and pink (9,1,4). The root of each is colour-coded to match.



Figure 12. Major triad steering via (0,3,6,9) tetrachord. The four root-position triads match those in figure 11.

MAJOR FLAT 5 TRIAD STEERING

(0,4,6) \Longrightarrow (0,3,6,9)

(0,3,6,9) tetrachord = two (0,3,6) overlapping trichords.



Figure 13. Major flat 5 triad steering via (0,3,6,9) tetrachord. Colour-coding identifies the root-position triads: red (0,4,6); purple (3,7,9); green (6,10,0); and pink (9,1,3). The root of each is colour-coded to match.



Figure 14. Major flat 5 triad steering via (0,3,6,9) tetrachord. The four root-position triads match those in figure 13.

STEERING APPLICATION: TETRACHORD PERMUTATIONS

Decisions can next be made how to apply triadic pathways that are interesting and various. In the same way that McLeod's compositions explore triadic steering in an imaginative manner, we may diverge from the abstract ascending/descending scale forms implied in figures 7–14.

Given the steering tetrachord has four members, there are 24 possible orderings of the keys.¹³ A few of these orderings are applied to the root position major triad in figure 15.



Figure 15. Major triad steering variations via (0,3,6,9) tetrachord, created by reorderings of the tetrachord.

Figure 15 maintains the major triad in root position, but in the third variation illustrates that a mixture of ascending and descending directions create further possibilities.

Table 2 lists all 24 permutations of the (0,3,6,9) and (0,3,4,7) steerings as integers. These can be practiced with the relevant triad qualities identified earlier in this chapter.

	(0, 3, 6, 9) steer	ing permutations	
0369	0396	0936	0963
0639	0 6 9 3	3690	3609
3906	3960	3069	3096
6390	6309	6093	6039
6903	6930	9036	9063
9360	9306	9630	9603
	(0, 3, 4, 7) steer	ing permutations	
0347	(0, 3, 4, 7) steer 0 3 7 4	ing permutations 0 4 3 7	0 4 7 3
0 3 4 7 0 7 3 4	(0, 3, 4, 7) steer 0 3 7 4 0 7 4 3	ing permutations 0 4 3 7 3 4 7 0	0 4 7 3 3 4 0 7
0 3 4 7 0 7 3 4 3 7 0 4	(0, 3, 4, 7) steer 0 3 7 4 0 7 4 3 3 7 4 0	ing permutations 0 4 3 7 3 4 7 0 3 0 4 7	0 4 7 3 3 4 0 7 3 0 7 4
0 3 4 7 0 7 3 4 3 7 0 4 4 7 0 3	(0, 3, 4, 7) steer 0 3 7 4 0 7 4 3 3 7 4 0 4 7 3 0	ing permutations 0 4 3 7 3 4 7 0 3 0 4 7 4 0 3 7	0 4 7 3 3 4 0 7 3 0 7 4 4 0 7 3
0 3 4 7 0 7 3 4 3 7 0 4 4 7 0 3 4 3 0 7	(0, 3, 4, 7) steer 0 3 7 4 0 7 4 3 3 7 4 0 4 7 3 0 4 3 7 0	ing permutations 0 4 3 7 3 4 7 0 3 0 4 7 4 0 3 7 7 0 3 4	0 4 7 3 3 4 0 7 3 0 7 4 4 0 7 3 7 0 4 3

Table 2. The 24 permutations of (0,3,6,9) and (0,3,4,7) steerings.

STEERING APPLICATION: INVERSION PERMUTATIONS

So far, each triad quality has been applied in a consistent inversion, even if the melodic order has been changed as in figure 15. Further pathways can be extrapolated by applying different triad inversions. Standard circular permutations yield three possible positions of the asymmetric triads available in the octatonic scale – root, first and second inversion. General permutations yield six possible positions of each triad.¹⁴

For a series of four triads appearing in any one steering there are 81 pathways based on change of triad inversion alone.¹⁵ Such triad sequences range from the consistent root-root-root-root, through to mixtures such as 1st-2nd-root-2nd. If general permutations are considered, this figure increases to 1,296.¹⁶

- 15 3x3x3x3 = 81.
- 16 6x6x6x6 = 1,296.

 $^{^{14}3! = 1}x2x3 = 6.$

If these triad inversion permutations are combined with the steering permutations, the grand total pathways through the octatonic scale in this approach yield 1,944 for circular permutations (conventional inversions), or 31,104 for general permutations, per triad quality.¹⁷ For the purposes of this paper, I have not tabulated all these, suffice to say that the methodology outlined should illuminate an exhaustive and systematic approach to creating various melodic lines through the octatonic scale. Figure 16 illustrates an example of how this methodology may be applied to the major b5 triad quality in the B octatonic scale.



Figure 16. Melodic example featuring Major b5 triad with steering and inversion variations.

 $^{^{17}}$ 24 x 81 = 1,944. Or 24 x 1,296 = 31,104. This figure quadruples if one considers the four triad qualities available.

STEERING APPLICATION: CHORD PROGRESSIONS

The focus thus far has been on creation of melodic lines based on steered triads. The steering concept can also be applied to four-note chords. With knowledge of the respective steering, the spontaneous transposition of a chord quality can be realised whilst retaining the flavour of the octatonic scale.

Taking the four triad qualities available and their respective steerings, figure 17 identifies the duplicate pitches produced.¹⁸



Figure 17. Diminished, Minor, Major and Major b5 triad with steerings in B octatonic, notated with unique and duplicate pitches indicated using open and filled-in note-heads, respectively.

Figures 3 and 4 indicate an approach to steering two 4-note chords of identical structure that tile the octatonic scale with no duplicate pitches. Progressions of four-note chords within the octatonic scale that avoid instances of duplicated pitches such as those indicated in the triads of figure 17 whilst maintaining chord quality must reconcile the intervallic design of the chord with the octatonic scale's symmetry.

¹⁸ Whilst duplication of pitches in a chord progression is not in itself problematic, it is proposed that avoidance of duplication will create more movement, through the increased number of voice-leading options. Triadic steering is not possible without such duplication, but four-note chord steering is possible in octatonic in that it creates a pure tiling.

The following figures reveal the available options using geometry and notation. Four keys of two-chord steerings are available in each quality due to the four degrees of symmetry in the octatonic scale. The geometric diagrams reveal a true tessellation of the octatonic scale, with each scale degree sounded a single time. These diagrams also reveal the bilateral symmetry present in the structure of all the applicable four note chords. This symmetry is a prerequisite for this kind of tessellation.¹⁹ The axes of reflection corresponds to members of the chord for some of the qualities (diminished seventh, dominant seventh flat 5), but not the others.

¹⁹ Analysis of the intervals of each four-note chord reveals patterns such as the diminished seventh <3-3-3-3>, major sharp 9 <3-1-3-5>, dominant seventh flat 5 <4-2-4-2>, major 6 <4-3-2-3> and its related inversion minor seventh <3-4-3-2>.

DIMINISHED SEVENTH CHORD STEERING

The diminished seventh chord is steered by a semitone. Being the chord most native to the octatonic scale,²⁰ the diminished seventh chord's inherent symmetry makes degenerate chord progressions, whose transpositions via the (0,3,6,9) tetrachord are consequently inversions of each other.

[0,3,6,9] ⇒ **(1)**



Figure 18. Diminished seventh chord steering in octatonic.

²⁰ This is due to the congruence of the scale's interval generator with the structure of the diminished seventh chord.



Figure 19. Diminished seventh chord progressions in octatonic, with steering interval indicated in semitones from each transposition.

MINOR SEVENTH CHORD STEERING

The minor seventh chord is steered by a tritone, and uses the (0,3,6,9) tetrachord to progress four transpositions through the octatonic scale.

[0,3,7,10] ⇒ (6)







Figure 21. Minor seventh chord progressions in octatonic, with steering interval indicated in semitones from each transposition.

MAJOR SIXTH CHORD STEERING

The inversional relationship of minor seventh to major sixth chords leads to a congruence of steering and transposition.

[0,4,7,9] ⇒ **(6)**



Figure 22. Major sixth chord steering in octatonic.



Figure 23. Major sixth chord progressions in octatonic, with steering interval indicated in semitones from each transposition.

MAJOR SHARP 9 CHORD STEERING

The octatonic scale is often used for the bluesy application where a minor third degree opposes a major third. The following chord progression shows this effect, and offers an alternative progression based upon the major/minor triad. The chord is again steered by a tritone, and uses the (0,3,6,9) tetrachord to progress four transpositions through the octatonic scale.

[0,3,4,7] ⇒ **(6)**







Figure 25. Major sharp 9 progressions in octatonic, with steering interval indicated in semitones from each transposition.

DOMINANT SEVENTH FLAT 5 CHORD STEERING

The dominant seventh flat 5 chord is steered by a minor third, and uses the (0,3,6,9) tetrachord to progress four transpositions through the octatonic scale. Figure 26 reveals the inversional symmetry present in the chord structure, such that (0,4,6,10) = (6,10,0,4).





Figure 26. Dominant seventh flat 5 chord steering in octatonic.



Figure 27. Dominant seventh flat 5 chord progressions in octatonic, with steering interval indicated in semitones from each transposition.



This paper sought to illuminate interval combinations within the octatonic scale that might offer new and fresh perspectives for the performer, improviser, and composer. Through its systematic mathematical approach at rendering common triadic formations, it possibly addressed some voids within the reader's understanding of the potential of the octatonic scale to produce at once ambiguous and meaningful sounds, constrained by the scale's symmetric limitations.

It is acknowledged that any theoretic approach to develop content in this manner is not music itself, but may perhaps facilitate the creation of more sophisticated and varied content within a language that is consistent, rich and expressive. If the author has aided a step towards this aspiration, this paper has humbly achieved its purpose. Thank you for reading and sharing.



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