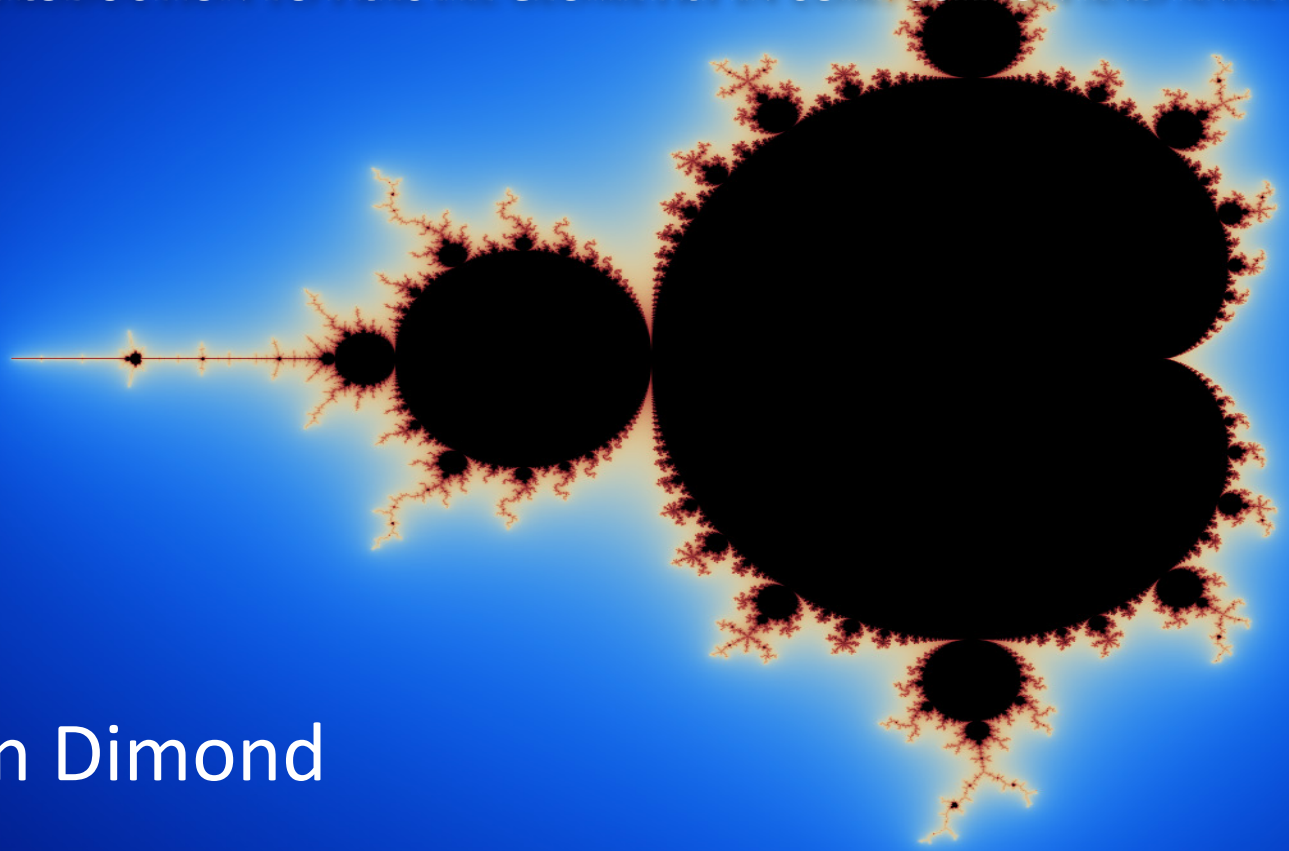
Self-similarity in music

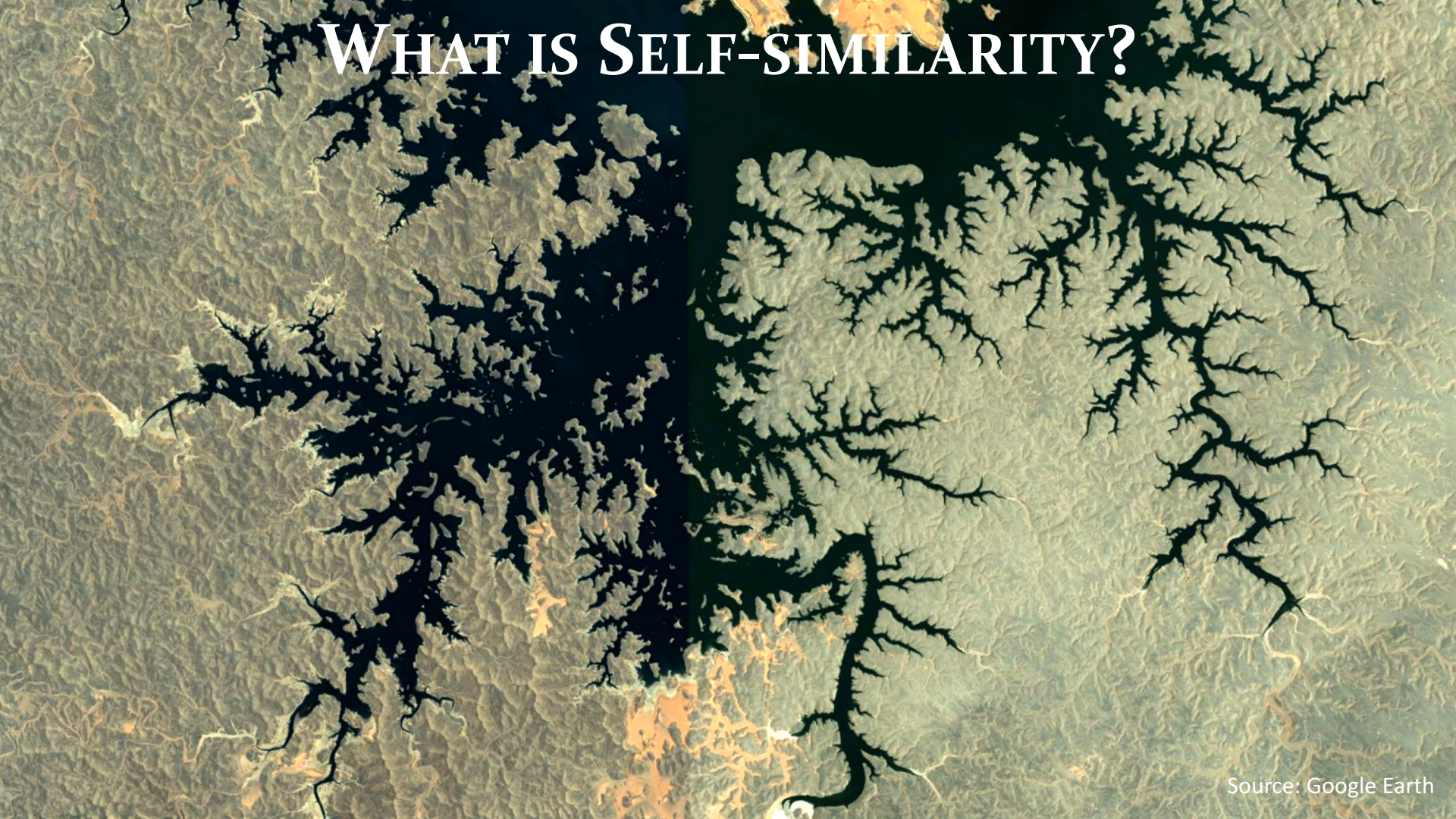
An introduction to fractal geometry in composition and analysis



Jonathan Dimond

April 2017

What is Self-similarity?

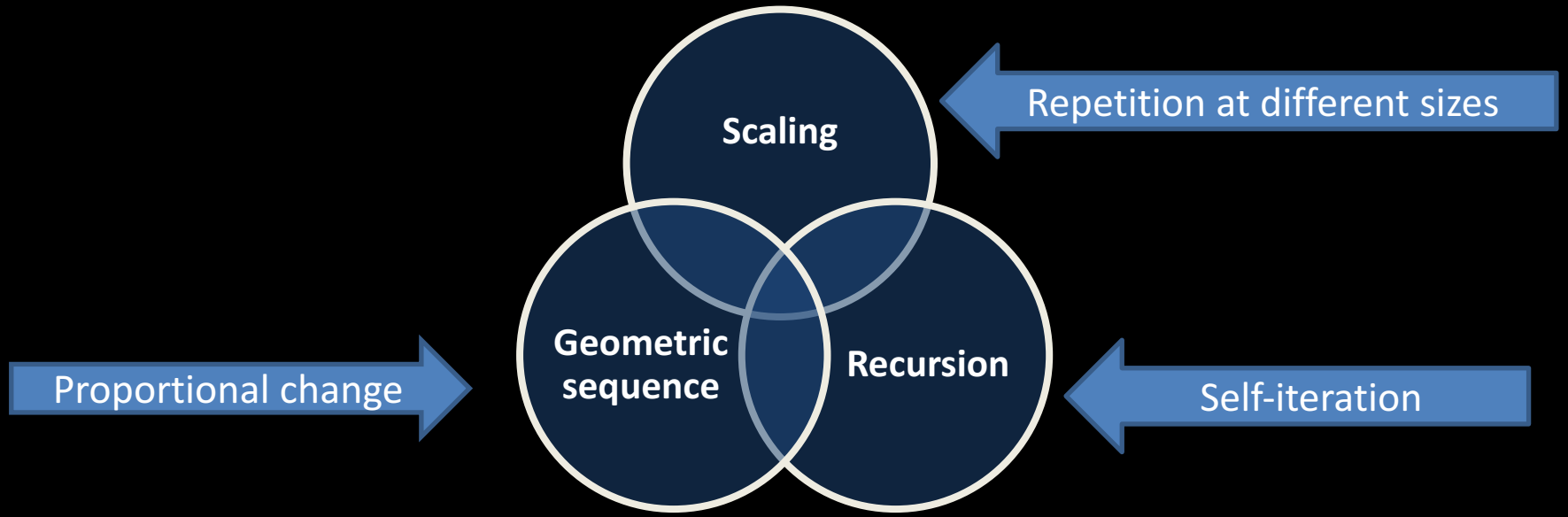


Source: Google Earth

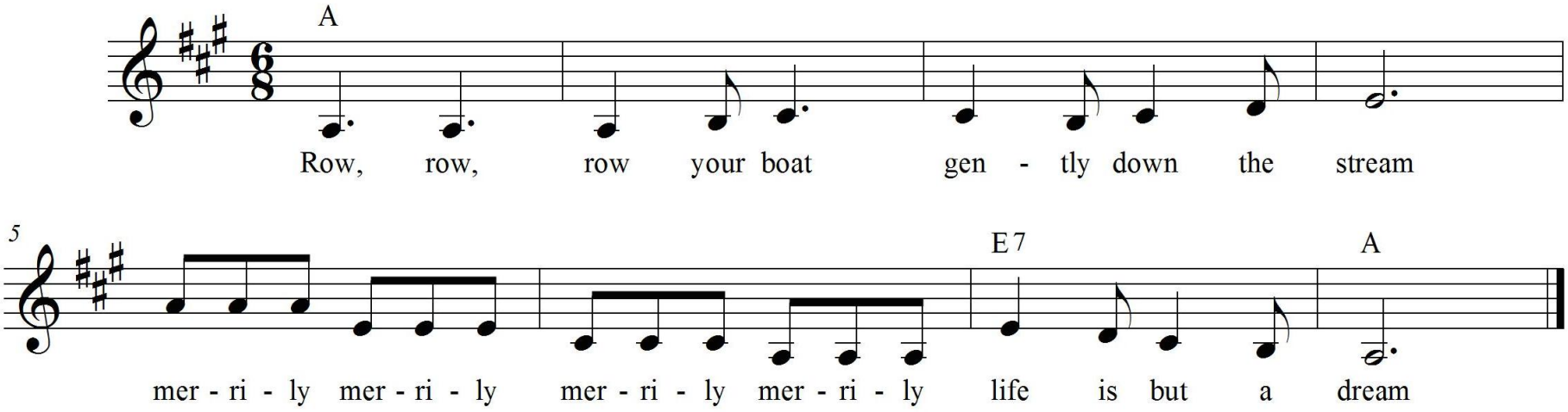
Towards a Musical Interpretation of Self-similarity and Fractal Geometry

Spatial relationships of fractal geometry translate to musical parameters – e.g. pitch and time.

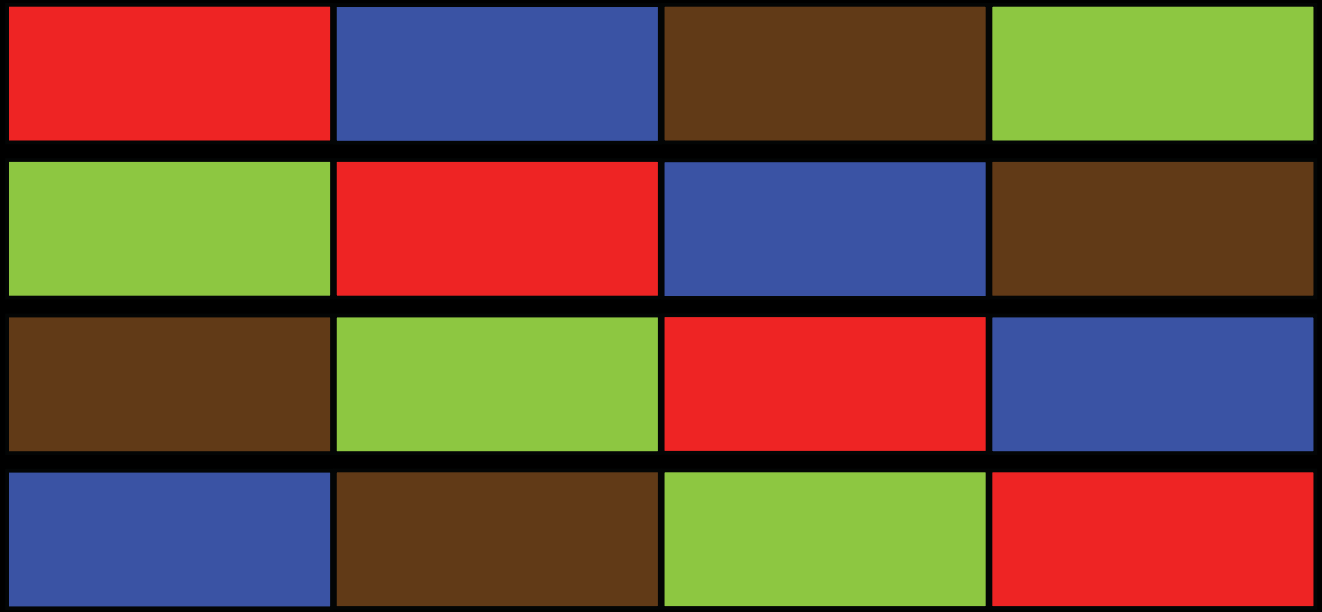
Indicators of Self-similarity



LET'S SING...


A
Row, row, row your boat gen - tly down the stream
5
E7
A
mer - ri - ly mer - ri - ly mer - ri - ly mer - ri - ly life is but a dream

That sure was fun...
but was that Self-similarity?



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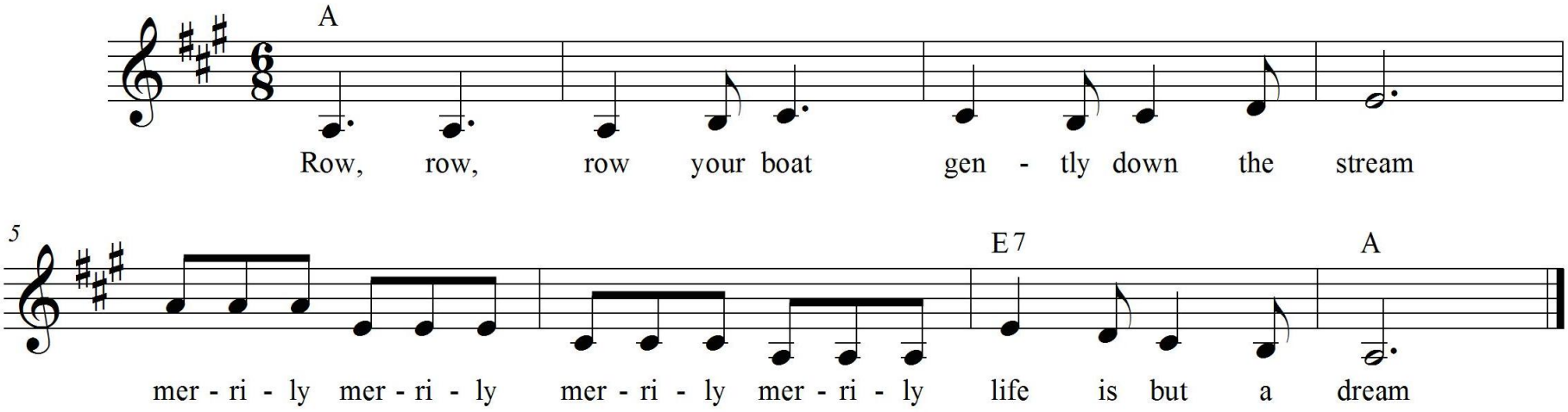
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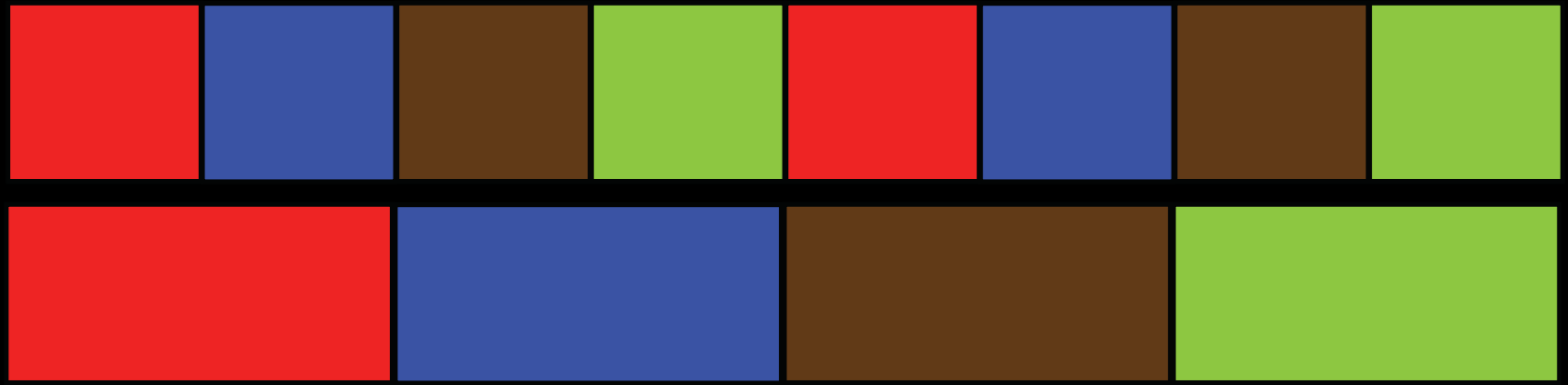

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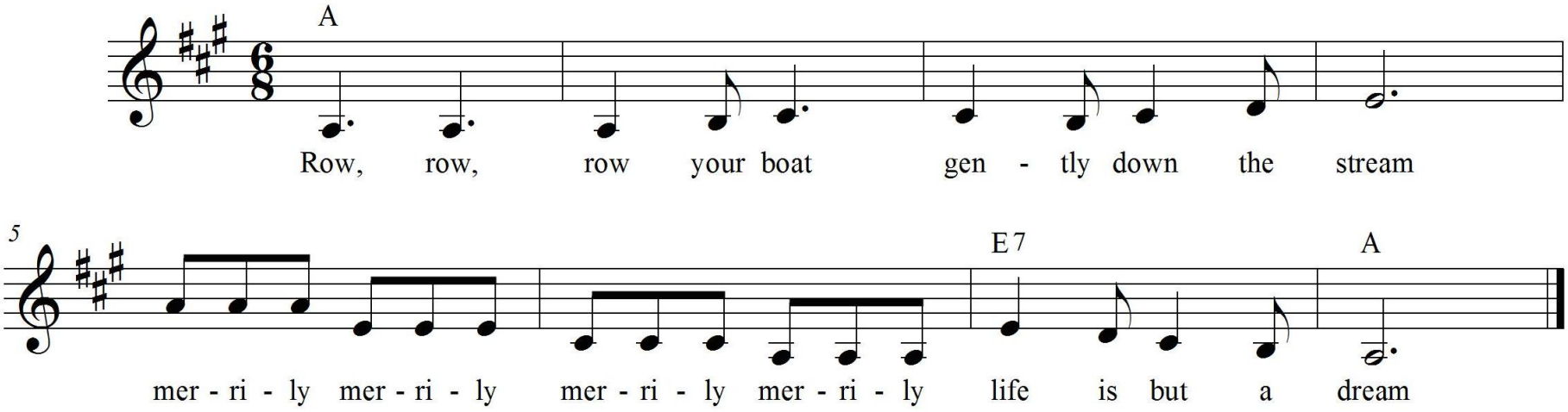
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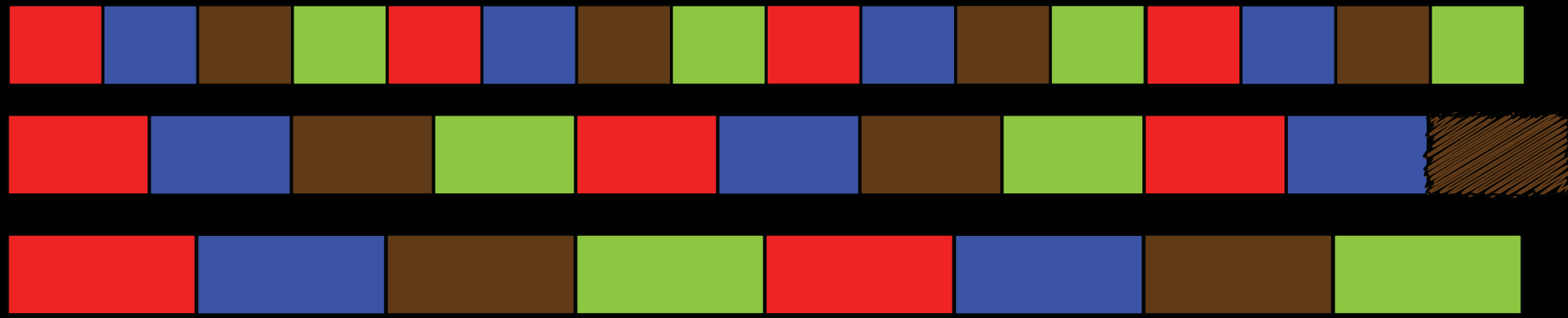
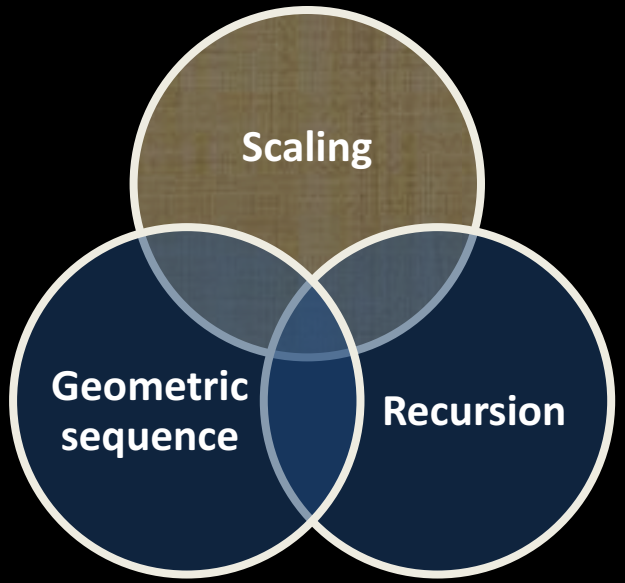
That sure was fun...
but was that *also* Self-similarity?

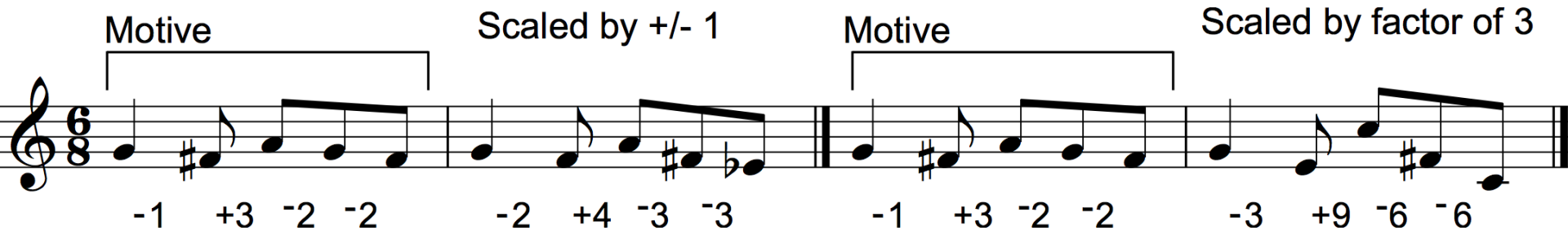


Self-similar Technique: Rhythmic Augmentation / Diminution

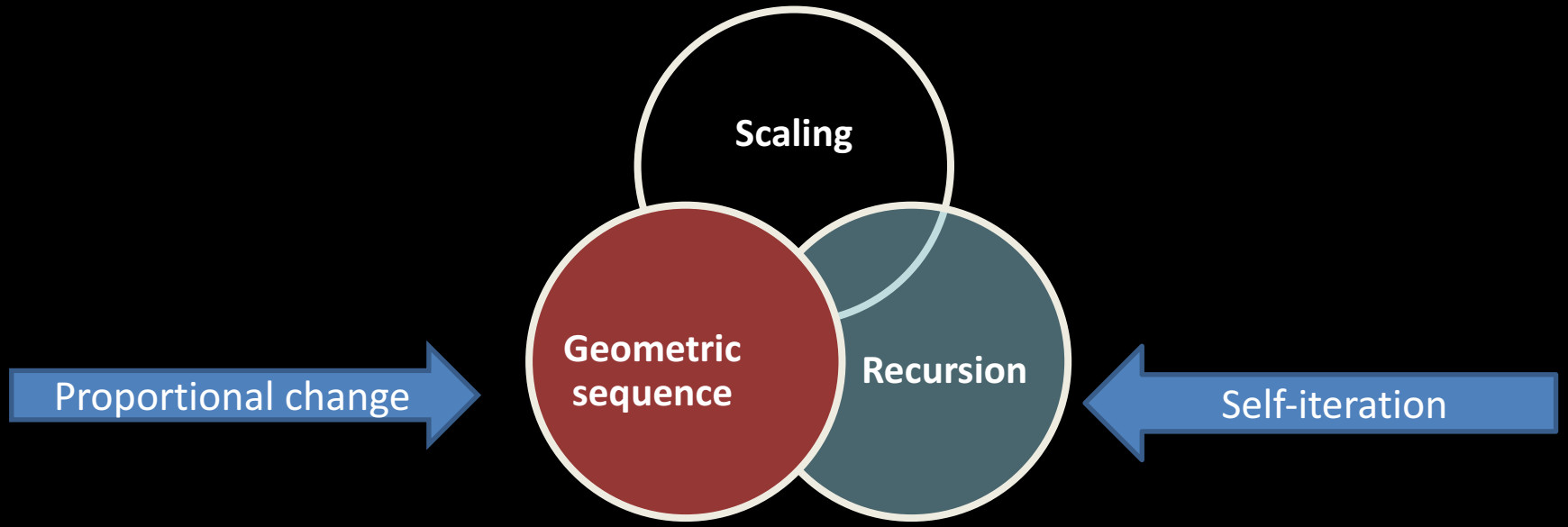


Self-similar Technique: Interval Scaling

Interval scaling - proportional (or arithmetic) change in interval size

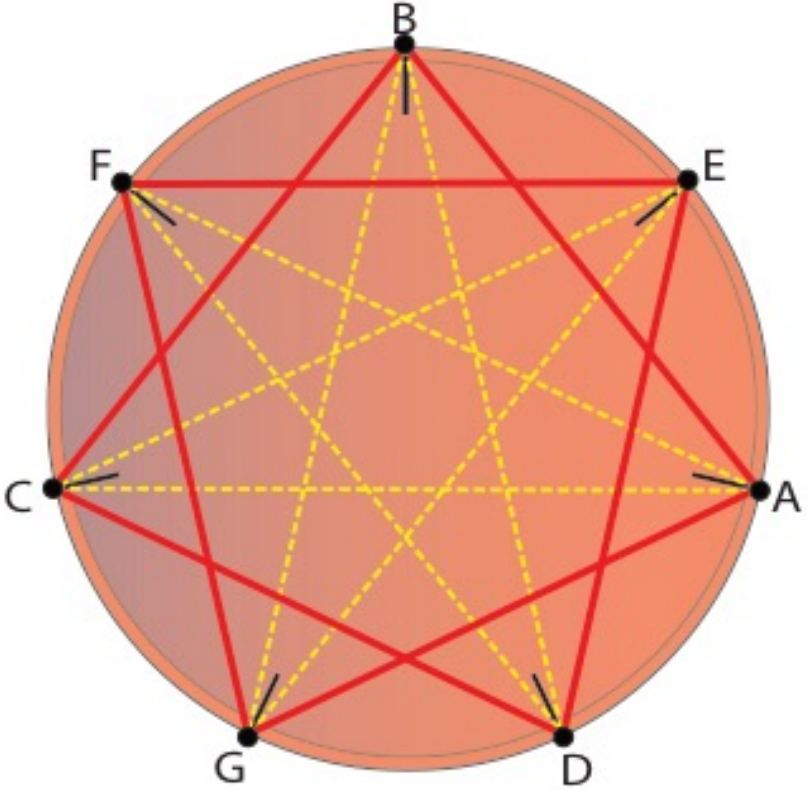


Focus on other features of Self-similarity...



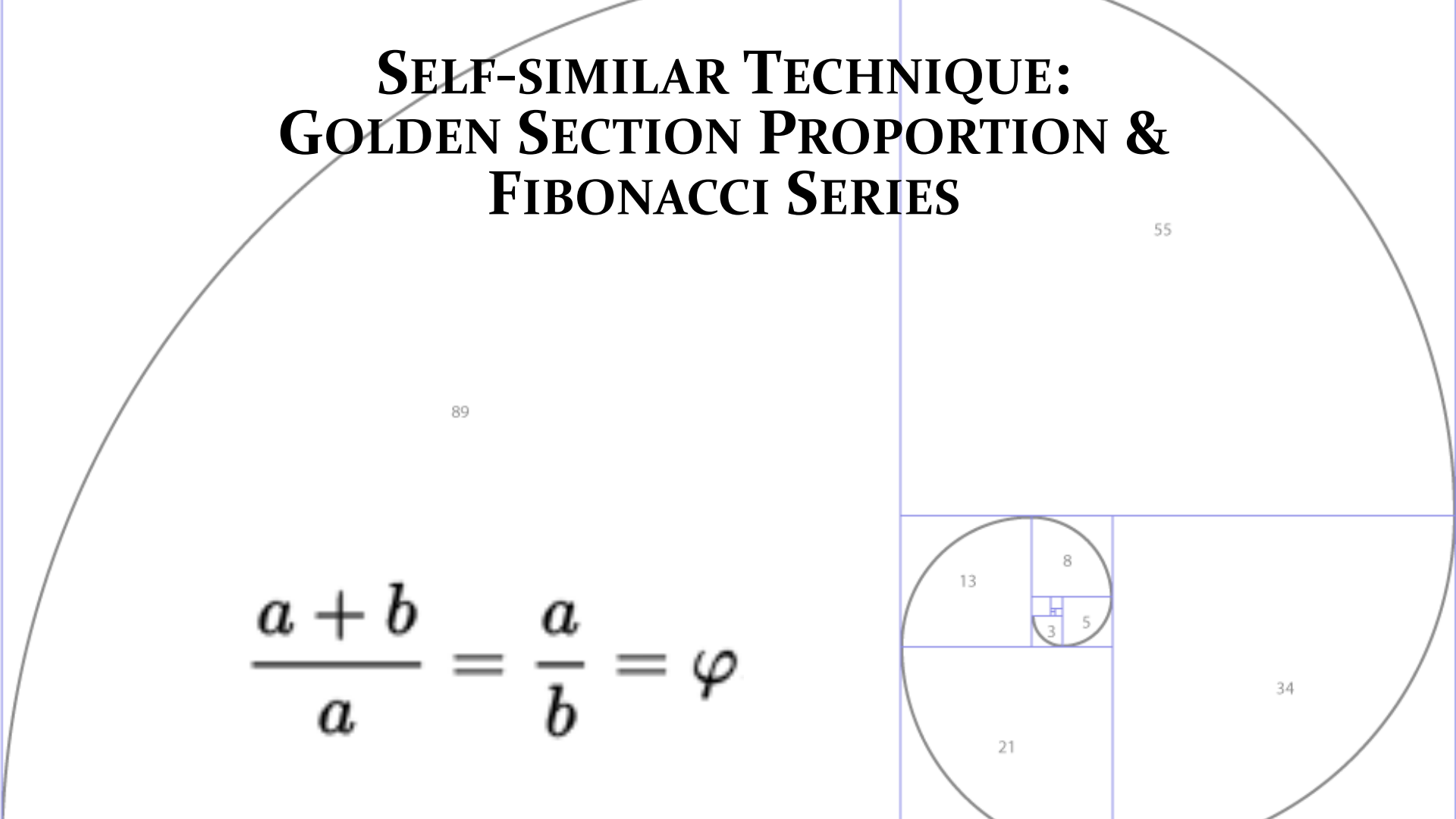
Self-similar Technique: Self-Similar Scales

Self-similar scales – built from reiteration of an interval



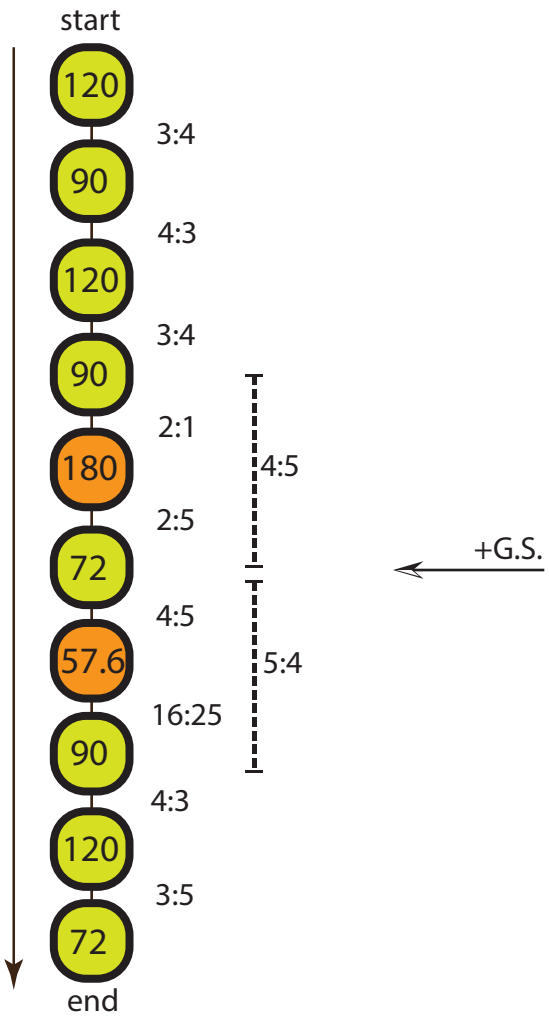
"Well-formed" scale:
Carey and Clampitt
Perspectives of New Music, Summer 1996

Self-similar Technique: Golden Section Proportion & Fibonacci Series

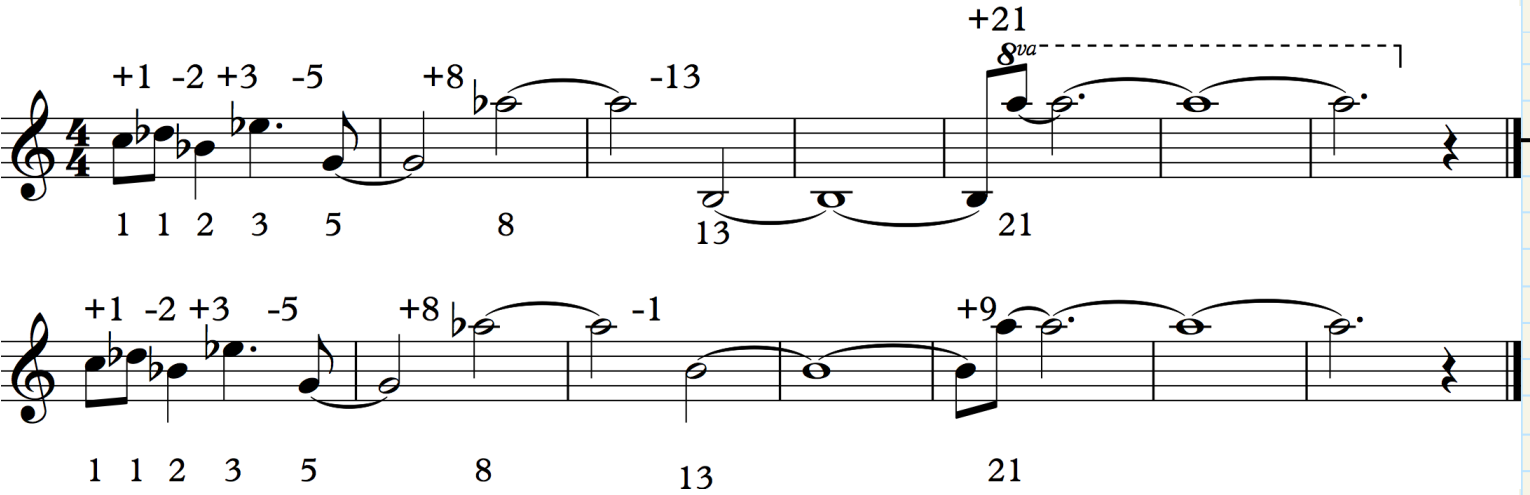


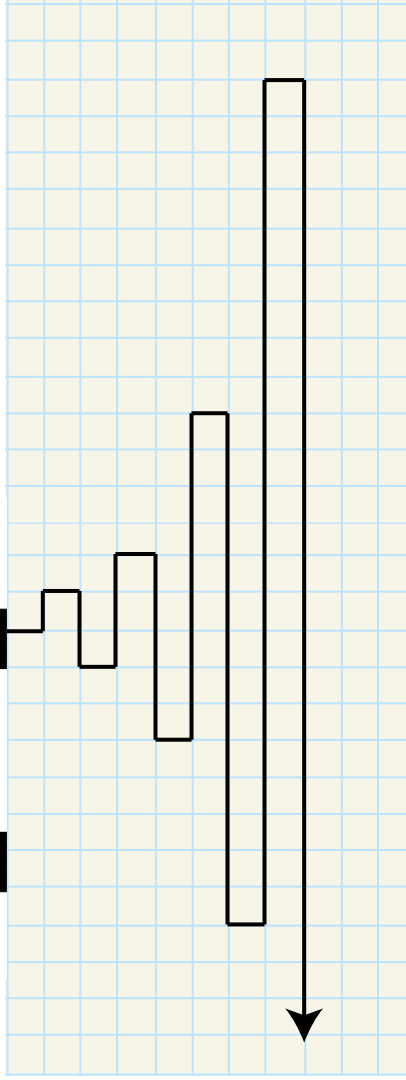
$$\frac{a+b}{a} = \frac{a}{b} = \varphi$$

Self-similar Technique: Golden Section proportion in Structure



Self-similar Technique: Golden Section proportion in Melodic contour





Self-similar Technique: Recursive Melodies

Built from reiteration of cell idea

May use an automaton

Self-similar recursive melody

What types of counting do you know that contain levels, or are "nested"?

- Counting bars rest (*1*234, *2*234, *3*234, *4*234, …)
- Simple counting in a base (1, 2, 3, 4, …, 10, 11, 12, 13, 14, …)

Let's consider ways of creating self-similar patterns from simple counting

(1, 2, 3, 4, …)

Self-similar recursive melody

Tom Johnson* defines self-similar categories of counting:

Accumulative counting (in the normal counting sequence, 2 becomes 12 and 3 becomes 123, etc)

1

12

123...

*Johnson, Tom. *Self-similar melodies.*

Self-similar recursive melody

Accumulative counting is used in South Indian rhythmic patterns called *Yati*.

Srotogata yati	*Gopuccha yati*	*Mridanga yati*
1	12345	1
12	1234	12
123	123	123
1234	12	1234
12345	1	12345
		1234
		123
		12
		1

Self-similar recursive melody

Repetitive counting (in the normal counting sequence, 2 becomes 22 and 3 becomes 333, etc)

1

22

333...

Self-similar recursive melody

Two-level self-similar counting.

Repetitive accumulative counting sequence:

1

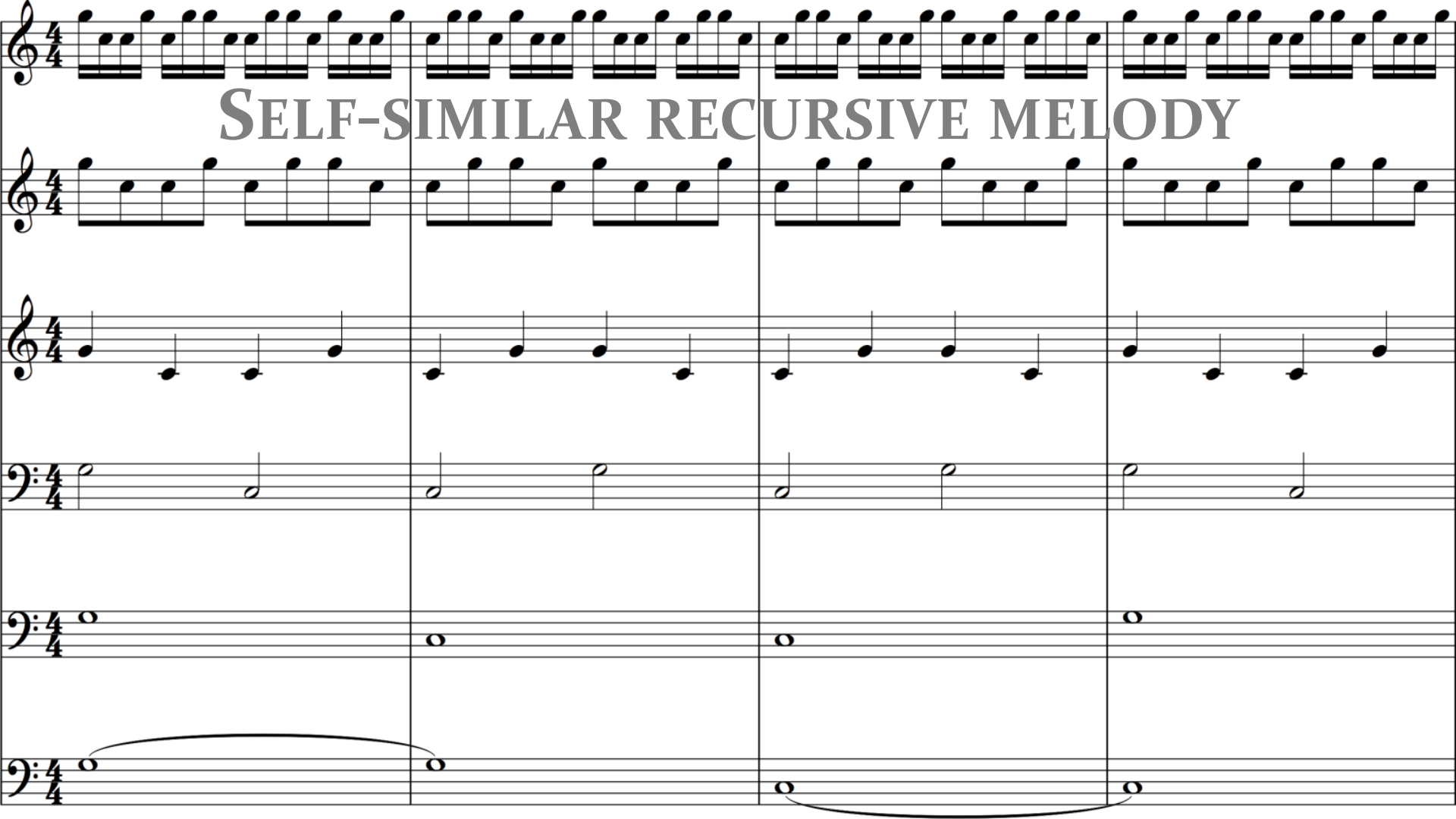
1 22

1 22 333...

0 1

1

Self-similar recursive melody



Self-similar recursive melody

Thue-morse sequence

Voice 1: 0110 1001 1001 0110 1001 0110 0110 1001

Voice 2: 0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0 1 0 0 1 … etc

Voice 3: 0 1 1 0 1 0 0 1 1 0 … etc

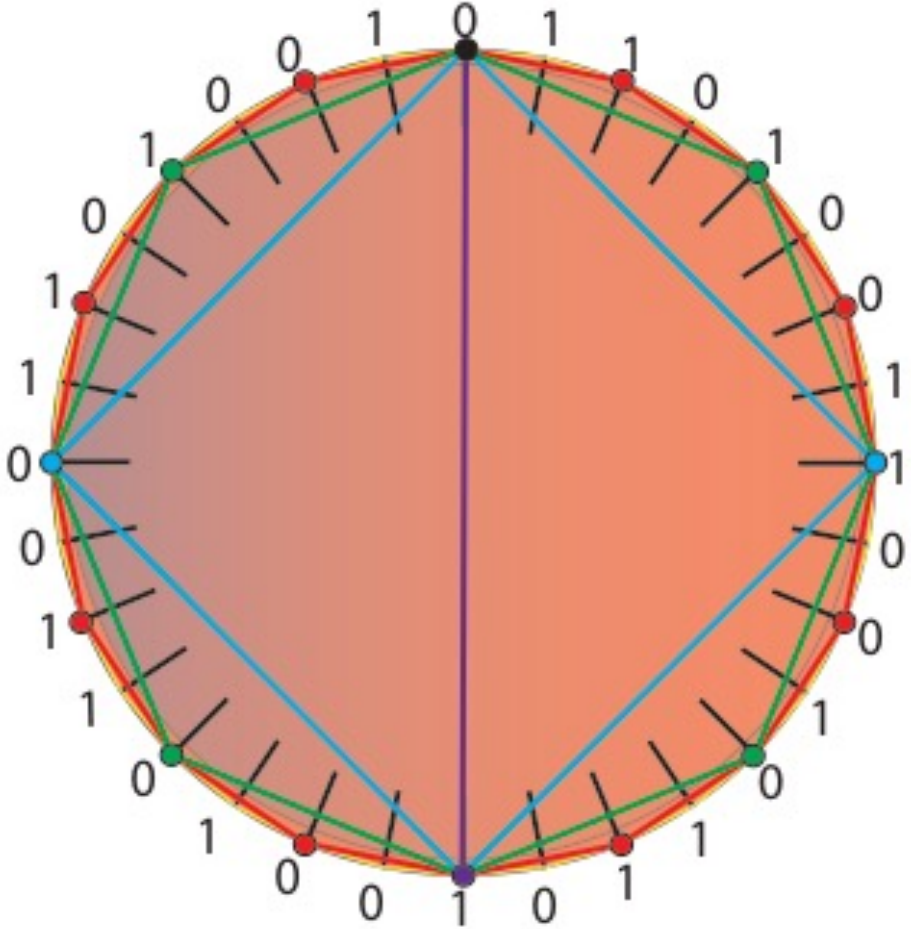
Voice 4: 0 1 1 0 1 … etc

Voice 5: 0 1 1 … etc

Voice 6: 0 1 … etc

Automaton: 0 ->01 & 1 -> 10 (digit seed of 0)

Self-similar recursive melody



Thank you…

If we contemplate the method of Nature, we see that everywhere vast results are brought about by accumulating minute actions.

Herbert Spencer. *Principles of Sociology* (1896), p.366.