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Folio of Compositions with Critical Commentary:

An exploration of intercultural influences in contemporary composition

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Abstract

This Doctor of Philosophy submission consists of a folio of original compositions with an accompanying critical commentary. Through practice-based research, the theoretical, creative and intercultural nature of the author's contemporary compositional practice is revealed and examined.

The critical commentary commences with an in-depth study of the author's salient techniques used for creation of material and post-compositional analysis. Diagrams—many drawing from geometry—are utilised in order to demonstrate the nature and potential of these techniques.

This work demonstrates the potential of a range of contemporary compositional techniques that seek to fuse intercultural approaches with jazz and Western art music, resulting in an hybridized contemporary style.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Publications included in this thesis

Compact Disc Recording

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Submitted manuscripts included in this thesis

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Other publications during candidature

No other publications.

Contributions by others to the thesis

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Statement of parts of the thesis submitted to qualify for the award of another degree

No works submitted towards another degree have been included in this thesis.

Research Involving Human or Animal Subjects

No animal or human subjects were involved in this research.

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Music Composition, Australian Music, Intercultural Music, Jazz, World Music, Contemporary Music.

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Chapter 1
Introduction

The contemporary composer ought to be a discoverer,

an inspirer and dictator of aesthetic taste.

Boguslaw Schäffer, *Introduction to Composition* (1976), p.12.

1.1 The origins of intercultural hybridity in my music

I was born in the U.K. and raised in Australia. My first exposure to music came from my parents, who were avid and eclectic listeners. My ears were repeatedly exposed to the sounds of Welsh choirs—my heritage—alongside popular radio programming of the 1970s, and my father's library of jazz LPs that ranged from the traditional small-group sounds of Louis Armstrong, the spectacular big band sounds of Stan Kenton, through to the chamber jazz sounds of the Modern Jazz Quartet. My young ears and inquiring mind sought to understand the array of musical languages to which I was exposed. As a budding trombonist, it felt natural to me that a rigorous grounding in Western art music should be combined with studies of improvisation and jazz, and my training from my teens onward reflects that belief. I never allowed specialisation to stifle opportunity for breadth, and my studies of North Indian classical music, multiple instruments, electroacoustic and instrumental composition all seeded new disciplines within my skill set by my early twenties. Having persisted in the study and practice of Indian classical music now for over 25 years, and having maintained an active interest in other genres popularly mixed with jazz (which itself is a melting-pot of influences), my creative voice has naturally evolved with an intercultural openness that stems from the stylistic eclecticism of my parents' listening habits.

Within this dissertation and creative folio of work the reader will detect three broad genres most identifiable in my personal style. These include Western art music (particularly drawing from the 'contemporary classical' music of the twentieth century), Indian classical music (both northern and southern), and jazz (namely Afro-American, but also jazz with a distinct European influence). The music of Cuba and Brazil have had a direct influence on jazz particularly since the 1960s, and have influenced my personal style both indirectly and through direct contact with bands and teachers specialising in these genres.

One of the earliest terms used to represent the hybridization[1] of Western classical music and jazz was *Third Stream*. The term was coined by the conductor/composer/performer Gunther Schuller in 1957, and sought to combine "...the improvisational spontaneity and rhythmic vitality of jazz with the compositional procedures and techniques acquired in Western music..." (Schuller 115). It is notable that by the early 1970s Third Stream came to represent not only the merger of classical music and jazz, but also the music of other cultures. Bands that demonstrated the Third Stream philosophy at that time included *Oregon*, a genre-defying ensemble of multi-instrumentalist composers whose output successfully met the challenges of intercultural hybridity. A raft of other composers, leaders and bands further promoted the Third Stream school's message, including Charles Mingus, Jimmy Giuffre, Miles Davis, the Modern Jazz Quartet, George Russell and Dave Brubeck, to name a few. Third Stream musicians and their projects were one of my earliest and most far-reaching influences, and led to my adoption of the Third Stream pedagogical approach in my teaching.

Though the territory that Third Stream inundated was generically broad, and coincided with the birth of the *world music* trend,[2] its premise and intention goes beyond indiscriminate borrowings from exotic cultures and genre fusion for the sake of novelty. In 1973 Schuller, as President of the New England Conservatory of music in Boston, founded the Department of Third Stream Studies with Ran Blake as its Chair.[3] Blake is a disciple of Schuller and a visionary performer in his own right. In his book *Primacy of the Ear*, Blake addresses the central tenets of Third Stream. These revolve around listening, memory, and the development of personal style. In an article of the same title, Blake talks of Third Stream involving a "personal synthesis process" and a goal for his students to "forge a unique personal improvisational style" (Blake, "The Primacy of the Ear" 1). "Thus, in order to understand what Third Stream is, one must focus on the philosophy and teaching methodology rather than a set of stylistic characteristics" (1).

Blake penetrates the superficiality of a musician's genre-breadth, where stylistic flexibility is seen as mundane as "changing [one's] clothes" (Blake, "Primacy of the Ear"

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1 Refer to Kraidy for a discussion of hybridity in the context of cultural works.

2 Feld, "A sweet lullaby for world music" 146.

3 It is no accident that I came to complete two degrees at New England Conservatory in the Department of Contemporary Improvisation, formerly Third Stream Studies.

3). In contrast to the world music trend, and the diluted nature of what some critics mistook Third Stream to represent, Blake maintains that "…a composer's style is connected directly to the ability to focus, even limit, the work" and suggests that a composer "has to draw and redraw his portrait as a compositional template, based on his personal life view" (4).

Long-term memory forms the foundation upon which the hierarchy of Third Stream skill acquisition is based (see Figure 1). Blake recognises the primary role that the ear plays in musical long-term memory, alongside interactive sensitivity (also called *real-time awareness*) as a prerequisite for its success (Blake, "Primacy of the Ear" 7). He also takes a holistic view of memory as integrating the body and mind, not just in terms of a performer's motor neurone and muscle-memory patterns, but states that "…you actually record the information in your body, and not just your brain" (8).

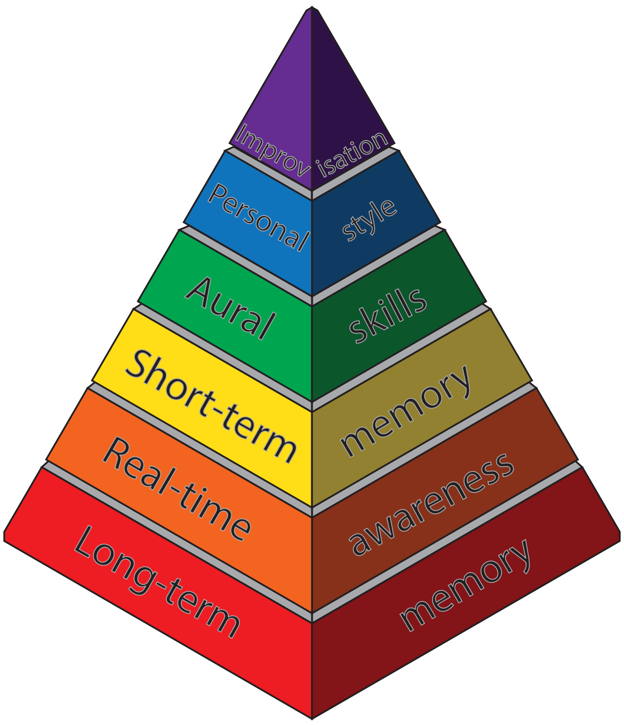

Figure 1: Third Stream skill acquisition hierarchy.

At the apex of the Third Stream skill acquisition hierarchy sits improvisation. An ability to improvise within a language represents mastery of the requisite linguistic tools for self-expression. The innovative improviser Derek Bailey is one of many practitioners who have referred to improvisation as "instant composition", and composition as "slowed-down improvisation" (Bailey 2), so the two take on a relative position in the hierarchy. Bailey postulated that "…there is no musical activity which requires greater skill and devotion, preparation, training and commitment" (5), and that the reason musicians are often reluctant to use the term ‘improvisation’ is due to the implication that “improvisation is something without preparation and without consideration, a completely *ad hoc* activity, frivolous and inconsequential, lacking in design and method” (5). Clearly, Third Stream seeks to denounce that implication, and in my own creative work I naturally embrace the foundation and apex of the Third Stream skill acquisition hierarchy, and all the levels in between.[4]

1.2 Global context

The effects on music and culture that have resulted from the expansion of the Western empire (especially since the late nineteenth century) are a focus of cultural theory, postcolonial analysis, and ethnographic studies. These effects are fuelled by international power relations, technological advances and the surge in social, capital, and informational exchanges, and have been examined by many authors.[5]

Born and Hesmondhalgh untangle the complex relations of identity and hybridity of Western art music with ‘other’ musics during the twentieth century music in musical-aesthetic discourses. Their investigation includes the study of ideological difference between music of the West and that which falls under such categories as Oriental, and critiques and questions the establishment of division and hierarchy through such categories in order to foster a more flexible, globally-inclusive conception that thinks across these fictive and divisive categories.

Robert E. Brown is attributed to coining the term *world music* in the early 1960s with the goal “to promote cultural harmony and understanding” (Williams). Although the term intended to reduce the way the term ‘music’ was seen as being synonymous with Western European art music, it actually enforced such a division, at least in a scholarly context (Feld, “A sweet lullaby for world music” 147). The 1980s saw the transformation of world music from a term with academic overtones to one with distinct marketing potential. Stokes reviews musical globalization from the late 1980s when ‘world music’ gained commercial traction as a profitable category for record production and touring artists. Peter Gabriel’s WOMAD festivals and recording label Real World Records are examples

4 Figure 1 is an original adaptation of concepts presented in Blake’s *Primacy of the Ear*, as well as personal notes from the spring semester 1994 Third Stream Methodology course at New England Conservatory, Boston.

5 Refer to Stokes, Aubert, Appiah, Slobin, Erlmann, Connell & Gibson, Born & Hesmondhalgh.

of the commercial beneficiaries of the redefinition of the term, through their successful innovation of how global musical resources were being curated, recorded, marketed, advertised, and promoted (Feld, "A sweet lullaby for world music" 151).

As a composer/performer professionally active since the early 1980s, globalization has challenged me to develop and maintain an integrated compositional approach despite an unprecedented diversity of influences. This melding of influences through the physical and virtual interactions of diverse peoples has given rise to a profusion of terms used to categorise the artistic process and result of such blends.

Categories and terminology used to describe the process and result of such blends includes intercultural music, transcultural music, fusion, hybridization, cross-cultural music, and bi-musicality.[6] Some contemporary composers have even invented their own terms to scaffold their approaches, such as *manifold composition* (Green 1). The occurrence and comprehension of these terms depends on the background, professional context and even geographic location of the practitioner. *Intercultural music* is commonly understood to involve the integration of music from multiple cultures (Kimberlin & Euba 2), and is the term adopted in this dissertation.[7]

A discussion of the interplay between genre and style is required. *Genre* is characterised by what an art work is intended to express and *style* is how it is actualised (Moore, "Categorical Conventions in Music Discourse" 441). For the purposes of this research, I distinguish genre from style by identifying style as the personification of influences, manifesting in a creative artefact that is identifiably unique. The Third Stream philosophy embraces the idea that these influences are not only those that lie outside of an artist's control, occurring perhaps unconsciously, but can also include those consciously and intentionally programmed into and stored within one's subconscious (Blake, "Primacy of the Ear" 36). Such a process lies at the heart of Third Stream pedagogy, and involves working with psycho-epistemology—a skill Ran Blake maintains "is the most important

6 The reader is directed to authors Hood, Slimbach, Wren, Evans, Feld, and Tenzer & Roeder for discussions of these terms.

7 The origin of the term is not certain, but early examples include *Intercultural Music Studies*, a book series published by Max Peter Baumann in 1990, and the 1988 *Symposium of the International Musicological Society and Festival of Music* in Melbourne Australia directed by Margaret Kartomi (Kimberlin & Euba 2).

part of learning, and all of man's other capabilities depend on how well or poorly he learns it" (36).

Genre by contrast concerns categories of music often having geographic and cultural associations. Cultures change over time through a myriad of influences, both direct and indirect, forced and somewhat consenting (Kottak 39). The forces of capitalism and globalization don't simply erase genre specificity, but rather permeate cultural fabric in such a way to create derivative multiple capitalisms and modernities, each different yet entangled (Connell & Gibson 191).

1.3 Risks and effects of intercultural hybridity

Born and Hesmondhalgh interrogate the exploitation of non-Western others' creative and intellectual property by international recording corporations. Such appropriations are often in the form of recorded sound objects, whose extraction and commodification are facilitated by recording and sampling technology. Aubert discusses the hegemonic relationship between commercial recording studios and their ethnic employees, suggesting that "world music is hybridisation elevated to dogma" (55). Arom consequently partitions the 'world music' category into 'synthetic music' (original products manufactured for the world music market) and 'recycled music' (indigenous music existing or having existed outside the world music market but included in it for commercial gain) (Tenzer & Roeder 390).

The role of recording technology has led to such phenomena as *disjunction*, where the sounds of people and their culture have been disassociated from the genre in which they arose (Arjun Appadurai qtd. in Tenzer & Roeder 391). Disjunction of electroacoustically captured and transmitted sound from its original sound source was a trend foreseen by Schafer, who coined the term *schizophonia* in 1969 to describe it (90). In contrast to this implicitly derogative term, Feld offers a contemporary and creative view of the phenomenon, offering the term *schismogenesis*[8] to describe current practice in a way that is not entirely destructive, but may in some cases may actually lead to the preservation of indigenous music (Keil & Feld 273). Arom also identifies that disjunctions naturally lead to new conjunctions (Tenzer & Roeder 391).

Bartók's pioneering work in the ethnomusicological study of traditional folk music in

8 Feld revised this term in 2000 with *schizophonic mimesis* (Born & Hesmondhalgh 263).

Hungary and Romania, and the resultant adaptation of this music into his compositional style is well documented. Hungary is intrinsically ethnically diverse, and as a Hungarian Bartók's philosophy and creative work was entangled with opposing and changing perspectives on the role and validity of music of the rural peasants versus that of the urban Gypsies (Born & Hesmondhalgh 120). In 1942 he wrote of the positive effects of intercultural hybridity by stating that "a complete separation from foreign influences means stagnation: well assimilated foreign impulses offer possibilities of enrichment" (Suchoff 31).

However impressionable musical genres may be,"foundational features of musical style and meaning endure even in the face of far-reaching musical and cultural change" (Bakan xxviii). Also optimistically, it is worth contemplating the role that change plays in even the oldest traditions of the world. Bakan argues that tradition is "a process of creative transformation whose most remarkable feature is the continuity it nurtures and sustains" (xxi).

The creation of intercultural hybrid forms can therefore be seen as both a natural outgrowth of cultural globalization, integral to the propagation of tradition, an inevitable part of cultural change, and also a conscious experiment on the part of the willing performer/composer. Navigating cultural differences in a global community is however fraught with risks and dangers. In their book of the same title, Glover and Friedman look at *transcultural competence* from a broad anthropological and psychological perspective, with an emphasis on the recognition of and respect for cultural differences as a prerequisite for avoiding cultural traps. These tend to arise when any of a number of cultural virtues, values or behaviours become maladaptive to their current environment (Glover & Friedman 130).

In anthropology the terms *emic* and *etic* are established categories that distinguish between research that is orientated with an indigenous perspective, versus that which is orientated with the perspective of the inward-looking scientist, respectively (Kottak 53). These terms are relevant to the contemporary Western performer/composer engaged with intercultural music. An emic approach requires the performer/composer to engage with a *cultural consultant* to teach them about their culture and give them a perspective on how they perceive and categorise the world (53). For a musician, this requires engaging teachers from the particular tradition, and often requires travel to different countries to experience the music *in situ* with its cultural environment. By contrast, an etic approach emphasises what the performer/composer deems important from their own scientific perspective, and works to offer an objective viewpoint with limited personal bias (Kottak 53). As a student engaged with formal Indian music training since 1993, and having travelled to India to study in the *guru-shishya parampara*[9] model, I am sufficiently anchored in the music to derive emic conclusions about its nature, as well as to employ and judge the etic research of my own and that of others.

Agawu investigates the music of his heritage, bridging analyses from the perspective of a European academic and that of a West African native. It is illustrative of the emic/etic paradigm that the etic approach proclaims additive rhythm,[10] for example, to be the "hallmark of African music" whilst a cultural analysis, rooted in African musicians' thinking shows no evidence of this structural analysis (Agawu, "Structural Analysis or Cultural Analysis?" 11). Agawu argues that the African musicians' implicit mastery of the mathematical properties we associate with the additive rhythmic system are founded in the etic orientation of musicologists' analyses, and ignore the qualitative measures of "holism over atomism, integration over separation and an across-the-dimensions over a within-a-dimension tendency" ("Structural Analysis or Cultural Analysis?" 6).

Wren's research centres on the aural and cognitive understanding that arises dynamically between improvising musicians in hybrid intercultural ensembles, and acknowledges the problematic nature of intercultural hybridity, with the goal of arriving at a "thoughtful and equitable framework for talking about musical hybridity" ("Improvising Culture" 18). Wren works towards "a methodology through which we can critically engage with the products of interculture" which he terms *discursive interculturality* (87).

The following risks and dangers were acknowledged during the creation of my folio of intercultural creative works. This is not to say that any friction between the component genres in my hybrid compositions is absent, possible or even desirable to eradicate. As Moore states, "much of the interest in music comes from the realization of friction between awareness of stylistic conventions that appear to be relevant to a particular piece of music, and the sonic experience itself" ("Categorical Conventions in Music Discourse" 442). My musical experience is rooted in cross-cultural reality, and I am constantly involved in

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9 Literally 'teacher-student lineage', this pedagogical model forms much of the basis of training in the arts in India.

10 Refer to Chapter 3.4 for a discussion of this term.

practical negotiation of such frictions. Contemporary hybridity is a natural outcome of such frictions and their negotiation (Stokes 60).[11]

Through my creative process and research, a number of risks of intercultural hybridity have been drawn to my attention. Follows are a list of these risks and subjective procedures I engage to mitigate them:

- Disjunction.

I avoid extracting sounds from their culture, and do not employ clichéd sound-bites to create an effect of generic exoticism. My practice involves immersion in performance discipline, which requires long-term and total dedication to development of language and craft.[12]

- Enthnocentrism.

I do not place *a priori* superiority of Western culture's music over any another, based upon the strength of the West's perceived intellectual economy (Agawu, "Structural Analysis or Cultural Analysis?" 12). My works celebrate the equal potential of music to contribute to human culture regardless of genre, country or ethnicity. An ethnocentric approach, by contrast, is associated with an inability to see the adaptive value of others' perspectives, and is considered a legacy of colonialism (Glover & Friedman 130).

- Simplification.

I recognize the complexity and richness of the musical traditions, and avoid insensitive reductionism that results in exotic-sounding yet superficial sound tokens. This may mean that intercultural effects are working upon deep-level compositional structure with no obvious superficial artefacts.

- Dissolution.

I avoid the disunion of elements and concepts from others that are strongly interrelated or codependent. For example if a coupling of formal structures is considered integral to communicating meaning in a particular genre, I retain them as a unit.

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11 Appiah discusses the values of cosmopolitanism, and denounces "creed, colour, country and culture" as pitfalls—where conceptual mistakes undermine morality.

12 The misuse of sampling technologies is a common cause of disjunction, facilitated by the relatively free and instantaneous transportability of exotic sound objects.

- Misrepresentation.

I avoid promoting any interpretation of traditional music as an 'authentic' representation either in composition or performance practice. This is most critical when presenting to Western audiences, who may have little else in the way of authentic comparison.

- Fission.

I avoid the division of intercultural composition or performance into two or more disjunct parts—that of the Western and that of the 'other'. Instead of the superficial overlaying of some aspect of a traditional music upon a Western layer, or of the creation of a temporal stratification through a series of 'them/us' splices, I seek the formation of deeper connections through procedural, structural, rhythmic, melodic and/or harmonic integration. This requires of the musicians a certain flexibility of conception and openness to learning each other's language.

1.4 Rationale and methodology

Rules can only be imposed by the work itself.

Iannis Xenakis, qtd. in Rahn, Perspectives on Musical Aesthetics 1994, p.158.

Schäffer wrote “the most important factor of contemporary music…is the change of material and its change in relations” (13). Yet this changeability is neither unique to contemporary music nor does it refer to random or total change. The striking juxtapositions used by Stravinsky in his dramatic Russian ballet music may create momentary surprise or even shock for the listener, however over the course of the performance the delayed resolutions and interlocking connections of the material unite to create aesthetic pleasure (Cone, “The Progress of A Method” 19). Schäffer proposes that change is a “governing principle”, and furthermore that the “essence of change in music depends on the presence of internal changeable relationships” (Schäffer 14). With change as new music’s constant, the contemporary composer turns to the journey as the preeminent process, and still aspires to secure a sense of unification and comprehensibility by the listener. “Thus, artistic value demands comprehensibility, not only for intellectual, but also for emotional satisfaction” (Schoenberg, “Style and Idea” 215).

How does a composer of new music, with its essential changeability, achieve such unification and comprehensibility? In my aesthetic there is a priority placed on integrating relationships of parts to the whole, with an expectation of this leading to satisfaction for the composer and listener. Deliberate defeat of expectation through use of contrast then serves the holistic goal of integration (Cone, “The Uses of Convention” 22). Schoenberg advocates for the organisation of formal elements in an organic manner, and for fulfilling the demands of logic, coherence and comprehensibility (Schoenberg, “Fundamentals of Musical Composition” 1–2). The antithesis of this concept exists when the “undergirding structure is in contradiction to the surface, if it does not come from the same creative spirit as what is readily apparent” and we consequently “experience the discontinuity as falseness, pretentiousness, or failure of the work itself” (Shawn 296).

This interdependence and relationship of parts to the whole is of special interest to me in composition. Self-similarity and symmetry are introduced as organising principles, both as part of my retrospective analysis methodology as well as in a more prospective approach in compositional planning. It is through this type of holistic analysis of my compositions—both retrospectively and congruently with the compositional act—that I am able to reflect upon the meaning of the material, to engage with it critically and to edit my ideas, and to direct and organise my research in a focused and systematic manner. “The inspiration, the vision, the whole, breaks down during its representation into details whose constructed realisation reunites them into the whole” (Schoenberg, “Style and Idea” 107).

My critical commentary employs a constructionalist analytical methodology that focuses on the relationships between musical events and the musical structure itself, in order to reveal such features as symmetry, balance and proportion (Bent & Drabkin 79). My goal is to transcend the observables in some way, providing insight into some of the processes behind my compositions in an objective manner (Jay Rahn, 24).

The employment of geometry to create geometric structural representations of music is demonstrated by Escot to be not only a versatile and insightful analytical approach, but one that seems immune to boundaries of genre and era. Escot shows that geometry, proportion and symmetry are universally applicable principles that can create meaningful correlations and illuminate unique compositional features (Escot 76). Tymockzo also demonstrates that music can be understood geometrically, and uses geometry to show the inner workings of harmony, its voice-leading and scale types (Tymockzo 19).

Practice-based research is one of several popular methodologies adopted by creative arts practitioners, as it offers a fitting alternative to the more orthodox social sciences or scientific research paradigms (Grierson & Brearley 5). Through my adoption of the practice-based research approach, I am at once the research practitioner and the subject of research. These two aspects of researcher and practitioner combine to produce an artefact—the folio of compositions—that is enhanced and illuminated by the research presented in this dissertation (Winter 3). Consequently, the theories presented in the critical commentary are the result of practice, rather than *vice versa*, and more importantly are intrinsically part of and contribute to the societal and cultural creative forces to which I belong (Regelski 113).

Heidegger coined the term *praxial knowledge*, which encapsulates this concept of ideas and theory being born of practice (Winter 3). Practice-based research is a process, and a reiterative one at that, which involves the continual feeding-back of ideas, concepts and theories between the artist as researcher and the artist as practitioner. This is very much a part of my compositional process, and has been referred to as an “iterative cyclic web” (Smith & Dean 19).

Borgdorff examines the challenges of couching artistic research within academia, and endorses and illuminates the practice-based research methodology. He reveals that artistic research articulates the “re-reflective, non-conceptual content of art” and therefore “invites unfinished thinking” (Borgdorff 143). What Borgdorff describes as “thinking in, through, and with art” corresponds with the aforementioned reiterative nature of a process-centred research approach (143).

My rationale for composing is simple. I write and play what I enjoy hearing. I share Schoenberg’s sentiment in this regard, who wrote that “a real composer writes music for no other reason than that it pleases him” (Schoenberg, “Style and Idea” 54). I also enjoy expanding the limits of what I understand and appreciate, which means that performing and composing is part of a greater act of lifelong learning. I think this is what Stravinsky meant when he was asked for whom he composed, and answered “The Hypothetical Other” (Stravinsky, qtd. in Rahn, “Perspectives on Musical Aesthetics” 151). In terms of the epistemological aspect of this critical commentary, I have sought to open myself up and expose the nature and limits of my beliefs and habits, and thus embrace what Foucault refers to as a “permanent critique of ourselves” (Foucault 43; Grierson & Brearley 21). Ultimately, my quest has always been to “find my own music”, and as Mathieu explains, that quest draws upon an internal “village” of musical experience that is at once culturally derived and idiosyncratic (Mathieu 74).

1.5 Research questions

This dissertation is a study of my contemporary compositional practice that aims to expose the theoretical, creative and embodied intercultural nature of its formative process. My practice-based research began with the over-arching question “what are the organising forces that inform creative ideas in my composition?”. The organising forces that arose most notably from my research were intercultural music, symmetry and self-similarity.[13]

The analytical topics featured in Chapters 2 and 3 examine the principles of symmetry and polyrhythm. I argue that polyrhythm can be considered a manifestation of symmetry, and so it is symmetry and intercultural music that feature as enduring universals in my music, and also of that upon which it is inspired.

It is through my dissertation that I aim to illuminate meaning, order and intention through the use of reflective writing, analytical tools and diagrams. It is my aim to contribute to the body of knowledge by translating retrospective and forensic analysis of composed work into proactive theoretical tools that can be exploited creatively by composers. In this manner I expose the reflexive nature of analysis and composition that parallels the practice-based research paradigm.

One may question whether the intercultural influences in my compositions are being transformed by my own aesthetic in such a manner as to create an original hybrid form that honours its sources, whilst being emancipated from their original cultural and musical references. My goal is to integrate ‘otherness’ into ‘wholeness’ void of differences of legitimacy, status and power (Born & Hesmondhalgh 21). The reader can determine my successful achievement of this goal partly through the mitigation of the risks of intercultural hybridity listed in section 3 of this chapter. It should be noted however that a thorough discussion of cultural considerations falls outside of the scope of this dissertation.

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13 My research into self-similarity in music developed into a voluminous topic that will appear in a publication separate to this dissertation.

1.6 Critical commentary structural overview

This dissertation is intended to be read in conjunction with the provided folio of compositions and their recordings. The chapters may be read in any order, but it is recommended that the first three chapters be read initially, as they detail the analytical and compositional tools used in the remaining chapters and applied in the folio.

Chapters 4 and 5 survey occurrences of symmetry, and polyrhythm (respectively) in five of the compositions from my folio, being:

- *Locked-In*
- *Paco*
- *Binary Times*
- *A Kayak*
- *Perfect Storm.*

These chapters feature diagrams as the sole means of expounding compositional principles in the key analytical areas studied in chapters 2 and 3. Short descriptions accompany each, designed to be read in conjunction with the relevant portions of the score and recording.

The inspiration for this approach lies in the work of Johnson & Jedrzejewski (*Looking at Numbers*) and Escot (*The Poetics of Simple Mathematics in Music*). In their work, diagrammatic illustrations sometimes take on an elevated importance and an aesthetic pleasure is educed that does not depend on the music from which they were born. It would be an honour if even one of my diagrams had this affect.

Each diagram stands at that liminal position between the real and the abstract, and it may considered fortuitous if they provoke more questions and creative ideas than the descriptive answers they provide.

Chapters 6 and 7 analyse the remaining two folio compositions in more detail, being:

- *Mod Times*
- *Birder.*

1.7 Critical commentary limitations

Musical analysis is a potentially never-ending task that may combine mathematics, aesthetics, phonology, syntax and semantics. The scope of this dissertation will therefore exclude many topics that are in fact integral to the conception, production and contextualization of my music. These include (but are not limited to) the role of improvisation, collaboration, acoustics, texture/timbre, microtonality/temperament, and the musicological areas of semiotics and cultural studies. Rather, I focus on rhythm, pitch and structure, and the isomorphic relationships that these parameters propagate to each other within my composition.

The study of exemplars in the field of intercultural hybridity and their characteristic features is an interesting related research question, but also falls outside of the scope of my dissertation.

1.8 Rhythm, metre and issues with Western notation

Rhythm is often the parameter of preeminent importance in my composition. Its representation in Western notation is sometimes problematic however. Procedures that span distances of time consisting of varied and multiple bar lengths commonly oppose the conventions of Western notation, its beat hierarchy and metric order, in preference for the primacy of the phrase, motive and procedure itself. One cannot always “see the beat” in any bar, and beam-groups tend to contradict the bar line when used in a phrase-centric manner.

An Indian rhythmic cycle (*tala*) of 16 beats for example may be a placeholder for one cycle, but the four 4-beat bars (*vibhags*) that partition each cycle are rarely clean containers for the all phrases in all parts. Rather, musical phrases of 3, 5, 6, or 7 beats in length can readily appear in opposition to the prevailing metre. The sort of Indian rhythmic processes I adopt in my music such as *addha*, *yati* and *korvai* are inherently phrase-centric (see Chapter 3).[14]

The precedence of phrase over beat to which I refer is evidence of an opposition or paradox between metre and rhythm, and something that is philosophically addressed

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14 Davidson recognizes the same issue with his *cell-based rhythms* (8). Reina recognizes the same dilemma in truthfully expressing South Indian polyrhythmic concepts using Western notation (“Applying Karnatic Rhythmical Techniques to Western Music” 46).

by Hasty in his rhythmic theory of metre. Hasty challenges the "customary devaluation" of metre as predictable and periodic, and offers a theory that treats metre "as an integral part of rhythm" (Hasty 6–7). This perspective on metre is congruent with my experience of *tala*, complete with its beat hierarchy and phrase centricity.

In order to address this conflict in the notation of my compositions, and to assist the performer in their realisation, I annotate numerals underneath important groupings to show where phrases commence, and to identify their duration in the prevailing subdivision, such as in the following extract from *Birder*.

Figure 2: Notation with grouping annotation.

This issue is made more severe in situations where I overlay different processes in multiple voices within a common rhythmic cycle. One solution would be to write different parts in different metres, but multiple simultaneous metres are not practically realizable in notation software such as *Sibelius*,[15] and represents an anomaly not catered for by standard Western notation. Instead, I may trade metric clarity for one musician in preference for another, such as in the final section of *Birder* (see Chapter 7). In such cases I may mitigate bias through the use of dotted bar lines to illuminate different metres that coexist.

The mathematical process of extrapolating grouping patterns from within cycles of time is a musically rich and intellectually stimulating pursuit that has firm roots in Indian classical music. Sequences of phrases following an internal logic of durational progression are the norm, especially South Indian music, and I adopt this approach. In working on phrases in a composition in 6/4 metre, for example, I may consider the time cycle as 72 semiquavers (3 bars) and explore partitions of this cycle length that feature the evolution of a motive, such as the sequence illustrated in Figure 3.

15 Steinberg's newly-released software *Dorico* offers promise in this regard.

2 (24) 8 (24) 14
6 (21) 10 (21) 14
10 (18) 12 (18) 14
14 (15) 14 (15) 14
18 (12) 16 (12) 14
22 (9) 18 (9) 14
26 (6) 20 (6) 14
30 (3) 22 (3) 14

Figure 3: Sequence of eight cycles of 72, partitioned into a logical sequence featuring the cadential motive of 14.

This type of numeric representation often reveals underlying structure more readily than Western notation, and appears in this dissertation and as annotations alongside standard notation in my folio of scores.

1.9 Use of diagrams

Diagrams have been exploited as an analytical tool since Euclid and Aristotle (Johnson & Jedrzejewski xiii). This dissertation employs several types of diagram that serve as schematic representations, offering insight into the relationships and organising principles of my music. The technique of diagrammatic representation can be considered a type of *inter-semiotic translation,*[16] whereby music and its mathematical features may be graphically represented (Pareyón 1).

The representation of temporal features of music using clock-like diagrams originates with the thirteenth century Persian theorist Safi al-Din al-Urmawi's *Book of Cycles* (Sethares 28). For my purposes, a *timeline* is both a specific periodic strand of time, and also the diagrammatic representation of such a strand of time.[17] Timelines are featured as clock-like geometric diagrams in Toussaint's *The Geometry of Musical Rhythm.*[18] Unlike

16 Inter-semiotic translation occurs when a system of signs transmuted into another different system of signs (Pareyón 105).

17 The term *timeline* has slightly different meanings between authors (see Anku, Agawu, "Structural Analysis or Cultural Analysis?" 1–3, and Leake, "Master Drummers of West Africa" 187).

18 The use of binary necklaces has its origins in the mathematical field of combinatorics, possibly first appearing in Leibniz *Ars Combinatoria* of 1666.

Toussaint, I always designate the start of the timeline as pulse one, rather than zero.[19]

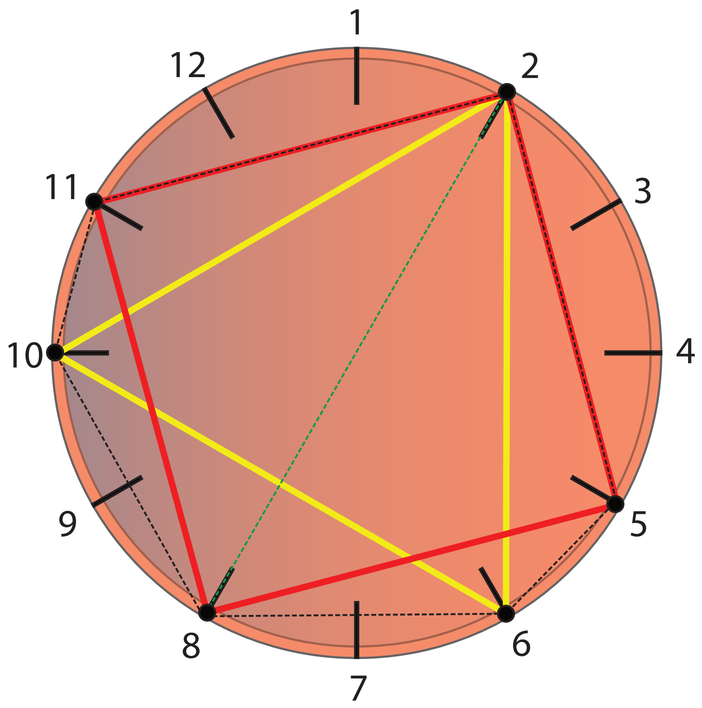

Figure 4: A timeline diagram.

The scale of a timeline could be microscopic (at extremes, a beat), moderate (a bar, phrase, or ostinato), or macroscopic (an entire rhythmic cycle, phrase, section or piece). As in any structural analysis, the choice of resolution is relevant, in that different levels of magnification have the potential to yield different insights into organisation and relationship. Timelines in this dissertation are typically periodic rhythmic phrases, and therefore are usually of a moderate length. Timeline diagrams always represent strands of time chronologically in a clockwise direction, although non-linear and non-contiguous relationships are often revealed.[20]

Sometimes I represent multiple rhythmic strata on a single timeline diagram,

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19 Though a zero onset at twelve o'clock has some mathematical advantages, I appreciate my audience are likely to be most familiar with standard Western notation and its counting methods, where sequences commence with one (e.g. "1 e & a 2 e & a…"), and would find it more intuitive to partition, say, a 16 beat cycle into four quarters at divisions 1, 5, 9, and 13 than 0, 4, 8 and 12.

20 Crystallography employs the same approach for analysis, where an essentially periodic strand of time that can be represented easily in one dimension as a line segment is turned into a circular form called a *cyclotomic set*, in order to derive further meaning from all the geodesic relationships contained (Toussaint, *The Geometry of Musical Rhythm* 36).

using colour coding to distinguish the layers. On occasions I may also add a middle concentric circle to represent pitch, when an isomorphic relationship with rhythm is under consideration. Following is an example of one of my *composite time-pitch wheels*.

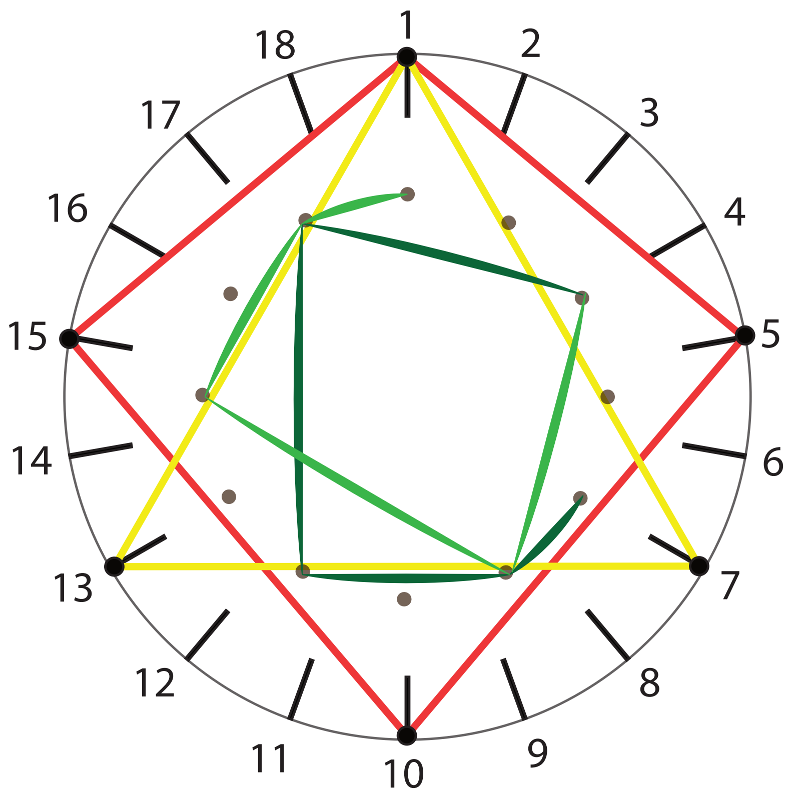

Figure 5: Composite time-pitch wheel.

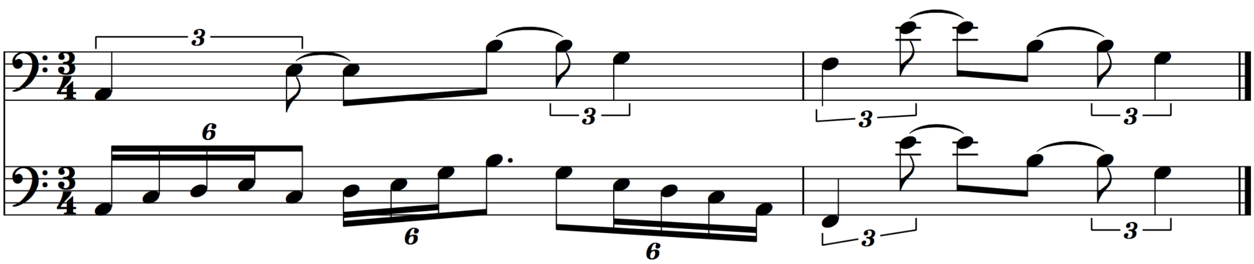

Figure 6: Notational source of Figure 5 (excerpted from *Mod Times*).

The parameters of my composite time-pitch wheels are as follows:

Outer temporal wheel

- Cycle direction is clockwise.
- All the pulses in a cycle are represented by equidistant metrics, which are numbered from 1 through *n*, being the length of the cycle.
- Rhythmic onsets are represented by black dots upon the circumference at the relevant pulse point, corresponding to the point that the polygon intersects the circumference.
- All timelines have at least one polygon, but more often two (as is necessary in polyrhythmic music). These are colour-coded. Occasionally, dotted polygons are used to assist when a metre is implied but not articulated.

Inner pitch wheel

- Pitch ascends in a clockwise direction.
- There are always 12 equidistant metric dots representing the 12-semitone octave.
- The metric dot in the 12 o'clock position is C by default, but the 12-semitone pitch-space may rotate to favour a different pitch of centricity, or to better illuminate symmetries.
- Paths plotted are colour-coded to identify different instruments. Colour coding may also be used to show additional (secondary, tertiary) pitch patterns and relationships.

I borrow from Forte's pitch-class set theory and other tools belonging to post-tonal theory to analyse the properties of rhythmic structures represented in timeline diagrams (such as in Chapter 3). The reader is referred to texts by John Rahn and Joseph Straus in the bibliography for relevant theoretical background.[21]

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21 Allen Forte's *The Structure of Atonal Music* (1973) is the original resource for the methodologies used to describe and quantify pitch-class sets, their properties and inter-relationships.

Triangular brackets <a-b-c> specify a series of *inter-onset intervals* as they appear in a timeline, much like an ordered series of pitch class intervals. Double triangular brackets <<g,h,i>> are used to distinguish *interval vectors* (also called *interval-class vectors*) from an ordered series of inter-onset intervals.[22] Chapter 3 applies these analytical tools and represents them diagrammatically.

The choice of circular diagrams to illustrate Indian time cycles is not without precedent. In *Mrdangam Mind*, Nelson employs circular diagrams to aid the understanding of temporality of South Indian music (see Nelson chapter 1). The legendary tabla player Pandit Taranath's *Taalchakra* employs concentric circles to illustrate time cycles in North Indian music (see Figure 7). Anku states that African music is perceived in a circular (rather than linear) manner, and employs circular diagrams to aid the definition of structural sets (Anku 1). The suitability of concentric circles for realising complex rhythm was also recognised by the synthesiser pioneer Don Buchla, whose *Polyphonic Rhythm Generator* serves as a powerful alternative to conventional drum machines.[23]

The visual appeal of representing music—and particularly cycles of rhythm—on wheels with polygons almost justifies the effort, without any corresponding analysis. However, the clarity in which patterns are illuminated, and the multiplicity of perspectives offered to the composer-turned-analyst makes these kinds of diagrams powerful tools for understanding music, its patterns and development.

22 *Full interval vectors* show all of the smallest possible distances between all onsets (including those that are non-contiguous), being analogous to the interval-class vectors of post-tonal theory. The difference between full and adjacent interval vectors is discussed in Chapter 3.

23 Refer to <https://buchla.com/product/252e-buchla-polyphonic-rythm-generator>.

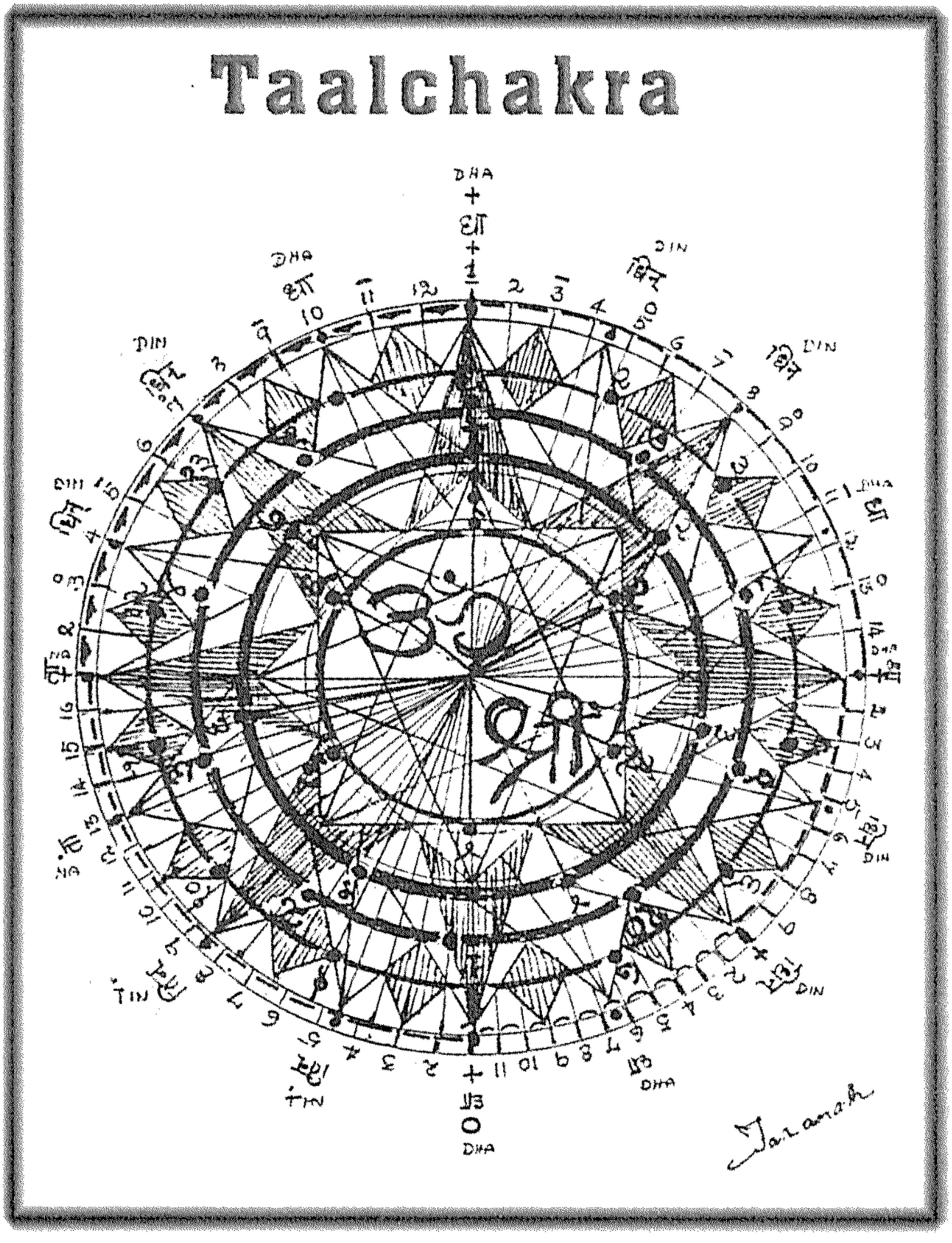

Figure 7: *Taalchakra* of Pandit Taranath.[24]

24 Figure from Naimpalli (31). Used by permission of the publisher.

1.10 Overview of folio of compositions

Locked-In

Locked-in syndrome (also known as *pseudocoma*) is a type of near-total paralysis that leaves a person aware and awake, and able to move their eyes, but otherwise lacking the ability to move or communicate.

The song's lyric recounts the one-way conversation a person may have held with their locked-in partner, conjuring the love, the reminiscence and the frustration such a conversation may involve.

This work is scored for the following instrumentation:

- soprano voice
- accordion
- electric bass guitar
- trumpet
- bass trombone.

Paco

This piece was written after the death of guitarist Paco de Lucía in 2014, and was written as a sort of elegy. It is not meant to be literally flamenco in style, though some flamenco concepts can still be observed. Paco de Lucía would often perform solo, complete with percussive taps to the instrument's body, and when accompanied, the traditional clapping (*palmas*) would be present. In my piece there is a clapping section as the penultimate section of the work that renders a 7-layer canonic figuration. This symmetrical reflection rhythm, augmented by foot pedals played by the bassist and trombonist and ultimately the drum kit, is not dissimilar in effect to the interlocking effect of *palmas*. The 3:4 polyrhythm and its rotations are also a common motif throughout the work.

This work is scored for the following instrumentation:

- bass trombone and percussion
- electric bass guitar and percussion
- drum set and percussion.

Binary Times

Intended as the sequel *Mod Times*, to be performed as a two-movement sequence. Its initial harmonic material departs from the closing moments of *Mod Times*. The work features self-similarity, mostly derived from the *Thue-morse sequence* to 32 bits, which plays out in the harmonic progression, rhythm, form and melody.

This work is scored for the following instrumentation:

- bass trombone
- electric bass guitar and percussion
- vibraphone
- drum set.

A Kayak

This composition takes inspiration from Karim Ziad's *Houaria*.[25] The structure of Ziad's work was studied and adapted to make way for the Afro-Cuban and Indian elements that contributed to the grooves, rhythmic phrases and modes of my piece. The number of characters in the title is also symbolic, as 6 plays a significant role in the metre, rhythm, pitches and phrases of my composition.

A Kayak incorporates bitonal pentatonic sounds inspired by the North Indian *Raga Marwa*. *Marwa* uses degrees 1, b2, 3, #4, 6, 7, which corresponds to the major pentatonic (first mode) based from the major sixth degree. In *Marwa*, the rules of the *raga* work to create a bimodal tension from the treatment of the sixth degree (Bor 114). In *A Kayak*, the major pentatonic (first mode) is superimposed a tritone away from the parent scale tonic (E), where a minor pentatonic (third mode) scale is placed.

This work is scored for the following instrumentation:

- bass trombone
- electric bass guitar and percussion
- drum set.

25 Karim Ziad, *Houaria*, perf. Karim Ziad, Intuition, 2007.

Perfect Storm

The goal of this work is to assimilate aspects of South Indian *raga* and *tala*, and Balinese solo drumming (*kendang tunggal*) into form based on North Indian *khyal*.[26] The mode is inspired by *raga Megh*, a monsoonal *raga* (hence the title). The composition incorporates the rhythmic techniques and practices of each genre, including the following.

North Indian classical: *Dadra tala*, *tihai*, *tabla* solo upon *lehara*, interweaving of *mukhra* catch-phrases.

Balinese *kendang tunggal*: Interlocking phrases of *kotekan* that tile the line, *ombak* tempo variations, *angsel* structural cues that dictate where cadences and contrasting sections occur.

South Indian classical: *Tani avartanam* percussion solo that features logical development of rhythmic motives in a set rhythmic cycle, *yati* and *korvai*.

This work is scored for the following instrumentation:

- bass trombone and *mridangam*
- electric bass guitar and *tabla*
- drum set and *kendang*
- optional solo *tabla.*

Birder

This work is scored for the following instrumentation:

- violin
- viola
- soprano saxophone
- double bass
- piano
- electric guitar
- electric bass guitar
- drum set.

Refer to Chapter 7 for details.

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26 *Khyal* (Arabic for “imagination”) itself is an amalgam of many different styles, but is broadly recognized as a contemporary style featuring improvisation and more freedom than *dhrupad*.

Mod Times

This work is scored for the following instrumentation:

- bass trombone
- electric bass guitar
- drum set.

Refer to Chapter 6 for details.

1.11 Conventions adopted

Being published in Australia, I conform to the British system of duration nomenclature, being semiquavers (1/16 notes), quavers (1/8 notes), crotchets (1/4 notes), minims (1/2 notes), and semibreves (whole notes).

Any reference to octave placement considers middle C as C4. Tuning in the recordings is the standard A=440 Hz.

Nomenclature from post-tonal theory uses Rahn as a model except where stated.

Neutral key signatures are used, yet my folio of compositions are not atonal. Pitch collections may be referred to being “in a key”, meaning there is a hierarchy around a principal note. However the pitch collection itself may not derive from diatonic or conventional modal sources, and its corresponding melody may use different notes from the pitch collection as temporary points of arrival and departure. Furthermore, harmony in one voice may be opposed by other contrasting pitch collections in another or undergo rapid change, thus reducing the effectiveness of key signatures, which would obfuscate ready analysis, and impede realisation by the performer.

Tempi are referred to as being beats per minute (b.p.m.), and the terms ‘bar’ and ‘measure’ are used synonymously, as are ‘metre’ and ‘time signature’.

Indian classical music is referenced frequently as a principal intercultural influence, and the different facets of North and South Indian classical music and their influences are identified accordingly. In other research, the classical music of these regions are categorised as *Hindusthani* and *Carnatic*, respectively. I have chosen the former designations.

Transliteration of non-English terms is fraught with challenge and contradiction. I have opted to use a consistent approach within this dissertation, basing my spelling choices on those most prevalent in the literature cited in the bibliography, and that of my teachers.

Chapter 2
Symmetry

Symmetry is the universal principle of nature, the principle permeating
the whole universe and revealing its unified picture
from atomic nuclei and molecules to the solar system…

Voloshinov, *Symmetry as a Superprinciple of Science and Art*, p.109.

2.1 Background

The terms *symmetry* and *proportion* vary widely in usage and scientific pellucidity, and may even be considered loose similes in arts discourse, often used to connote an aesthetic aim that eludes quantification. Artists have historically used symmetry to comprehend and create order, beauty and perfection, with nature's state of equilibrium as a model (Weyl 25).

One early artistic correlation of beauty and symmetry is in Polykleitos' sculpture (fifth century B.C.E.), where *symmetria* represented mathematical proportion and balance. Other notable examples include da Vinci's *Vitruvian Man* (1490) and Dürer's *Four books on Human Proportion* (1534). To Plato, symmetry is considered an innate facet of the physical world, intrinsic to nature and naturally occurring in art, with mathematical laws as their source (Weyl 8).

Asymmetry seldom indicates a complete absence of symmetry (Weyl 13). The two can in fact be viewed as a dualistic monism[1]—essential forces that together create a universal frame of reference. Musical analysis may be conducted within this frame, aiming to reveal connections and congruencies—the symmetrical forces of gravitational attraction that balance the asymmetrical forces of resistance—that together combine to make the whole[2] (Csapó 199).

The tension between symmetry and asymmetry is examined by Feynman, McManus, Frey and Weyl. According to Frey, symmetry represents rest and binding, order and law, formal rigidity and constraint; whilst asymmetry represents motion and loosening, arbitrariness and accident, life, play and freedom (Frey, qtd. in Weyl 16). Harmony models

1 The *yin–yang* symbol of Chinese philosophy is an example.

2 This dualism resembles the two inadequate worlds of Plato, being the world of ideas (exact symmetry in mathematics) and the world of things (approximate symmetry in nature) (Voloshinov 112).

this opposition, with the neutrality and balance of symmetrical scales contrasting the hierarchical goal-orientation of asymmetrical tonality. There exists an “ebb and flow” that takes place as melodic transformations “create motion away from, and back to symmetry” (Pearsall 34).

Wilczek writes “Symmetry means you have a distinction without a difference” (58). This simple yet appealing definition corresponds with the concept of development or variation in music—that grey area between contrast and repetition. This liminal position of likeness with difference is the concept that underpins narrativity in music (Lundy *et al.* 278). Music employs the structural principles of repetition, contrast and development to articulate time and thus create form, conventionally represented by letters such as A A’ B A (Wallerstedt 49; Kempf 155). Essential to these three principles is the notion of likeness, and requires the listener/analyst to differentiate diachronic material. The application of repetition, contrast and development to musical material by the composer contributes greatly to the music’s sense of flow, direction, and journey for the listener, for it is in the juxtaposition of these principles, along with the listener’s memory, anticipation and apprehension of likeness and difference, that musical form is created (Lissa 532). In post-tonal music such as that in my folio, symmetry plays an important role in the generation of material, but it is through the connections and changes that occur in that material over time—often represented by the development of musical motive—that the essential musical idea and process are revealed (Pearsall 33). Schoenberg is quick to point out the primacy of the motive and its development. “Everything depends on its treatment and development” (Schoenberg, “Fundamentals of Musical Composition” 8).

Wilczek uses the term *nontrivial symmetry* for situations where distinctions don’t make any difference, and demonstrates that the knowledge of the presence of symmetry in an object or set of objects enables us to deduce the properties of other objects or sets of objects.[3] Wilczek uses the term *local symmetry* for situations where each place and moment defines its own symmetry. In contrast to this scenario of independent symmetries is the concept of *global symmetry*, where changes happen universally in lock-step. Thus in considering the presence of symmetry in music, or indeed during any process of structural interrogation, one has to be ready to adjust the scale or scope of one’s attention, as

3 Einstein’s *Special Theory of Relativity* can be considered a postulate of symmetry in this manner (Weyl 127).

symmetries may operate on microscopic levels or on more over-arching, macroscopic levels, and upon different musical parameters.

The parametric extent of musical symmetry and the suggested criteria for its measurement are proposed by the composer Gyula Csapó, who recognises the theoretical challenges faced by the analyst that “arise from the complex, often interactive, simultaneous and temporal, i.e. multidimensional manner in which musical parameters behave” (183). In his paper, Csapó presents a number of approaches to symmetry, accompanied by examples and related issues. Some of these include symmetries of:

- duration (rhythmic series)
- scale (pitch series)
- melodic impulse (upsurge) and resolution
- harmony (chord construction)
- registral distribution
- functional tonal relationships (tonic/subdominant and dominant/tonic versus prolongations)
- number of attacks (e.g. irregular number of onsets within phrases of equal lengths)
- spatial distribution (across stereophonic auditory space)
- dynamics (as juxtapositions of loudness/softness)
- form
- timbre (as overtone structures observed through spectral analysis).

Symmetry in twentieth century musical composition has been broadly recognized in the works of Schoenberg, Crumb, Debussy and Bartók (Pearsall 33, 39).

There are many occasions in my work where symmetry is observable pan-parametrically. Such *isomorphic*[4] applications of symmetry include the simultaneous application of a symmetrical pattern upon pitch (intervals measured in semitones) and rhythm (duration measured grouping of pulse). Messiaen is one composer who exploits such isomorphic symmetries, and unites his symmetrical *modes of limited transposition* with his symmetrical *nonretrogradable rhythms* in a way that he hopes will lead the listener to a certain emotive and transcendental sensation—the “charm of impossibilities” (Messiaen 13, 21). Symmetry and proportion can be deliberately applied in composition

4 The term *isomorphic* comes from the Greek words meaning ‘equal form or shape’.

and analysis. In composing, I employ principles of symmetry in composition in pursuit of balance and unity through logical and pleasing interrelationships, and find that they promote innovative ideas by fostering both freedom and transformational process (Pearsall 39). Schillinger demonstrates the potential of melodic symmetries by considering axial orientation and direction of travel (Book IV).

Post-compositional analysis reveals examples of symmetry in my work that have occurred intuitively. Our perception of structure in the world around us is a practiced and essential process that utilises discrimination, recognition and identification of all sensory input in order to learn and derive meaning, and we work to develop and improve the internal representations of the patterns we perceive over time (Pomerantz & Lockhead 11–14). It is not surprising, then, to find the result of subconscious symmetrical patternings during the creative act.

I will now introduce the types of Euclidean symmetries[5] utilised in my folio of compositions and their analyses. Refer to Benson (chapter 9), Straus (chapter 3), Toussaint, Tymoczko, and Weyl for supporting reading.

5 Euclidean symmetries can be represented using conventional geometry of one, two or three dimensions, as opposed to those fractal constructs that are not dimensionally concordant (Mandelbrot 14).

2.2 Bilateral symmetry

Also known as *mirror symmetry* and *reflectional symmetry*, *bilateral symmetry* is the most dominant in art and nature (Voloshinov 110). Bilateral symmetry in two dimensions shows invariance about a particular axis. It is said that one part is a *reflection* or *mirror-image* of the other when this type of symmetry is present. It is thus a precise and measurable phenomenon, illustrated easily using geometry.[6]

Bilateral symmetry may occur about a vertical axis (y-axis) or horizontal axis (x-axis), as illustrated geometrically in Figures 1 and 2, respectively. In musical terms, symmetries about the vertical (y-axis) in music are temporal, whilst symmetries about the horizontal (x-axis) tend to concern pitch. Note that this *axis of reflection* is perpendicular to the direction of travel. The formula for determining the location of the bilateral axis of reflection (*a*) in any sequence of members (*m*) is as follows:

$$\frac{a = (m + 1)}{2}$$

This formula may be applied to symmetrical constructions in the domain of pitch or time. The formula shows that the location of the axis of reflection may not correspond to a member of the specific collection (rhythmic onset or pitch), depending on whether the number of members are odd or even. All symmetrical diagrams in this chapter indicate the axis of reflection as a pink dotted line. (Timeline diagrams in other chapters may feature a corresponding green dotted line.)

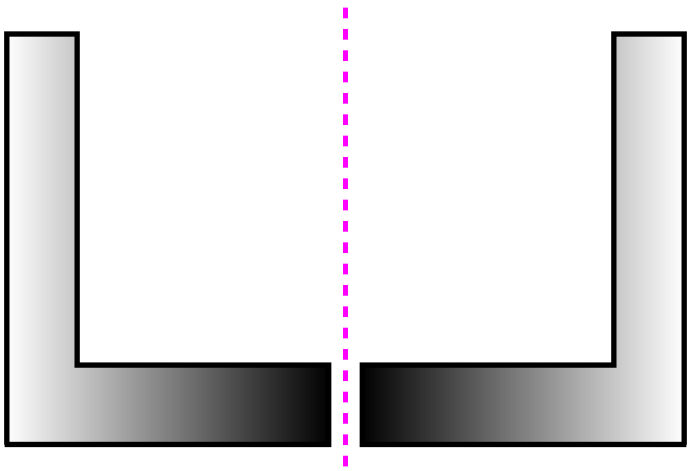

Figure 1: Bilateral symmetry about the vertical (y) axis.

6 Refer to Chapter 3 for numerous geometric examples of bilateral symmetry in polyrhythm.

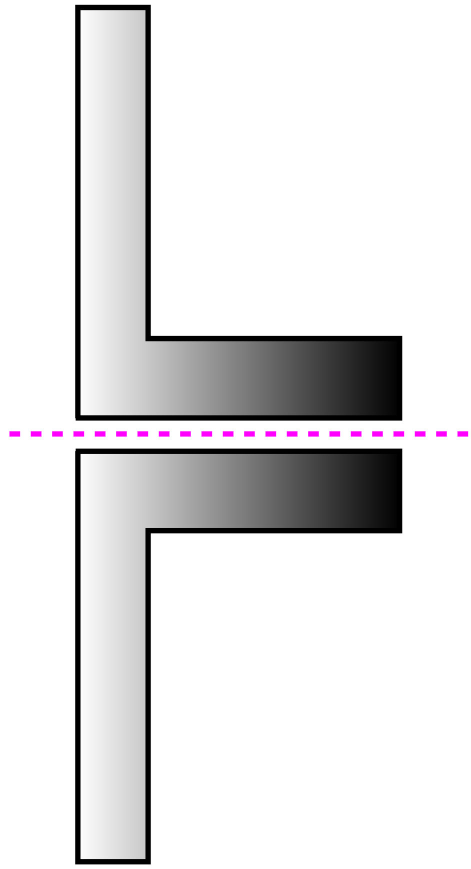

Figure 2: Bilateral symmetry about the horizontal (x) axis.

The *interval of reflection* applies to both axial arrangements, and concerns the distance between the reflected forms. The nature of this interval has an effect on the surrounding material and the overall effect of symmetry. The more prominent the bilateral division, the more emphasis is placed on the neighbouring parts.[7] This is idea is supported by the application of bilateral symmetry in Indian rhythmic structures, and the role of the *karvai* (structural gaps) will be discussed later in this chapter. It is also supported by the deliberate violation of symmetry in endings and cadences through harmonic opposition in composition from the Classical and Romantic eras (Kempf 158). Even in post-tonal music of Crumb, analysis reveals deliberate "degeneration of symmetry" to demarcate passages (Pearsall 34).

Temporal symmetries about the vertical (y-axis) are often called *palindromes*. With any symmetry that deals with duration, it is worth noting that it is the onset that marks the point of measurement in time, rather than the (mathematically correct but inappropriate) middle point of the duration (Csapó 184). As with any temporal analysis,

7 Imagine the effect of closing the gap in Figures 1 and 2 such that the two portions adjoin, versus a widening of the white space around the pink dotted axis of reflection.

the aforementioned question of scale or resolution arises, and the analyst may need to adjust the scale of resolution to educe the most fruitful results in terms of pattern, meaning, and likeness to other parts. The extremes of such a continuum of analytical scales of resolution are termed microscopic (as in individual chords and motives) and macroscopic (such as formal structure). In the illustrative examples that follow, one should consider that symmetries are rarely found in all parts and at all scales at once (as in Wilczek's *global symmetry*). More common is the situation where different symmetries occur at different scales of resolution, in different parts, and at different times, creating a complex web of interrelationships.

Simple ternary form A–B–A is a macroscopic example of bilateral symmetry about the vertical axis, where the broad features of the A section are recapitulated after a contrasting bridge section. This symmetrical archetype can be found as the essential formal structure in art and architecture alike, from classic sonata form through to the triumphal arches of Baroque architecture. Bartók expanded the symmetrical A–B–A arch form to five sections A–B–C–B–A, and applied it to his fourth and fifth String Quartets, as well as his *Concerto for Orchestra*.

The following example shows bilateral symmetry on a microscopic level, applied to the rhythmic design of a single melodic motive.

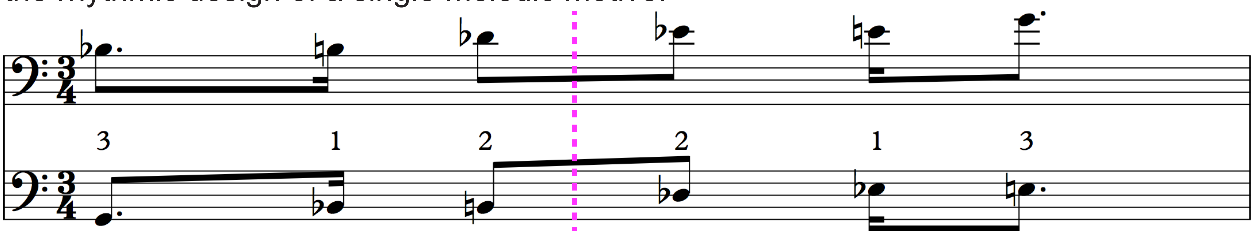

Figure 3: Motive from *Mod Times* demonstrating bilateral symmetry about the vertical axis.

This example is rhythmically palindromic, revealed by the series of numbers representing groupings of semiquavers <3-1-2-2-1-3>. As the excerpt contains six members in the sequence, the vertical axis of reflection for the bilateral symmetry lies precisely midway between the third and fourth members of the motive (on the third demi-semiquaver of beat two). This example also displays bilateral symmetry in pitch. The aforementioned grouping of semiquavers is applied isomorphically to pitch about the horizontal axis, where the intervals between the instruments contain the same <3-1-2-2-1-3> sequence (measured in semitones, ignoring octave displacement). See Chapter 6 for a full analysis of this motive in *Mod Times*.

The third movement of Haydn's Symphony no.47 "Menuetto al Rovescio" is an example of a large-scale palindrome where the themes are presented in reverse order in both pitch and rhythm after the midpoint axis of reflection. This is a rare example of mirror symmetry applied from the macro-formal structure down to the microcosm of pitches and phrases in tonal music, and succeeds in adhering to the strict rules of key relations and chord progression, tonal function and dissonance, metric hierarchy and rhythmic flow, whilst maintaining musical interest and aesthetic integrity (Kempf 159).

Such thorough approaches to bilateral symmetry became more common in post-tonal music of the twentieth century, where rules of tonality were relaxed and dissonance emancipated. Examples are found in Stravinsky's *Cantata* "Ricercar II: Tomorrow Shall Be", Schoenberg's *Pierrot Lunaire* "Der Mondfleck", and the variations in Webern's *Symphony* opus 21.

Symmetrical scales are examples of horizontal (x-axis) symmetry. Scales derived from the factorization of the 12-semitone octave into groups of 2, 3, 4, and 6 semitones have found broad application in the music of Debussy, Bartók, and Messiaen. The composer is not limited to these equal partitions, but may subsequently apply an *interval generator* to each of the partitions to create larger scales and a variety of derivative pitch collections. Messiaen's *modes of limited transposition* demonstrate the application of interval generators to create symmetrical scales (Messiaen 57). Figure 4 illustrates this procedure applied to the tripartite division of the octave to create what is known in jazz as the augmented scale.[8] An interval generator of 3+1 (traced by the yellow polygon) is applied to each major third (pitch interval of 4 plotted by the red triangle).

8 Messiaen would categorize this scale as a truncated version of his Third Mode .

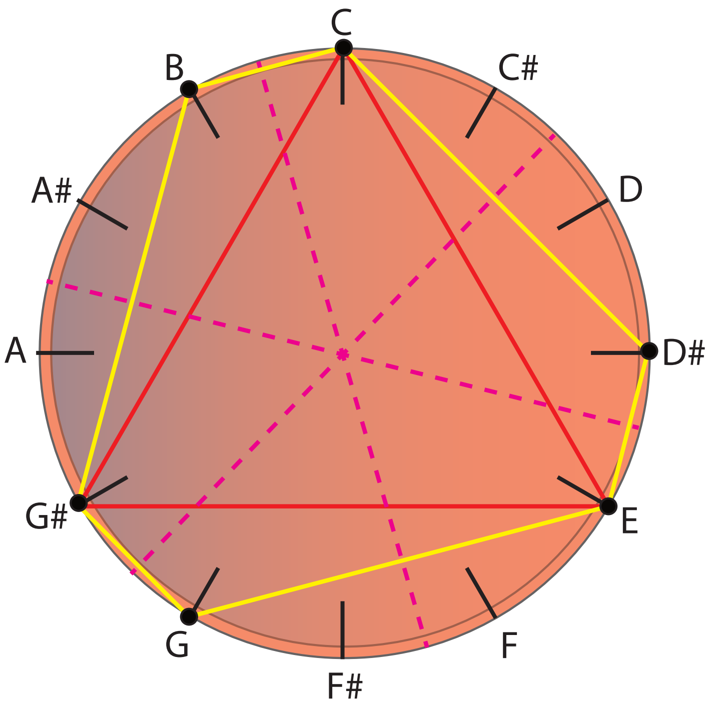

Figure 4: Symmetrical scale plotted on a pitch wheel (from C) showing equal partition of the octave into three major thirds (red triangle) and application of an interval generator of 3+1 to each major third (yellow polygon).

In geometric representations of symmetry, the axis of reflection may not be horizontal or vertical. Without the diagonal reflection lines in Figure 4 the bilateral symmetry would be less apparent. Toussaint proposes nine categories bilateral symmetry that consider the various combinations of axial orientation, including those that feature positive or negative diagonal slope as in this example ("The Geometry of Musical Rhythm" 229). Also notable in Figure 4 is the presence of not one but three axes of symmetry, none of which are orthogonal.[9]

The symmetrical division of gamuts larger than the octave also occur in my composition, and may or may not recur at the 24-semitone double-octave.[10] It is worth acknowledging that 12-semitone equal temperament is not the only symmetrical tuning system, and did not become the standard in Western European art music until the mid-nineteenth century (Chalmers 2). Other systems such as the pentatonic scale of Java and heptatonic scale of Thailand demonstrate a contrasting approach to symmetry in the

9 Group theory in mathematics proves that it is impossible to create two non-orthogonal axes of symmetry. Refer to Toussaint's attempts in "The Geometry of Musical Rhythm" (231).

10 Refer to Schillinger book II chapter 6, and Slominsky for various theories of symmetrical scales and approaches to their application.

temperament of intercultural music (Schillinger 145).[11]

Composers have experimented extensively with the property of inversional symmetry in harmony, and this principle has featured in the harmony of Schoenberg's compositions even before his 12-tone period (such as in the expressionistic works *Drei Klavierstucke* of 1910 and *Pierrot Lunaire* of 1912). Rahn states that "an inversionally-symmetrical set always has a canonical ordering whose interval series is its own retrograde…" (Rahn "Basic Atonal Theory" 91). This property can be seen in the following chord, where the pattern of intervals ascending from the bottom note mirrors the pattern descending from the top note. The chord corresponds to the symmetrical set [0,1,4,5,8,9].

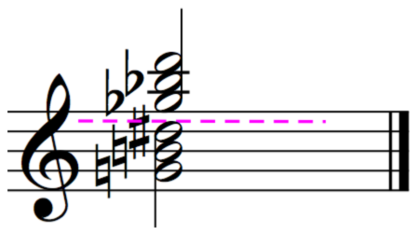

Figure 5: Inversionally-symmetrical chord showing reflection of two augmented triads about an axis of reflection of E quarter-tone sharp.

Figure 6 illustrates that the subject in Bach's fugue in D minor is presented alongside its inverted form.[12] This is a rare example of its kind in this genre (Kempf 161), but does illustrate the scope of inversional symmetry in the sense that intervals can be considered relatively in terms of contour and in harmonic context, and not just absolutely in terms of number of semitones per post-tonal theory.[13]

11 My studies in microtonality with Joseph Maneri during the early 1990s introduced me to the 72-note equal temperament of Ezra Sims. Refer to Chalmers for a full account of practitioners of equal temperament in microtonal music and associated theories, and to Helmholtz for a historic and acoustic account of tuning systems of the West and the East.

12 Goeth, Philip. "Well Tempered Clavier Book I: Prelude D minor." J.S. Bach's Well Tempered Clavier. 2012. Accessed 26 Apr. 2017 <http://www.bachwelltemperedclavier.org/pf-d-minor.html>.

13 See Straus (126) for a discussion of melodic contour in post-tonal music.

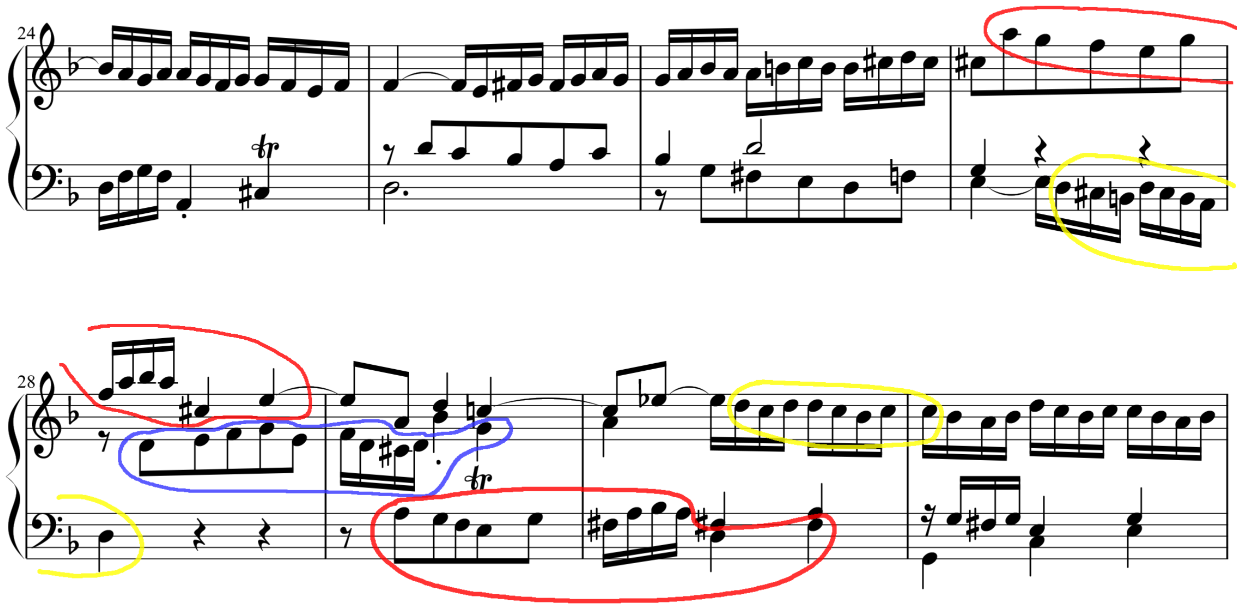

Figure 6: Inversional symmetry in Bach, fugue in D minor from the sixth movement of *The Well-Tempered Clavier* BWV851. Blue indicates the subject, red the inverted subject, and yellow the counter-subject.

Horizontal (x-axis) symmetry may be derived from more macroscopic features than individual chords, themes and scales. It may be found in pitch centres arrangement as structural landmarks in a composition. Examples include Coltrane's use of major third relationships in his augmented cycles as featured in *Giant Steps*, and Bartók's use of minor third relationships in his diminished cycle *axis system* in *Music for Strings, Percussion and Celesta* (as revealed by Lendvai).[14] Both composers utilise these symmetrical relationships as key areas, pitches of centricity, and as the basis of chord substitution.

Macroscopic mirror symmetry on the horizontal axis occurs in Strauss' *Elektra*, where D is taken as the axis of reflection in terms of the organisation of key areas for the leitmotifs and associated material over the course of the two-hour opera (Antokoletz 14).

14 Bartók's integration of symmetry and self-similarity in the use of Golden Section proportion in this work is significant, and is examined by Kempf (161).

2.3 Translational symmetry

Translation is an automorphism that leaves the structure of an object unchanged, relocating it a specific distance along a fixed straight line (Weyl 42; Voloshinov 113). Rhythm in time represents the most intuitive representation of the straight line, with its intrinsic homogeneity and constituent metrical units (Voloshinov 111). In this way translation occurs about a vertical axis as repetition.

The fundamental importance of repetition as a symmetrical principle in music is indicated by poetics in relation to linguistics.[15] According to Jakobson, "Only in poetry with its regular reiteration of equivalent units is the time of the speech flow experienced, as it is—to cite another semiotic pattern—with musical time." (7)

Figure 7 illustrates translational symmetry geometrically as three repetitions of the initial (left-most) object about a vertical axis. The green arrow indicates the direction of translation, being a vector (comprising size and direction).[16] The axis of reflection is again perpendicular to the direction of travel.

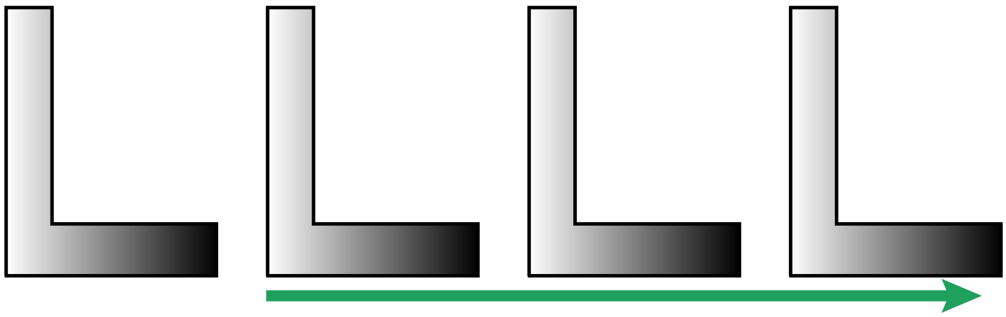

Figure 7: Simple translational symmetry about the vertical (y) axis.

Translation can also occur about a horizontal axis. Figure 8 illustrates translational symmetry geometrically as three repetitions of the initial object about a horizontal axis.

Horizontal axis translation in music is typically realized as transposition, such as the transposition of themes from one key to another. Octave equivalence is another example of horizontal axis translation in music. In post-tonal theory, pitch class 0 (C) is invariant under transposition by lots of 12 semitones (octaves). Modulo (mod 12) is applied to show such translation as octave equivalence. For example, ordered pitch interval +25 ≡ +1.

15 Refer also to Leonard Bernstein, "Lecture 3: Musical Semantics," *The Unanswered Question*, Harvard University, 1973.

16 Weyl states vectors are analogous to translation (45).

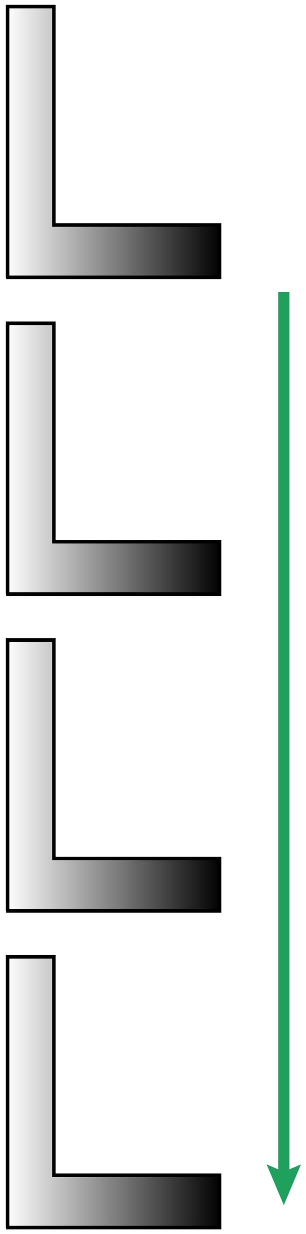

Figure 8: Simple translational symmetry about the horizontal (x) axis.

Just as the *interval of reflection* applies in bilateral symmetry, the *interval of translation* (or translation length) applies to both axial orientations of translational symmetry. A simple round is an example of translational symmetry about a vertical axis. The nursery rhyme *Frère Jacques* sung as a four-part round would typically have an interval of translation of eight crotchet beats. Figure 9 represents such a round as a *wallpaper diagram*.

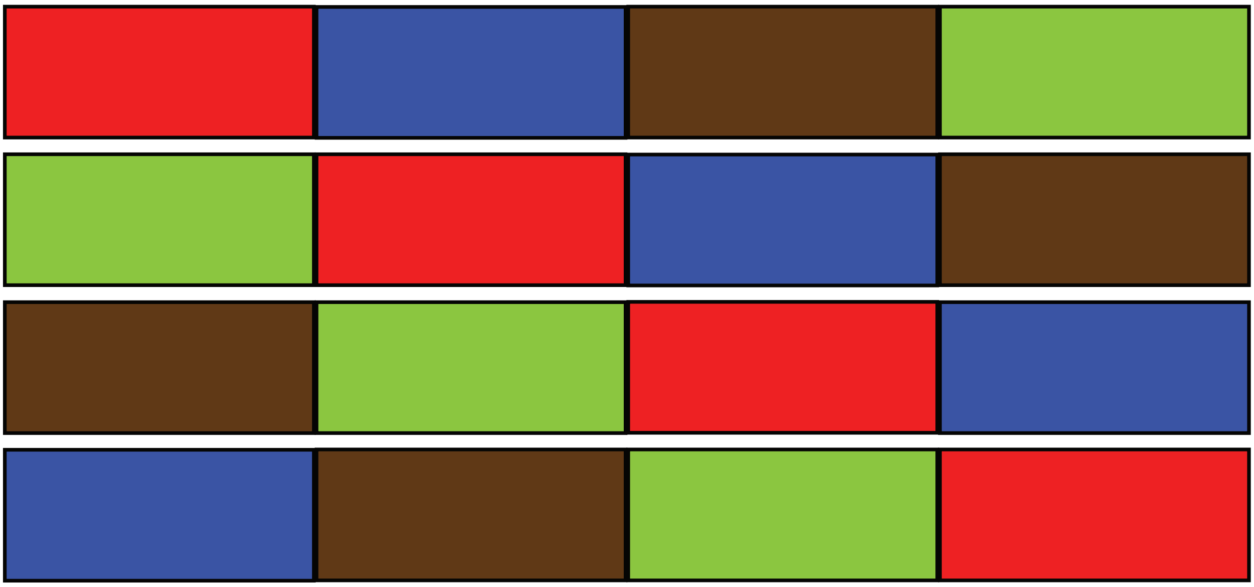

Figure 9: Wallpaper diagram of *Frère Jacques* displaying translational symmetry as a four-part round, with an eight beat interval of translation.

Vertical axis translation may appear as repetition of other musical parameters such as dynamics, instrumental groupings/orchestration types, lyrics, tempi and of other structural features.

Figures 10 and 11 provide musical examples of translational symmetry about a horizontal and vertical axis, respectively. Figure 10 has an ascending perfect fourth as the interval of translation, and Figure 11 a dotted minim.

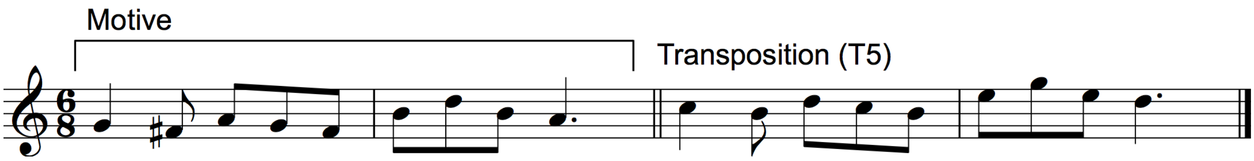

Figure 10: Translation about a horizontal axis (transposition).

Figure 11: Translation about a vertical axis (repetition)

2.4 Rotational symmetry

Rotational symmetry resembles translation, only instead of the vector of relocation being a straight line, the symmetry occurs about an angled path. When that path is circular, it is said the object has a specific number of *degrees of rotational symmetry*. This is calculated by 360÷*n* where *n* is the angle of rotation.[17] Figure 12 is built from the same object used in Figures 1, 2, 7 and 8, with rotary translation applied at 120° angles. The resulting object thus contains three degrees of rotational symmetry.

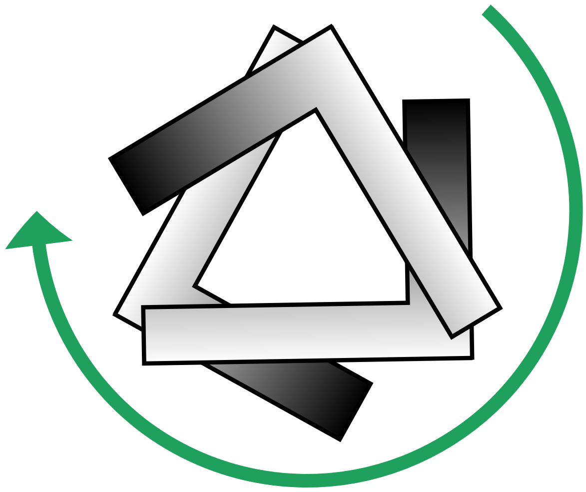

Figure 12: Rotational symmetry showing three degrees of rotational symmetry.

Rhythmic displacement, though having the appearance of rotational symmetry when geometrically represented on the circular timeline illustrations in this dissertation, is translation by definition. Benson and Solomon cite a number of musical examples of rotational symmetry that are also arguably translational in nature, as they involve repetition about horizontal (pitch) or vertical (time) axes.[18] If the definition of axial orientation is consistently maintained, the rotational symmetry of Figure 12 resists immediate musical analogy.

One musical analogy that conforms to the established pitch/time dimensions and has

17 The rotational symmetries that allow an object to be mapped to itself via rotation are also referred to as *cyclic order* in mathematical group theory. Note that the degrees or *orders* of symmetry are finite and require *n* to be a factor of 360, with the final result including the null rotation of 0°.

18 Such examples certainly have wave-like or cyclical appearance when diagrammatically or notationally represented, such as the Alberti bass line pattern, trills, and the four-note inversionally-symmetric ostinato from Ravel's *Rhapsodie Espagnole*. Refer to Weyl p.39–80 for numerous examples of rotational symmetry in art and nature, many of which combine bilateral and rotational symmetry.

rotational symmetry is the acoustic nature of pitch itself, as it involves periodic oscillation of sound pressure waves. This effect is observable via oscilloscopes, harmonographs and computer imaging software.

To avoid contradiction, Madden constrains the operation of rotation to ±90°, and applies the category to situations where chordal (simultaneous) material is reorientated so that rhythmic order (horizontal direction) is applied, and vice versa (29). In this scenario melodies would become chords, and chords would become melodies or arpeggios. Madden defines rotation of this type as *verticalisation* and *horizontalisation* (respectively), though other composers use the same terms to describe more self-similar approaches.

Composer Robert Davidson employs *verticalisation* of melodic/rhythmic material into harmonic, arrhythmic material, and the *horizontalisation* of pitch material into linear patterns in his compositional process (Davidson 6). In the isomorphic approach to verticalisation, melodic primary material “is gradually transformed into a single extended pitch by a process something like dissolving a substance in water until saturated” (7). Such parametrical transfers are distinct from localized occasions of symmetrical rotation, in that they operate on a deep and fundamental level, being the primary organizing principle of compositional form. Steve Reich’s *Come Out* illustrates verticalisation, whilst Davidson’s *Spin* illustrates horizontalisation.

If compositional procedure were to be considered in a less rigorous definition of *rotational symmetry*, then *isorhythm* of the fourteenth century Flemish composers (such as Machaut and Dufay) is a contender for the category. The isorhythmic procedure draws from two groups with different membership size, one group being the pitch content (*colour*) and the other rhythmic duration (*talea*). Melodies are created by drawing from these two out-of-phase groups.[19] This process is illustrated in Figure 13, with groups of five members (A through E) combined with groups of eight members (1 through 8). The two axes represent pitch and duration.

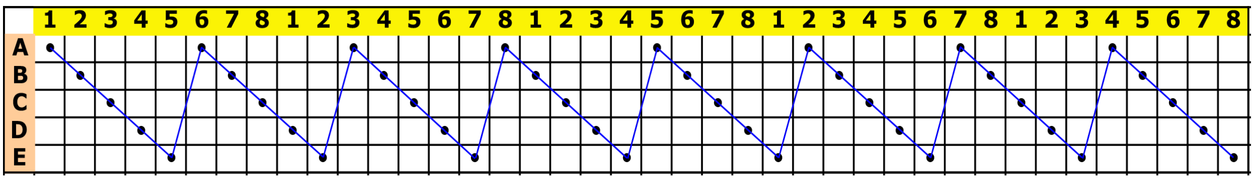

Figure 13: Isorhythmic procedure combining five members (A through E) with groups of eight members (1 through 8).

19 The sixth movement of Messiaen’s *Quatour pour la Fin du Temps* contains a modern example of the isorhythmic technique (in section F).

This diagram resembles a symmetrical *frieze pattern* (categorised as *p111*). Frieze patterns demonstrate translational symmetry, as they always involve repetition of a geometric pattern in one direction (Benson 303). (Frieze patterns are discussed in section 6 of this chapter.)

The rotational nature of the permutation of the two component cycles is more pronounced in the following diagram, which uses spirographic tools to trace the orbit of a 30-tooth cog within a 48-tooth cog to create the same 5:8 ratio illustrated in Figure 13.[20]

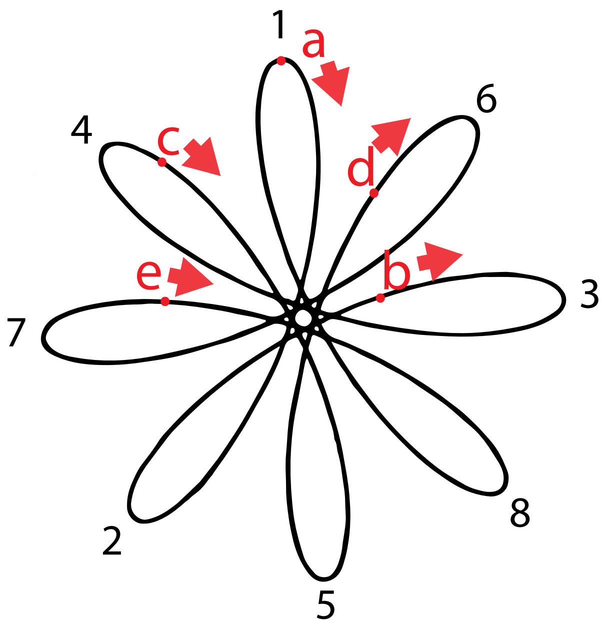

Figure 14: Isorhythmic procedure combining five members (A through E) with groups of eight members (1 through 8), illustrated as a spirograph.

Figures 13 and 14 reveal that the geometric representation of rotational symmetry often reveals other kinds of plane symmetry, including the reflection of bilateral symmetry and the repetition of translational symmetry.[21] Such *compound symmetries* are discussed in section 6 of this chapter.

In Johnson's *Self-similar melodies*, a number of approaches to counting in circles are presented that contain further examples of rotational symmetry in compositional method.

20 The inspiration for this type of diagram is derived from Ashton's book of *harmonographs*, being a visual tool that was invented in the nineteenth century for studying harmonic relationships. *Chladni patterns*, named after the German physicist Ernst Chladni (1756-1827) reveal the same phenomenon.

21 Weyl also recognises the natural blending of the categories of symmetry (41).

The repertoire and performance practice of the harp of *Nzakara* demonstrates another approach to rotational symmetry in music.[22] This African harp has five strings, always struck in pairs as dyads. The following diagram illustrates the possible mathematical combination of dyads.

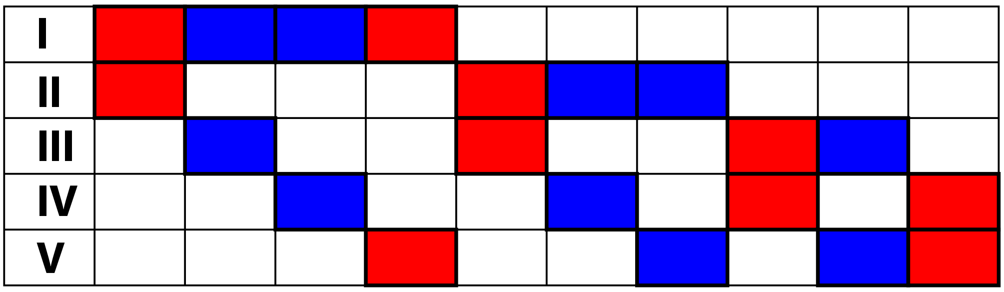

Figure 15: Dyad combinations possible on the five-string harp of *Nzakara*. Strings are labelled in roman numerals. Nonadjacent pairs are coloured blue.

The harp strings are considered to have a cyclic (rather than linear) order, such that the lowest string (5) is considered adjacent to the highest (string 1). Furthermore, the strings are only plucked in nonadjacent pairs, which reduces the number of dyad combinations from 10 to 5, illustrated by the blue colour coding in Figure 15. These allowable pairs are labelled 1 to 5 in Figure 16.

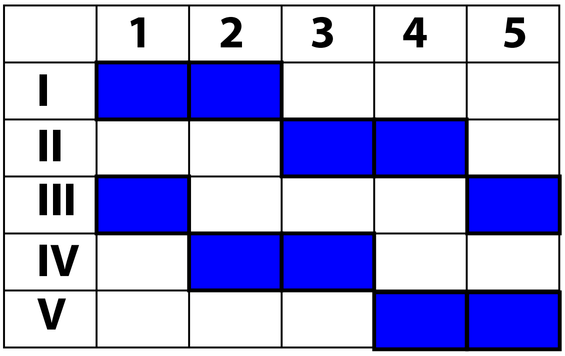

Figure 16: Five legal dyad combinations used on the five-string harp of *Nzakara*.

The following diagram interprets Chemillier's transcription of the *Nzakara* melody known as *limanza*, based on the dyad combinations identified in Figure 16.

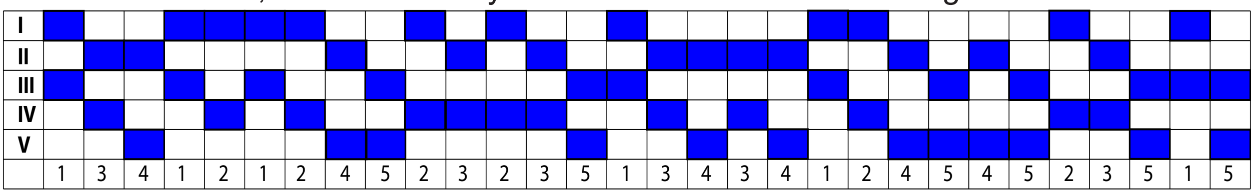

Figure 17: *Limanza* harp melody. The bottom row shows the dyad labels per Figure 16.

22 The *Nzakara* people populate the Central African Republic and Congo. For a detailed look at the *Nzakara* harp and the mathematics of its repertoire and performance, refer to Benson, Gregory, and Chemillier.

Chemillier observes that the texture created by this traditional harp melody is a canon (613). However the translational symmetry is not simply free imitation, but rather based upon a number of rules that led to rich symmetrical construction of a *cyclical ladder*.[23] These rules include the avoidance of repeated successive dyads, and the occurrence of unique dyads at the start of each cycle.

The symmetrical design is revealed by rotating the first two dyad pairs (1 and 3) shown in Figure 17 to the end of the series. Figure 18 adopts this necklace and then partitions the 30-dyad melody into five groups of six dyads, illustrated by the colour-coded dyad pair numbers in the bottom row.

Numerical analysis of the first group of six dyads in this rotation of the melody reveals that each successive group is created by transposing the string order up one string (+1 mod 5, considering the aforementioned cyclical string order).[24] Hence the original dyad motive <4-1-2-1-2-4> becomes <1-3-4-3-4-1> in the third iteration (+2 mod 5).

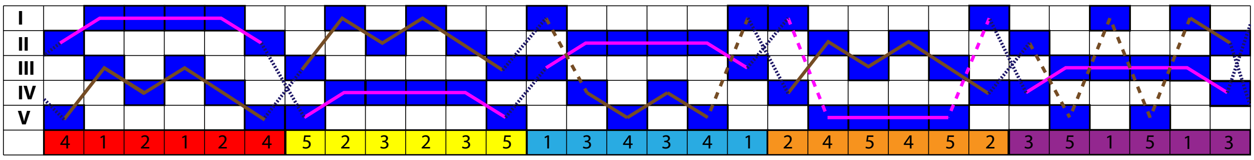

Figure 18: *Limanza* harp melody rotated and partitioned into five groups of six dyads, with plot-lines tracing the motives and their canonic treatment. The bottom row shows the dyad labels per Figures 16 and 17.

Figure 18 illustrates a number of symmetries. By considering the dyads as two voices, the lower voice can be seen trailing the upper voice by 6 attacks throughout, being the interval of translation of this canon. Furthermore, only two types of motive are used in the canon, illustrated by the brown and pink plot-lines.[25] This binary form is at odds with

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23 Chemillier found that each allowable core sequence of dyads is made up of a generator followed by four successive translations of the generator modulo 5. Gregory explains that as the pattern returns to the beginning of the core after five translations, each harp tune is a perpetual cyclical ladder of successive translations (Gregory 29).

24 The directions of ‘up and ‘down’ in this discussion align with the string numbers I through V, for sake of consistency, and not necessarily reflect the direction of pitch.

25 The dotted brown and pink plot-lines identify voice-crossing that results from the mod 5 operation that inverts the direction of movement. The dotted black lines are used to distinguish the five joins between the six-dyad motives of each type (coloured consistently brown or pink), which often also cross voices but maintain the canonic relationship throughout.

the five sections of the cycle, and the canonic imitation between the two voices infers a kind of seamless rotational symmetry through their combination, creating an effect not unlike the aural illusion of a *Shepard tone* scale. This effect is further enforced by the displaced starting point of this canon, rotated for clarity in this diagram.[26] It is notable that the <4-1-2-1-2-4> dyad motive itself contains a kind of rotational symmetry, as illustrated by the Möbius strip illustration of the sequence in Figure 19.

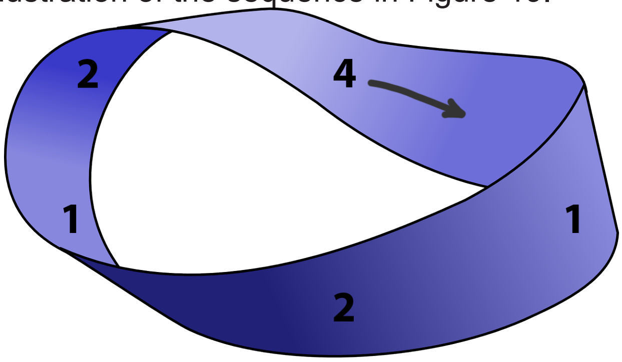

Figure 19: Symmetry of the <4-1-2-1-2-4> dyad motive illustrated upon a Möbius strip.

Gregory unveils the mathematical proofs behind the complex procedures behind the canonic *Nzakara* harp melodies. Though she acknowledges the impossibility of reducing the complexity of composition and its performance to a single algorithm, Gregory shows that of the 3,125 possible sequences of note pairs, the combined rules reduce the harpist's lexicon to twenty unique harp songs, assuming that one can commence a sequence at any point in the cycle (37).

Figure 20 illustrates the harp melody on a timeline to reveal the rotational symmetry that arises from the cyclic shift of the <4-1-2-1-2-4> generating motive. The same colour coding is applied from the bottom row of Figure 18.

26 Like so many ostinati in African music, the *Nzakara* harp melody is only meaningful as a repeated pattern, and furthermore introduces destabilizing elements in its fabric in such a way as to make it a dynamic, syncopated, dance-orientated pattern (Agawu 9). Chemillier reveals the *Lyndon words* that are the mathematical basis of such periodic musical structures (612). The overall patterns may consequently be considered *necklaces*. See Chapter 3.7 for a discussion about *necklaces* and *bracelets*.

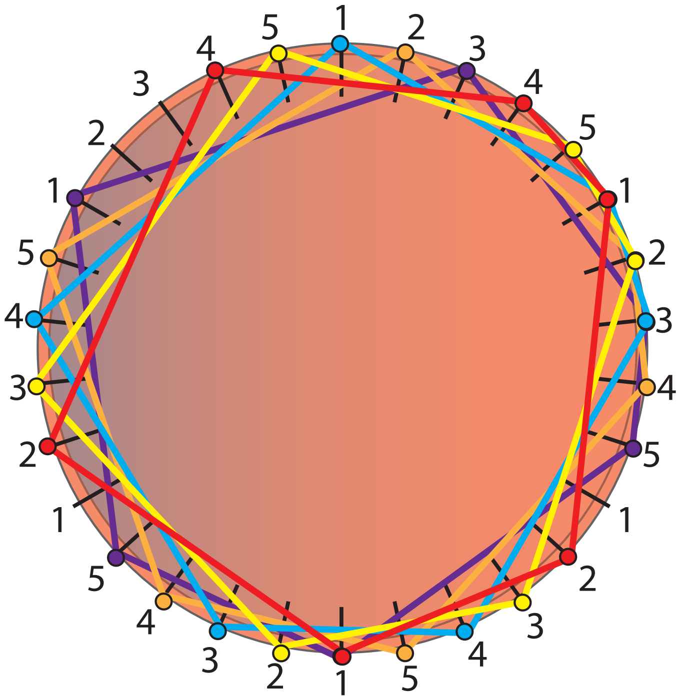

Figure 20: Timeline representation of the *Limanza* harp melody as a sequence of five groups of six dyads. In order, the sequences are colour-coded red, yellow, blue, orange and purple, with the dyad identifiers reading clockwise as six lots of five.

I have identified how the compositions and performance practice of the harp of *Nzakara* demonstrates another rich and creative manifestation of rotational and translational symmetries. Through the application of geometric tools I have illuminated the principles of proportionality, ambiguity of direction, cross-accentuation, thematic economy, and inertia arising from cyclical patternings—traits that are consistent with African music (Escot, "The poetics of simple mathematics in music" 36).

2.5 Shear symmetry

Shear symmetry exists when any of the aforementioned categories are applied in an approximate manner, with transformations occurring non-uniformly across the *x* and *y* dimensions. The following figure illustrates shear symmetry applied to bilateral and translational symmetries. Musical examples include when a melodic motive is transposed (translation about the horizontal axis) but some of the intervals are modified, such as when a step becomes a leap.

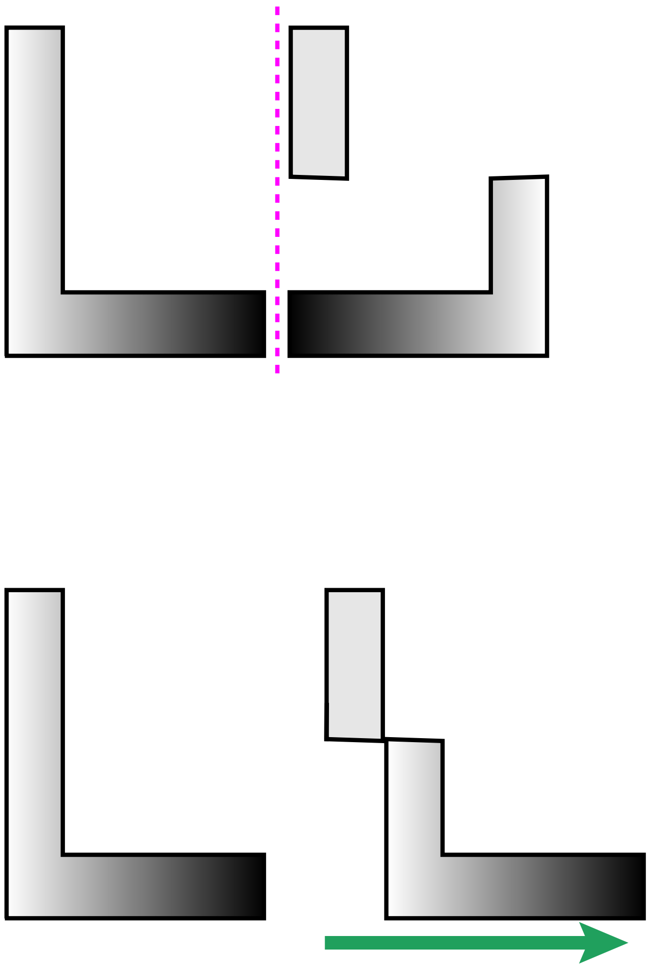

Figure 21: Shear symmetry applied to mirror reflection (top) and translational (bottom) symmetries.

Shear also applies in self-similarity where scaling transformations in the *x* and *y* dimensions are not uniform, also known as *affine transformations* (Madden 5). Musical examples include when rhythmic augmentation or diminution is applied in a non-uniform manner, per Messiaen's *inexact* category (Messiaen 19).

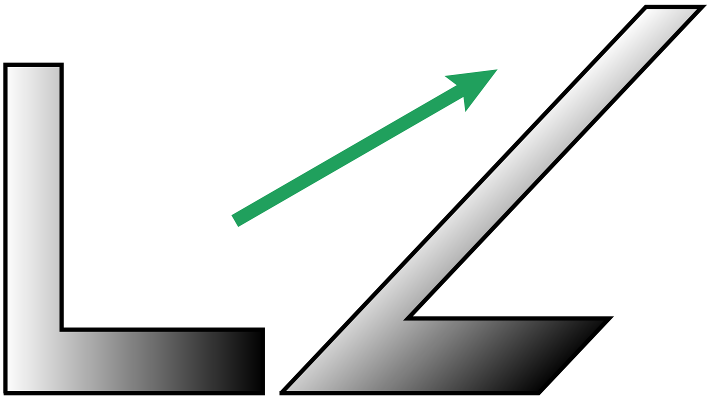

Figure 22: Shear symmetry applied to scaling.

2.6 Compound symmetry

Symmetry may be *simple* or *compound*. Compound symmetry combines multiple types of symmetry, such as *glide reflection*. In this category, bilateral reflection is applied and then translation is applied along the axis of symmetry. Musical examples are common, because operations typically occur across pitch and time. One example is when a melodic motive is followed by an inverted (reflected) version in successive bars, as in the subject and counter-subject in Figure 6. Aside from this example of counterpoint, melodic sequencing and parallel harmonic motion are examples of compound operations,[27] as translation is occurring on both the axes of pitch and time, and the result is arguably self-similar in nature.

Figure 23: Translation about vertical and horizontal axes (melodic sequence).

Compound symmetry is best summarised by the seven categories of *frieze patterns*, named after the symmetrical decorative architectural patterns found below roof lines.[28]

27 Solomon, Larry. "Symmetry as a Compositional Determinant." 2002. Accessed 27 Apr. 2017 <http://solomonsmusic.net/diss4.htm>.

28 The seven types of frieze pattern are categorised by the International Union of Crystallography. Clair, Bryan. "Frieze Patterns." Math and the art of MC Escher. 2012. Accessed 3 May 2017 <http://mathstat.slu.edu/escher/index.php/Frieze_Patterns>.

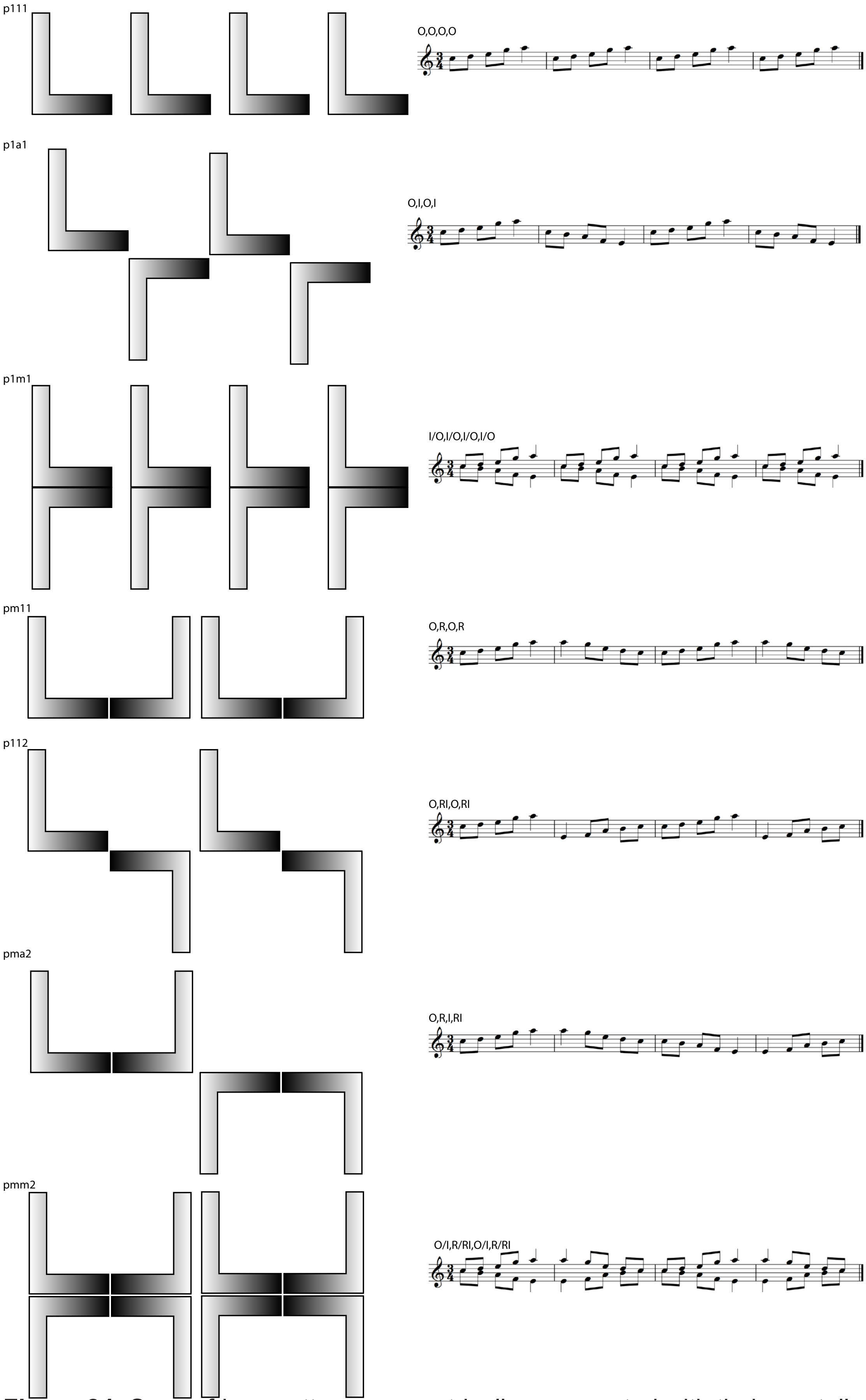

Figure 24: Seven frieze patterns geometrically represented with their crystallography categories, and melodically represented with their melodic transformation categories.

Frieze patterns feature bilateral and/or rotational symmetry, always combined with translational symmetry, as friezes inherently apply repetition of a geometric pattern in one direction. Figure 24 illustrates the seven frieze pattern categories geometrically alongside a melodic realisation. I have identified the relevant melodic transformation categories for each, where O stands for original, R for retrograde, I for inversion and RI for retrograde inversion.

2.7 Indian counting sequences

My study of Indian classical music has revealed that counting is intrinsic to the art and pedagogy, and the music requires that one negotiates complexes of counting that contain symmetrical and sometimes recursive properties. Whether they be taught explicitly supported by verbalised theory (such as *ginti* of the North and *korvai* of the South), or implicitly through rote learning and modelling from a *guru*, the rhythmic counting sequences of Indian classical music embody syntactical rules that are identifiably Indian. South Indian classical music has a particularly rich system of rhythmic procedures, coupled with a pedagogy that incorporates geometric representations, a systematic performance practice that explores the potential interaction of phrase scaling and rhythm cycle, and the onomatopoeic language of recitation known as *konnokol* (*bol* in North India).

Indian counting is premised on the concept that rhythm follows an orderly sequence of equidistant and atomic units of time that are grouped into words and sentences in a grammatical manner. This concept aligns with the additive rhythmic model of time.[29] Furthermore, Indian rhythm is inherently cyclical, and when such counting sequences are overlaid on time cycles, they effect the listener’s sense of time flow, inevitably changing it from the stable, linear, predictable effect of an unadorned ostinato or groove in a particular tempo, to a feeling of pushing and pulling, acceleration and deceleration, often with opposing forces and ambiguity as the counting sequences juxtapose the underlying metre or *tala*. Indian classical music refers to the sensation of rhythm and movement as *laya*, and although *laya* refers principally to tempo, the term broadly represents all the interrelationships that contribute to an inward orderly flow, even on a philosophical level (Sankaran 25).

Specific counting sequences such as North Indian *tihai* and other Indian rhythmic

29 Refer to Chapter 3.4.

structures are identified in this dissertation and in the accompanying scores using numerals that represent the value of each group. A grouping of seven may be annotated with the numeral 7 beneath the first onset of the group, or with any number of respective sub-groupings such as <2-5>, <2-2-3>, an so on. The inter-onset intervals <2-2-3> can also be thought of as a group of three onsets, one per sub-group, or interpreted with any number of subsidiary attacks such as <2-2-1-1-1>.

The point is that musicians trained in Indian classical music—and those that realise my intercultural contemporary compositions—are required to have flexibility to infer appropriate groupings of Indian counting sequences expressed often in generic syntactical form. Musicians trained solely in Western notation and European classical art music may naturally be inclined to interpret <2-2-3> as follows.

Figure 25: One realisation of a grouping of seven expressed as <2-2-3>.

However when it comes to Indian counting sequences, and Indian classical music in general, such a sequence must require such a flexible conception as to be able to be realised with a range of sub-groupings, applied in any subdivision, to any metre, and from any starting position. What follows is but one of a myriad of possible examples of the same <2-2-3> motive, demonstrating variation of duration, subdivision, metre and starting position. It can be seen that significant rhythmic acuity is demanded from the performer.

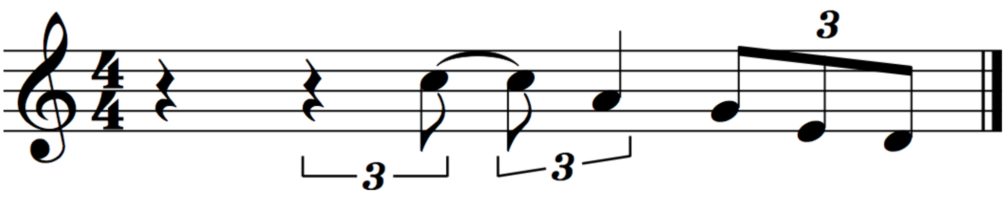

Figure 26: An alternative realisation of a grouping of seven expressed as <2-2-3>.

The following categories of Indian counting sequences commonly appear in my compositions, and are examples of symmetry that I have assimilated into my compositional voice through my Indian classical music studies. In order to conceive, differentiate and explain the structure of Indian counting sequences, one refers to the number of *stages* that form a sequence.[30] The stages of a counting sequence can be thought of as intelligent partitions that reveal the pattern and symmetry from the constituent lines.

Tihai / Mora

A *tihai* (North) or *mora*[31] (South) is a cadential phrase sounded three times, which can be formalised simply thus:

A
A
A

The simple tripartite translational symmetry of this form conceals the potential complexity of its inner structure and application. *Tihai* can be of various durations, from small gestures that fit within a few beats, through to huge multi-cycled self-similar structures that last a hundred beats or more. Being cadential, *tihai* are calculated to span specific and structurally important periods of time, often culminating on a down-beat (*sam*).

Tihai, as with all material based on sequences of groupings, rely upon clarity of their groupings for structural intelligibility. Practical realisation of clear groupings is aided by the use of accent, dynamic contouring and timbre, and for pitched instruments, the choice of notes. Perceptibility of groupings is dependent upon discrimination in this regard, and in the case of such repetitive forms as *tihai*, critically so.

Structural intelligibility of groupings of notes is also enhanced when gaps or rests are interpolated into the material, known as *karvai* in South Indian music. *Karvai* act functionally as separations but are not necessarily silent, often being filled with a sustained sound or subordinate articulations.[32] The separation function relates directly to the concept of *interval of translation* discussed in section 3—an appropriate analogy

30 The term is found in Nelson, for example p.54.

31 The term *arudi* is used as a simile for *mora* by many authors and practitioners of South Indian music (refer to Sankaran; Nelson). It is differentiated in meaning by others, but remains a cadential rhythmic device with an internal logic.

32 Refer to Sethares chapter 3.2 for a discussion of ways hierarchy may arise from regular successions.

given the predominance of situations where *karvai* are applied to symmetrical structures. Such structural gaps may be conceptualized differently in North Indian music.

In this dissertation and in my composition folio, numerals in brackets represent the value of the gaps between primary groupings. The following is an example of a *tihai* with a total value of 32 and gaps of 4.

8 (4) 8 (4) 8

A South Indian interpretation would articulate the first pulse of each *karvai* or gap, whereas in a North Indian interpretation they are often considered wholly silent. Furthermore, the *tihai* of North Indian classical music tend to consider the final articulated pulse in the last grouping to be congruent with the destination pulse (often *sam* or beat 1), thus containing that arrival point in the sequence itself. The mental shift from the South to North Indian approach therefore requires the previous sequence to be reinterpreted as follows.

9 (3) 9 (3) 9

This sequence has a total value of 33 inclusive of the pulse that articulates the destination of the sequence, rather than 32 in the prior example. Remember, these are the same compositions![33] *Tihai* in my composition folio incorporate both approaches.

It is not uncommon to find *tihai* in traditional North Indian melodic and rhythmic phrases that contain these tripartite structures with smaller self-similar tripartite structures contained within each phrase.[34] Such a *chakradar tihai* can be represented thus:

ABBB
ABBB
ABBB

---

33 Having been trained in North Indian tabla performance and repertoire, I have historically tended to think in these latter terms. However, recent increased exposure to the South Indian music has won my appreciation of the former approach, and I am finding benefit in being multilingual in this regard.

34 Nelson defines the South Indian equivalent—*compound mora* (Nelson 53).

Sometimes colloquially referred to as a *no-hai* (*no* meaning ‘nine’ in Sanskrit), the leading material (A) can also be omitted from the *chakradar tihai*, to simply create:

AAA
AAA
AAA

Seemingly infinite varieties of *tihai* exist. Some *tihai* have gaps (*karvai*) between each part (e.g. *damdar*) whilst others run continuously from one repetition to the next (e.g. *bedam*). Some *tihai* are ascribed specific categorization (e.g. *farmaishi*) whilst others only intuitively correspond to the general definition of *tihai* as a cadential form, containing more than three macroscopic stages, for example.

The following two symmetrical sequences are likely to be categorized as *tihai* in the North, even though the first sequence varies the motive length and the second is a four-stage structure. Both compositions have a total duration of 48 pulses.

5 (15) 6 (15) 7

Formal representation:
A (B) A+1 (B) A+2

This sequence is self-similar (or arguably self-affine) due to scaling of the A motive. Its translational symmetry is thus shear.

3 9 15 21

Formal representation:
A 3A 5A 7A

This sequence is self-similar due to true (proportional) scaling of the A motive. It could be realised <1-1-1-3-3-3-5-5-5-7-7-7>.

Yati

Sequences called *yati* are patterns that are easily represented by simple geometry. These geometric representations exhibit bilateral symmetry, shear symmetry, translation, or a compound of symmetries. Whilst somewhat present in the practice of North Indian Classical music, the theory of *yati* belongs to South Indian music, and features a more rigorous and systematic approach to its application (Clayton 166). *Yati* categories bear names that correspond to their visual likeness. The integers in the following figures represent units of pulse, and may also have corresponding pitches. For example, 12345 may be articulated <2-1-1-1>, <3-1-1>, or with a run of semiquavers F-G-A-B-C with onsets <1-1-1-1-1>. (Refer also to Sankaran 29–32.)

1. *Gopuccha yati* (‘cow’s tail’).

The final onset in each line of the sequence tends to make a syntactical line towards the final 1, as does the first onset. Together, these lines of time form a goal-orientated cadence that ultimately focuses on the convergence on the final 1. *Gopuccha yati* resemble triangular figurate numbers and possess bilateral symmetry.[35]

In practice, this and all following categories of *yati* tend to incorporate a gap (*karvai*) of consistent length between each line. These gaps do not disturb the symmetry, but rather heighten the effect of the pattern by distinguishing one line from the next.

12345
1234
123
12
1

Figure 27: *Gopuccha yati*.

35 See Chapter 3.9 for a discussion of figurate numbers.

2. *Srotogata yati* (‘stream becoming a river’).

This sequence creates a goal-orientated cadence yet is more diffuse than its retrograde form, *Gopuccha yati*. This is because the goal orientation towards the final onset (5 in the figure below) tends to be less predictable, as the focal point of a such a growth sequence is potentially infinite, and therefore relies upon pitch and metric context for its syntactical logic. *Srotogata yati* also resemble triangular figurate numbers and possess bilateral symmetry.

1
12
123
1234
12345

Figure 28: *Srotogata yati.*

3. *Sama yati / Pipilika yati* (‘ant row’).

This sequence lacks the growth that comes with the pronic progressions of the previous categories, and features simple translational symmetry. *Tihai* (*mora*) are three-stage *sama yati*.

12345678
12345678
12345678
12345678
12345678

Figure 29: *Sama yati.*

4. *Mridanga yati* (barrel-shaped like the *mridangam* drum).

This sequence balances the goal-centred properties of *srotogata* and *gopuccha yati* in a palindromic sequence. Anticipation of the final convergence is established after progressing beyond the mid-point (12345 in the figure below). *Mridanga yati* possess bilateral symmetry. In terms of figurate numbers, their form resembles rectangular numbers (being two triangles).[36]

1
12
123
1234
12345
1234
123
12
1

Figure 30: *Mridanga yati.*

5. *Damaru yati* (hour-glass shaped like the *damaru* drum).

This sequence is an inversion *mridanga yati*, and also balances the goal-orientated properties of *srotogata* and *gopuccha yati* in a palindromic sequence. Anticipation of the final convergence is established after progressing beyond the mid-point (1). *Damaru yati* possess bilateral symmetry. In terms of figurate numbers, their form resembles rectangular

36 The shared diagonal axis (being the mid-point of the sequence) is shared by the two halves in this case. This means the total value of any *mridanga yati* (abbreviated MY, and excluding gaps) can be calculated by modifying the rectangular number formula MYn = (n(n+1)) - n. See Chapter 3.9 for a discussion of figurate numbers.

numbers (being two triangles).[37]

12345
1234
123
12
1
12
123
1234
12345

Figure 31: *Damaru yati.*

6. *Vishama yati* ('random').

This sequence balances the predictability of the *sama yati* with an unsystematic arrangement of durations. Used sparingly, it could introduce a requisite element of surprise. It possesses asymmetry.

37 The shared mid-point onset of the sequence (being the apex of the intersecting triangles) is shared by the two halves in this case. This means the total value of any *damaru yati* (abbreviated DY, and excluding gaps) can be calculated by modifying the rectangular number formula DYn = (n(n+1)) - 1.

12
1
12345
123
123456

Figure 32: *Vishama yati.*

The geometric representation of *yati* clearly illustrates the principles of contraction (rhythmic diminution), expansion (rhythmic augmentation), and symmetry (contraction-expansion, expansion-contraction, and stasis). These principles are universally applicable and not unique to Indian Classical music. Schillinger refers to similar geometric forms in his melodic theories, and also employs diagrams to represent their physical and musical forms. Schillinger uses the terms *centrifugal* and *centripetal* in his categories (287), and Nelson also talks of centripetal forces in his cyclic diagrams (16). In reference to the cadential nature of these types of sequences, Schillinger writes "...without resistance preceding such a climax, does not produce any dramatic effect" (283).

Yati can be scaled or adjusted in size to fit desired duration. Variables include the subdivision,[38] metre, and use of *karvai* to pad each line. Different *yati* can also be combined for more complex effects. The following example is a *gopuccha yati* that contains *sama yati* repetition on the microscopic level. The reduction takes place from 7 to 1. Note that internal repetition has no influence on the number of stages.

38 Subdivision can be adjusted uniformly for the entire pattern or can be varied at logical structural points to create tempo-modulation effects.

1234567
1234567
1234567
123456
123456
123456
12345
12345
12345
1234
1234
1234
123
123
123
12
12
12
1
1
1

Figure 33: 7-stage *gopuccha yati* with *sama yati* repetition on the microscopic level.

Any specific sequence can also be realised in a multitude of rhythmic ways. Sub-groupings that retain a consistent number of onsets, even for a portion of the sequence, yield a complex internal patterning that is easily perceptible by the listener. The 7-stage *gopuccha yati* illustrated above could be realised with the following sub-groupings:

<2-2-1-1-1> (three times)

<1-2-1-1-1> (three times)

<1-1-1-1-1> (three times)

<1-1-1-1> (three times)

<1-1-1> (three times)

<1-1> (three times)

<1> (three times)

Note that the decisive usage of sub-groupings maintains an identical grouping cardinality (of five) for the first three stages, despite the decreasing stage duration. South Indian classical music often features this kind of *pancha nadai*[39] phrasing, which is structurally recognisable and has an internal logic that arouses expectation and anticipation in the listener.

Korvai

Meaning "strung together", *korvai* compositions typically consist of a sequence of phrases that are not identical, but that follow a logical pattern of development. *Korvai* possess two sections, the first often creating rhythmic opposition to the metre (*tala*), and the second containing a repeated *mora* cadence.[40] The following 32-pulse composition is typical of *korvai* structure.

2 (2)
3 (2)
4 (2)
5 (1) 5 (1) 5

Figure 34: *Korvai* with 32 pulse duration.

39 Sanskrit for "five divisions", *pancha nadai* phrasing sees the basic *solkattu* for five "ta di ki na dom" transformed to accommodate larger groupings through insertion of rests between syllables.

40 In North Indian classical music, *tukra* and *paran* are similar in structure, though the first section tends to be more poetic, and does not necessarily follow a strict logic in its counting pattern.

Korvai resemble hybridizations of South Indian counting sequences (Figure 34 being a *srotogata yati* followed by a *mora*), and, like the *gat* compositions of the North Indian *tabla*, are poetic compositions often handed down from master teacher (*guru*) to pupil (*shishya*) (Naimpalli 58). Embedded in such compositions are the aesthetic values of the *guru* and subsequently their school (*gharana*) and lineage (*guru–shishya parampara*). Compositions such as *korvai* place primary importance on the intrinsic beauty and symmetry of their logic.

2.8 Balinese kotekan

In the process of analysing and performing the unique interlocking *kotekan* patterns of Balinese *gamelan gong kebyar* orchestra, I have discovered symmetrical and self-similar relationships that have influenced my composition. Some of these influences are apparent in my folio composition *Binary Times*. Essential to the superstructure of *kotekan* is the relationship of two parts—called *polos* and *sangsih*—that combine seamlessly to create the tapestry of the whole.[41] *Kotekan* presents a continuous stream of pulses usually performed at high speed, and *polos* and *sangsih* are typically arranged and executed in such a way as to share pitch material and a blend of on- and off-beat rhythms in a way that prioritizes the resulting combination over the individuality of the parts. That is, the two parts that comprise *kotekan* perform disjunct pitch material, whose combination makes a conjunct whole.[42] It is the symmetry of the tessellating *kotekan* patterns that is my present focus, rather than an in-depth look at the art-form.[43]

41 In the examples provided *polos* and *sangsih* are notated as the lower and upper voice, respectively. *Polos* refers to the “simple” part that tends to track the contour of the underlying *pokok* melody, whilst *sangsih* refers to the “differing” part that sits above and interlocks with *polos* (Tenzer 214).

42 Balinese music is not the only genre to exploit the illusory aural effects that arise from high-speed interlocking parts. Kubik identifies the so-called *inherent rhythms* intrinsic to the instrumental music of Eastern and Central Africa, featured in compositions for xylophone and guitar (Kubik 33).

43 Refer to Vitale, Hood, Spiller and Tenzer for a closer look at *kotekan*, its principles, context and technique. Also note that the musical examples, though based upon established models, are original and do not represent traditional modes nor tuning systems of Bali. The examples provided present one rhythmic application to categories whose names derive from pitch organisation, and though the two are intertwined, the examples should not be taken as rhythmic exemplars of the category in which they appear.

The cyclical orientation presented in all these Balinese examples—and also the African ones discussed previously—is chosen to educe maximal insight for the Western reader, yet native practitioners may conceive the timelines in a contrasting orientation. Steele discusses the "end-weighted" nature of pitch and rhythmic orientation within both Balinese and Javanese traditional music. Here, the weight of the gong resolution typically directs Westerners to perceive that primary moment in the cycle as beat 1, yet native practitioners consider it the final beat 8 (Steele & Tenzer 230). A discussion of psycho-acoustics and cultural conditioning that contributes to these discrepancies is beyond the scope of this dissertation, suffice to say that it is the ambiguities that arise from these multiplicities of perspective that are intrinsic to the nature of *kotekan* and my interest in it.[44]

Nyog cag

Sangsih presents a temporal translation of *polos*, in that the parts are rhythmically offset from each other by a single unit of subdivision, in a duple metre, with no shared onsets. The two parts are effectively complements of each other, and by belonging to the category of *complementary rhythms*, take on a number of other interesting properties.[45] *Polos* and *sangsih* typically cross voices with interlaced pitches, and, coupled with equal attack weight between the parts, fast tempi, and self-similar scaling to the underlying *pokok* ground melody, all sorts of combing effects result that may disorientate the unaccustomed performer and listener. The result is a kind of perceptual rivalry and

44 For a discussion of the role of perception and cognition of time in Balinese music, refer to McGraw. The psycho-acoustic nature of melody in gamelan are discussed by Sethares (69).

45 Toussaint shows that two complementary rhythms with the same number of onsets are *homometric*—i.e. they share the same inter-onset interval vectors. Refer to Toussaint, "The Geometry of Musical Rhythm" chapter 23 for more on the topic of *complementary rhythms*. It is also worth noting that the performance practice of the *gangsa* instruments requires that each performer use their free hand to mute each struck note upon each successive onset. In this manner, each player's limbs are engaged in mirror relation to the other's (one dampens whilst one strikes). Each performer's limb pattern thus resembles a *smooth toggle rhythm*, being a cyclic rhythm that demonstrates complete balance between left and right limbs. Refer to Toussaint, "The Geometry of Musical Rhythm" chapter 29 for more on the topic of *toggle rhythms*.

multiplicity in perception not unlike the temporal features of the music of West Africa and the West African diaspora (Pressing, “Black Atlantic Rhythm” 298). A visual analogy to this perceptual rivalry is the *double image* technique of visual art. Salvador Dali’s *Slave Market with the Disappearing Bust of Voltaire* is one example, whilst Davidson relates his related experiments in convergence of time’s cycle and direction to the work of M.C. Escher and the unstable perception created by the geometrical form of the *Necker cube* (Davidson 85).

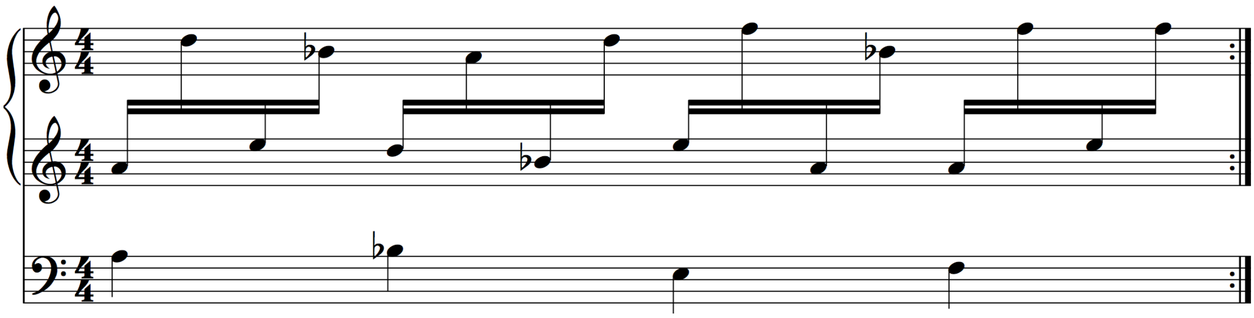

Figure 35: Example of *nyog cag kotekan* (top staves) with *pokok* (bottom stave).

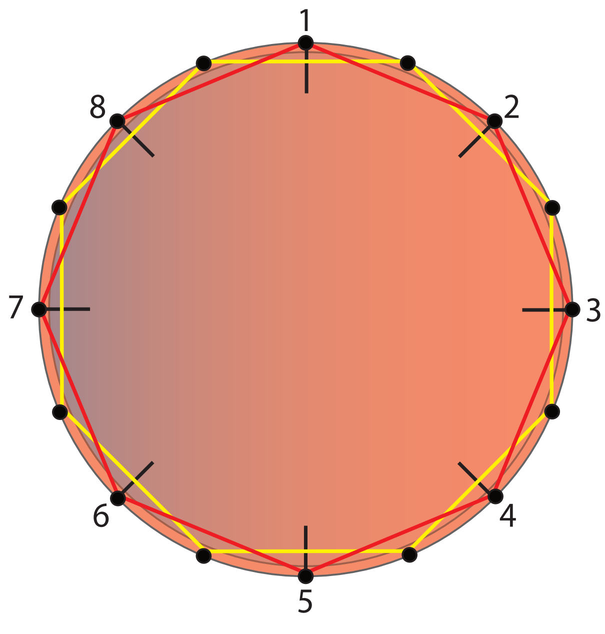

Figure 36: Translational symmetry of *nyog cag kotekan* represented on an 8-pulse timeline. *Polos* and *sangsih* are colour-coded red and yellow, respectively.

Nyog cag is a simple example of a rhythmic canon, used elsewhere in Balinese repertoire,[46] and found also outside of the genre in the music of Messiaen (see Messiaen 24). As all pulses in the cycle are sounded without unison onsets by the two voices, *nyog cag kotekan* is also a simple form of *rhythmic tiling canon*.[47]

Norot

Sangsih presents a temporal translation of *polos*, in that the parts are rhythmically offset from each other by a single unit of subdivision, in a duple metre, yet there are overlapping onsets/pitches due to the appearance of successive notes. Translational symmetry is still strictly present, though a different musical effect arises from the unisons, which typically envelope the underlying *pokok* melody. In the following example, *polos* and *sangsih* preempt the *pokok* pitch by three semiquavers every minim (Vitale 5).[48]

Figure 37: Example of *norot kotekan* (top staves) with *pokok* (bottom stave).

The timeline diagram below illustrates the bilateral symmetry of the <2-2-1-1-2> pattern of onsets in the *norot* example, and the translational symmetry arising from the displacement of *sangsih* by one pulse relative to *polos*.

46 *Kecak*, also known as the *Ramayana Monkey Chant*, is a famous example. Refer to the Folkways recording in the bibliography.

47 Refer to Toussaint, "The Geometry of Musical Rhythm" chapter 28 for more on the topic of rhythmic canons.

48 *Norot kotekan* typically sees *polos* and *sangsih* arranged in such a way as their pitches alternate between the *pokok* pitch and an upper neighbour pitch (Vitale 6).

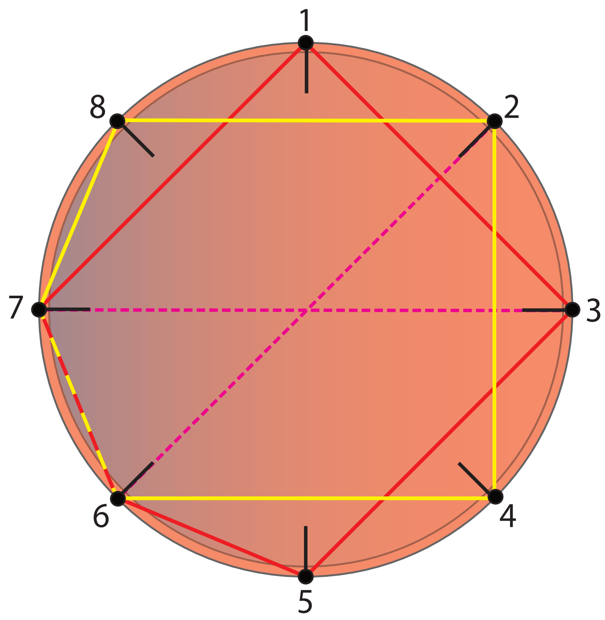

Figure 38: Translational and bilateral symmetry of *norot kotekan*. *Polos* and *sangsih* are colour-coded red and yellow, respectively.

Kotekan Telu

Meaning "three" in Balinese, the name of this category arises from the arrangement of three pitches in *polos* and *sangsih* around the underlying *pokok* melody. The example provided conforms with typical *telu* design in that the middle note (C) is shared between *polos* and *sangsih*, and each of these parts alternates a unison onset with the *pokok* melody (Vitale 6).

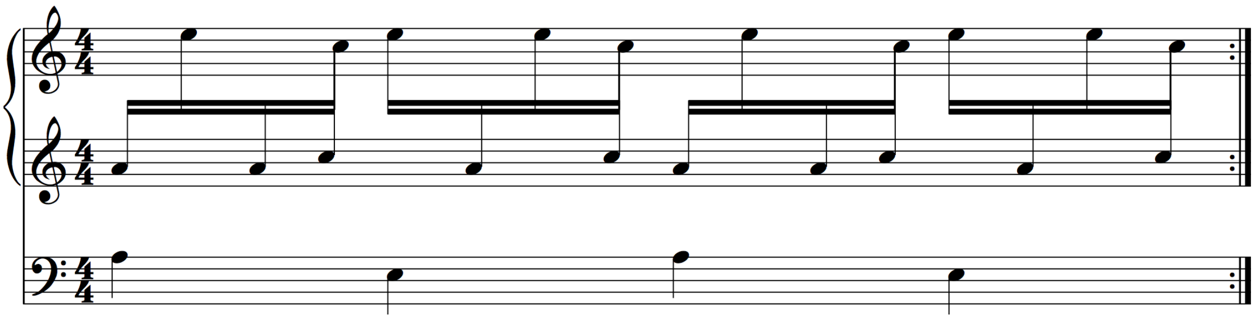

Figure 39: Example of *kotekan telu* (top staves) with *pokok* (bottom stave).

The symmetry of the rhythmic interlock of Figure 39 is notable. Figure 40 plots the *polos* and *sangsih* on a timeline and reveals that they are not merely translations of each other, but that the particular interlock arises from reflectional (bilateral) symmetry about a diagonal axis. The rhythms are effectively bracelets of each other. (See Chapter 3.7 for more on necklaces and bracelets.)

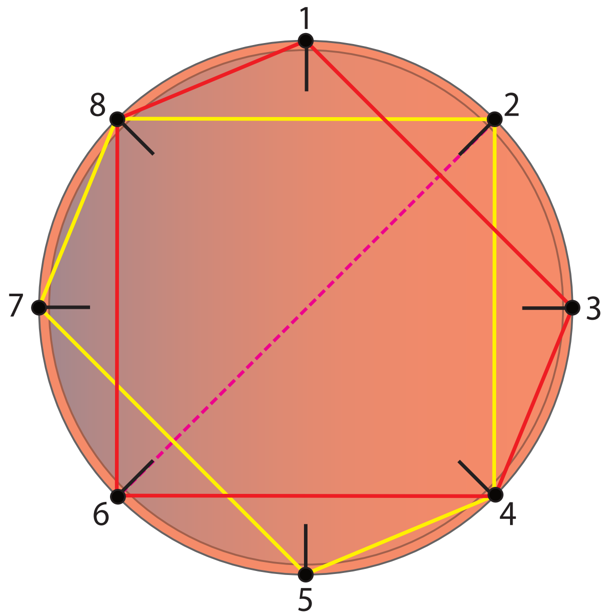

Figure 40: Translational and bilateral symmetry of *kotekan telu* represented on an 8-pulse timeline. *Polos* and *sangsih* are colour-coded red and yellow, respectively.

The inter-onset intervals of the two parts follow, listed from their first onset in the cycle (pulse 2 in the case of *sangsih*).

Polos <2-1-2-2-1>

Sangsih <2-1-2-1-2>

Given the cyclical nature of these rhythms as ostinati, one can consider the way the *polos* part wraps around on itself, much like the *Nzakara* harp melodies discussed earlier in this chapter. Though the duple nature of the cycle infers phrases with durations of powers of 2, I discovered a certain logic through partitioning a lengthy sequence of the <2-1-2-2-1> pattern into lots of three onsets, regardless of duration. Considered in this way, *polos* performs a periodic sequence of five 3-onset phrases, with a total period of 24 pulses (lasting three cycles).

<2-1-2>
<2-1-2>
<1-2-2>
<1-2-1>
<2-2-1>

Figure 41: *Polos* partitioned into five phrases of three onsets.

The nature of this cyclical ladder is highlighted in Figure 42, where the cells of the sequence are colour-coded to reveal the phases of <2-2-2-1-1> shared by each column.

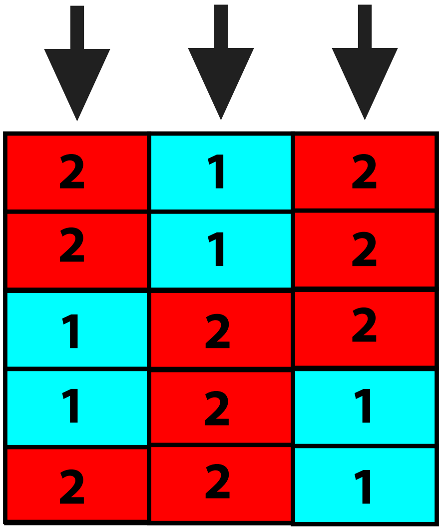

Figure 42: Box diagram of Figure 41 revealing the vertical translation of the inter-onset intervals <2-2-2-1-1> in *polos* across three columns.

Analysis reveals the *sangsih* line contains the same generating seed pattern as *polos*. Treating its inter-onset intervals <2-1-2-1-2> as part of a longer sequence built from 3-onset portions as per *polos* in Figure 41, the following pattern is revealed.

<2-1-2>
<1-2-2>
<1-2-1>
<2-2-1>

<2-1-2>
<2-1-2>
<1-2-2>
<1-2-1>
<2-2-1>

Figure 43: *Sangsih* partitioned into phrases of three onsets. Four phrases (totalling 19 pulses) precede the dividing line before congruence with *polos* (per Figure 41).

This comparison reveals a twisted kind of rotational symmetry in *kotekan telu* that sees *sangsih* perform a translation of *polos* that is displaced by 19 pulses (or two cycles plus 3 pulses). As I am treating the first onset of *sangsih* as the start of its sequence, yet it actually occurs on pulse 2 of the 8-pulse cycle, one would have to rest for 20 pulses (not 19) in order to perform *kotekan telu* effectively as a canon. The logic behind the symmetrical patterning is similar to the cyclical ladder of the *Nzakara* harp melodies discussed earlier in this chapter.[49]

Kotekan Empat

Meaning "four", the name of this category arises from the arrangement of four pitches in *polos* and *sangsih* around the underlying *pokok* melody, usually shared as two unique pitches for each part without overlap (Vitale 8). The simultaneities thus provide an opportunity for dyadic harmony (possibly triadic depending on *pokok*), and arpeggiation of four-note chords. I have found that this approach suits contemporary application to changing harmony.[50]

The examples provided are authored by band-mate Adam King, titled *Kotekan For Alit*. There are two players of pitched instruments (bass trombone and bass guitar, which appear as the top and bottom staves respectively). Each spontaneously selects a dyad pair for each section of the piece. King authored three approaches to *kotekan empat*, identified below as cycles 1, 2, and 3.

Figure 44: Cycle 1 of *kotekan empat* (from *Kotekan For Alit*, by Adam King).

49 A similar adjustment of phase was employed in the *Nzakara* harp example, where the first two dyad pairs were rotated to the end of the sequence before the periodic nature of the pattern was revealed. See Figures 17 and 18.

50 My *Tripataka* band-mates Adrian Sherriff, Adam King and myself have experimented with improvising in this manner upon jazz chord progressions.

The top (*sangsih*) line features a *reflection rhythm*. Reflection rhythms are created when a mirror-symmetric image of a rhythm (about some axis of symmetry) becomes the rhythm's complement (Toussaint, "The Geometry of Musical Rhythm" 207). This presence of this property can be verified through deconstruction of the *sangsih* line into its essential seed rhythm, being three onsets <1-1-2>. From here, following Toussaint's *alternation method*, the original line can be reconstructed by first distributing the three onsets between alternating hand-strikes, and then repeating the sequence with continued alternation in the following manner.

L		L			L		
	R			R		R	

Figure 45: Reconstruction of *sangsih* from *Kotekan empat* cycle 1 using reflection rhythm alternation method.

When this pattern is plotted on a timeline, bilateral symmetry is revealed about two axes. The same colour-coding is applied.

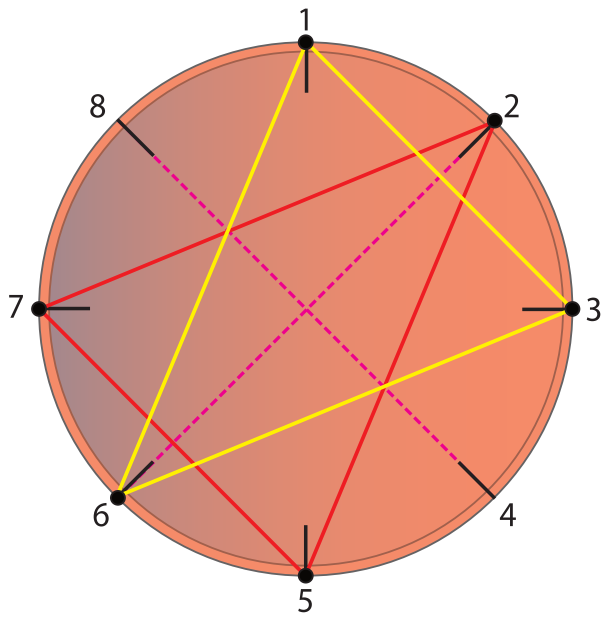

Figure 46: Translational and bilateral symmetry of *sangsih* from *Kotekan empat* cycle 1 represented on an 8-pulse timeline.

Figure 46 confirms the criteria are met for a *reflection rhythm*. Considering the yellow triangle represents the pitch B-flat and red represents pitch A, indeed this timeline matches the *sangsih* part precisely (see Figure 44).

Now that *polos* and *sangsih* in cycle 1 have been considered singly, some further observations about their combination can be made. The *polos* line is not intrinsically a reflection rhythm like *sangsih*. In *kotekan empat* the aforementioned dyadic simultaneities between *polos* and *sangsih* are harmonically important, and here in cycle 1 they appear as the tritone between E and B-flat.[51] Taking these simultaneities as structural cues provides the inter-onset interval series <2-3-3>, which when rotated as a necklace results in the Cuban *tresillo* <3-3-2>.[52]

Focussing on the arrangement of rests reveals a correspondence in both parts such that every semiquaver rest in polos aligns with pitch A in *sangsih*, and every semiquaver rest in *sangsih* aligns with pitch F-sharp in *polos*. I refer to this construct as the *gap dyad*, which combines with the harmonic dyad to contribute to the overall tapestry of the interlocking texture. The symmetry of these dyads is revealed in the following box diagram.

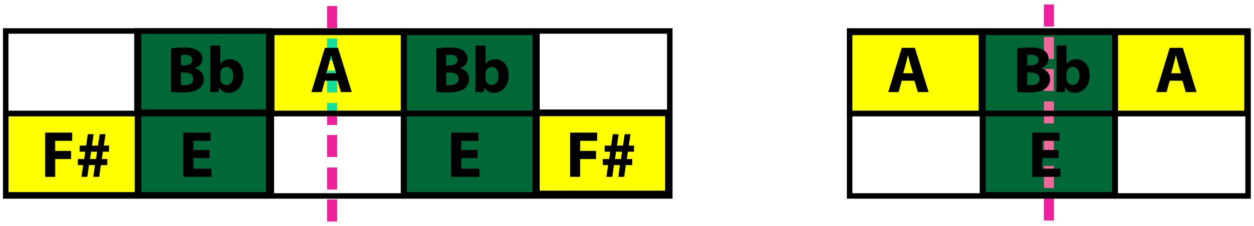

Figure 47: Box diagram illustrating bilateral symmetry of the *harmonic dyad* (green) and *gap dyad* (yellow/white) of *polos* and *sangsih* from *Kotekan empat* cycle 1.

Kotekan empat cycle 1 is rotated in this box diagram to commence on pulse 8 (with respect to Figure 44). The sequence is divided into two sections to further emphasise the bilateral symmetry of each, illustrated by the dotted pink line. The two sections are palindromes about the middle pulse.

The construction can now be examined further at a cellular level. The following legend defines the four shapes that comprise Figure 47, retaining the two cells containing the axes of bilateral symmetry as an integral design feature. Notice the L-shapes employ the types of bilateral symmetry about both axes examined in section 2 of this chapter (see Figures 1 and 2).

51 This span between the outer tones of the two parts is typically a fifth on the *gangsa* of the *gamelan gong kebyar* (Vitale 8).

52 Refer to Toussaint *The Geometry of Musical Rhythm* for more about the ubiquitous Cuban *tresillo* rhythm.

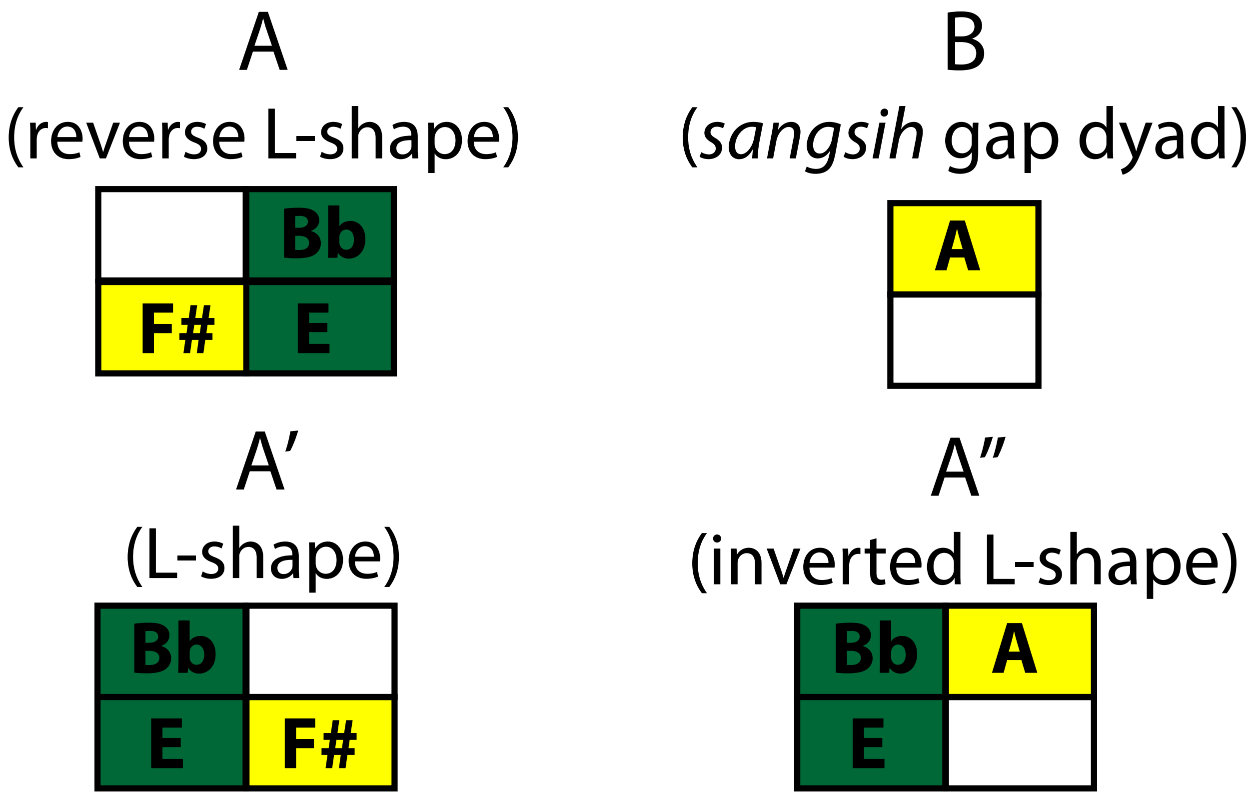

Figure 48: Legend of cell formations in *Kotekan empat* cycle 1.

Based on these cells, the formal structure of the combined parts in *Kotekan empat cycle 1* is:

A B A' B A"

The arch-form symmetry of this structure is a notable reflection of the bipartite arch-forms illustrated in Figure 47, and its binary toggle also resembles the movement of the *sangsih* line itself. The combination of multiple displaced axes of symmetry, both within the *sangsih* part and between the two parts at various levels of scale contributes to the perceptual rivalry and multiplicity in perception identified earlier in this section.

Cycle 2A is the second of the three cycles in King's *Kotekan For Alit*, and consists of the following phrase of 32 pulses.

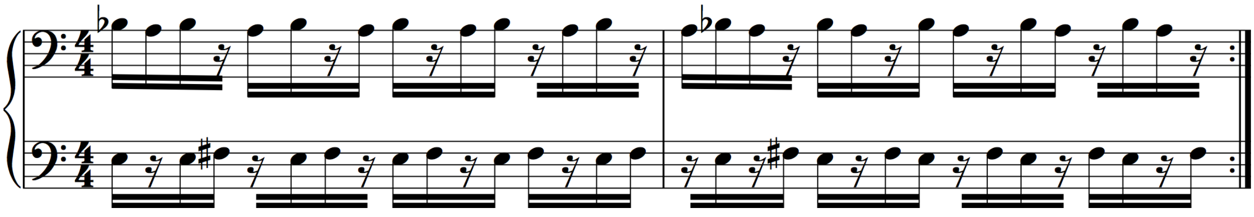

Figure 49: Cycle 2A of *kotekan empat* (from *Kotekan For Alit*, by Adam King).

The rules of the *harmonic dyad* and *gap dyad* of *polos* and *sangsih* are retained from cycle 1. The cardinality of the pitches and rests can easily be deduced. *Sangsih* has 11 onsets on B-flat and 11 on A, totalling 22 onsets in the 32 pulse cycle. This means there are 10 gaps, which account for the 10 F-sharps in *polos* to form the first *gap dyad*. *Polos* requires 11 onsets on E to match the *harmonic dyad* of 11 onsets on B-flat in *sangsih*. The remaining 11 pulses in *polos* must correspond to the aforementioned 11 onsets on A in *sangsih*, forming the second *gap dyad*.

Rhythmic symmetry of the two lines is revealed by counting their inter-onset intervals. The following considers a new group commences after any rest.

The top (*sangsih*) line is <4-3-3-3-3-4-3-3-3-3-3>. This splits the 32-pulse cycle in two halves. The phrase <4-3-3-3-3> undergoes translation.

The bottom (*polos*) line is <2-3-3-3-3-3-2-3-3-3-3-1>, which logically phases to start on the final pulse 32 to keep the grouping of 3 intact. The result is instead <3-3-3-3-3-3-2-3-3-3-3>. The bilateral symmetry of *polos* revolves around the group of 2, whose location is skewed with respect to the translational symmetry line of *sangsih*, appearing on pulse 18 of the top line in Figure 49. The bracelet form <3-3-3-3-3-2-3-3-3-3-3> shows this symmetry more clearly, with the axis flanked by five groups of 3 on either side. These symmetries are illustrated in the box diagram of Figure 50.

4				3			3			3			3			4				3			3			3			3		
		3			3			3			3			3			2		3			3			3			3			3

Figure 50: Box diagram illustrating translational and bilateral symmetry of *polos* and *sangsih* from *Kotekan empat* cycle 2A. *Polos* and *sangsih* are colour-coded red and yellow, respectively.

This diagram hints at a symmetry of the two parts' combination, which is further revealed in the following diagram that reintroduces the pitches of both parts and colour-codes them as cellular units.

Bb	A	Bb		A	Bb		A	Bb		A	Bb		A	Bb		A	Bb	A		Bb	A		Bb	A		Bb	A		Bb	A	
E		E	F#		E	F#		E	F#		E	F#		E	F#		E		F#	E		F#	E		F#	E		F#	E		F#

Figure 51: Box diagram illustrating bilateral symmetry arising from the combination of *polos* and *sangsih* in *Kotekan empat* cycle 2A.

This diagram reveals that the pink T-shape focussed around pulse 18 is the axis about which the combination of *polos* and *sangsih* revolve. This axial pulse is preceded by five Z-shape cells (led by the yellow *gap dyad*) and followed by five reflected Z-shape cells (or S-shape, ending with a yellow *gap dyad*), with the last *gap dyad* overlapping the first at pulse 2 for a seamless effect.

Cycle 2B is the last of the three cycles in King's composition, and consists of the following phrase of 32 pulses.

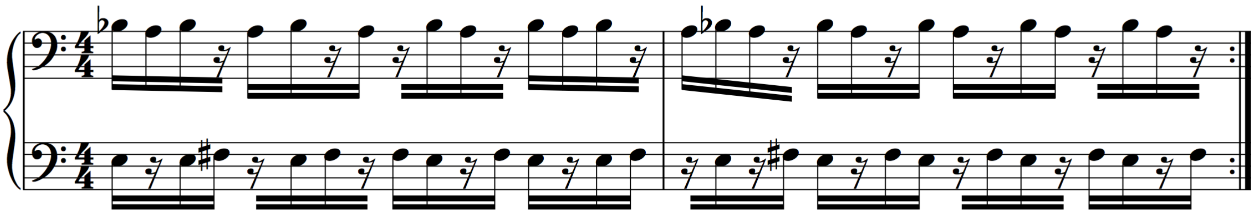

Figure 52: Cycle 2B of *kotekan empat* (from *Kotekan For Alit*, by Adam King).

This variation of cycle 2A begins its divergence via the F-sharp on pulse 9 in *polos*, which requires a rest in *sangsih* as it is a *gap dyad* pitch. This disturbance is resolved by *polos* on pulse 15, when it reiterates E to revert to the sequence as per cycle 2A. These disturbances of course have an effect on the symmetry. The diagram that follows colour-codes the cellular units in the manner of Figure 51, with three Z-shape cells (led by the yellow gap dyad) and seven S-shape cells (ending with a yellow *gap dyad*), with the last *gap dyad* overlapping the first at pulse 2 for a seamless effect.

Bb	A	Bb		A	Bb		A		Bb	A		Bb	A	Bb		A	Bb	A		Bb	A		Bb	A		Bb	A		Bb	A	
E		E	F#		E	F#		F#	E		F#	E		E	F#		E		F#	E		F#	E		F#	E		F#	E		F#

Figure 53: Box diagram illustrating bilateral symmetry arising from the combination of *polos* and *sangsih* in *Kotekan empat* cycle 2B.

The effect of the anomalous F-sharp on pulse 9 in *polos* results in a fractured symmetry that can no longer be seen as bilateral across both parts for the entire 32-pulse cycle. The pink T-shaped wedges in the box diagram represent the fulcrum points about which the prevailing symmetrical patterns are refracted, creating a bifurcation of axes of symmetry. The following diagrams reveal the overlapping cyclical nature of these bifurcated symmetries.

Figure 54 illustrates the pattern for the 21-pulse segment from pulse 30 through 18, which is rotated for clarity such that the axial pulse 8 (pitch A) is at 12 o'clock.

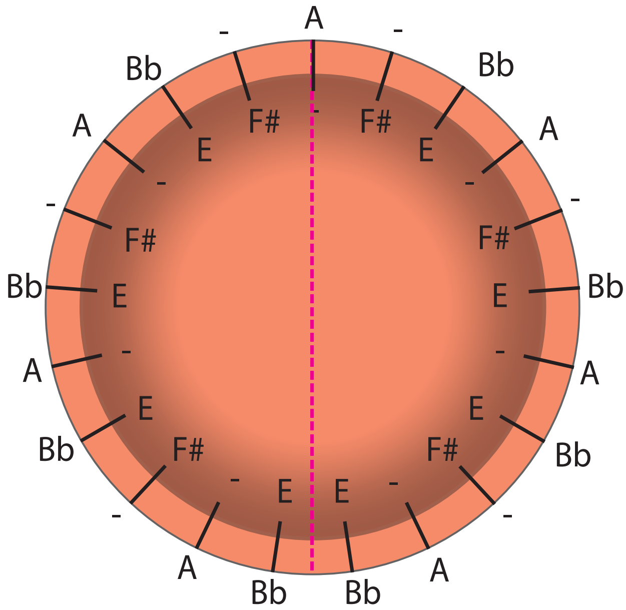

Figure 54: Timeline illustrating bilateral symmetry during pulses 30 through 18 (centred around pulse 8) arising from the combination of *polos* (inner circle) and *sangsih* (outer circle) in *Kotekan empat* cycle 2B.

Figure 55 illustrates the pattern for the remaining 11-pulse segment from pulse 19 through 29. The colour coding reveals the bilateral symmetry about the axis corresponding to the *harmonic dyad* (in pink), whilst the surrounding *gap dyads* and *harmonic dyads* are colour coded accordingly.

A		Bb	A		Bb	A		Bb	A	
	F#	E		F#	E		F#	E		F#

Figure 55: Box diagram illustrating bilateral symmetry during pulses 19 through 29 (centred around pulse 24) arising from the combination of *polos* and *sangsih* in *Kotekan empat* cycle 2B.

The compound symmetry present between *polos* (lower line) and *sangsih* (top line) can be seen in this diagram. *Polos* sounds the retrograde inversion of *sangsih*, in that it consists of the palindrome of the onsets, with intervallic direction inverted. This combination is not listed in the crystallography categories of frieze patterns outlined in section 6 of this chapter, but would be represented by reiterations of the following diagram, being a simultaneous variation of category *p112*.[53]

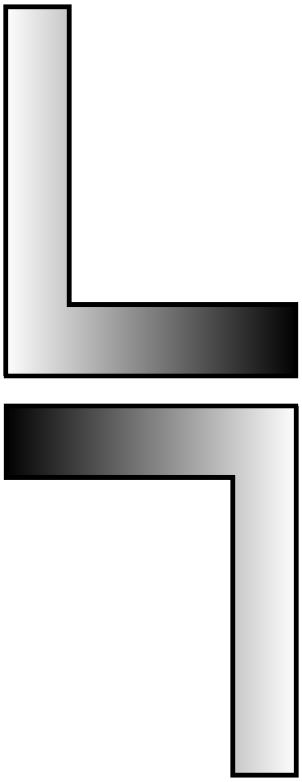

Figure 56: Compound translational and reflectional symmetry.

Figure 57 illustrates the total pattern as a timeline, which is rotated for clarity such that the axial pulse 8 for the 21-pulse segment (illustrated in Figure 54) is at 12 o'clock, whilst the axial pulse 24 for the 11-pulse segment (illustrated in Figure 55) is at 6 o'clock. The colour coding matches Figure 53.

53 This diagram could alternatively be viewed as featuring rotational symmetry.

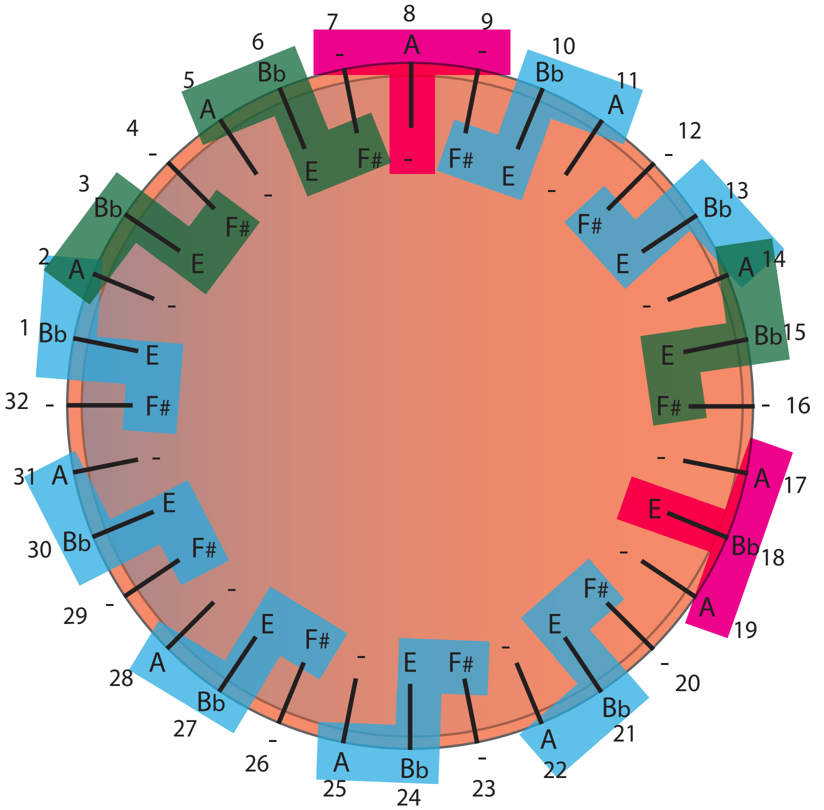

Figure 57: Timeline illustrating bilateral symmetry arising from the combination of *polos* (inner circle) and *sangsih* (outer circle) in *Kotekan empat* cycle 2B.

It can be seen that the construction of *kotekan empat* in King's composition exploits manifold symmetry that traverses scale from the macro to the micro level of structure, arises from the organisation of the parameters of pitch and duration both separately and in combination, and envelopes the *polos* and *sangsih* parts both singly and together. The employment of bilateral symmetries about decentralised axes appears a consistent trait of the symmetries discussed in the music of Bali. They demonstrate a recursive approach to the conception of symmetry in circular time that is reminiscent of the African harp of *Nzakara* melodies.

2.9 Perception of symmetry

In section 3 of this chapter I cited octave equivalence as an example of translational symmetry in music. In most musical genres this symmetry would be perceived as true, evidenced by the congruent use of harmony and melody in voices and instruments across different octaves of the pitch spectrum. However the following diagrams reveal the logarithmic scaling present in the sequence of octaves, and thus the geometric nature behind the acoustic reality of octave equivalence.

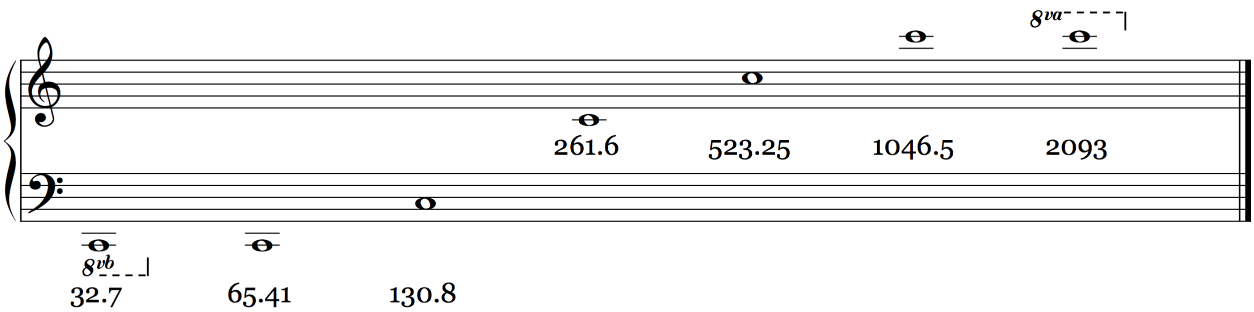

Figure 58: Octave sequence from C1 to C7 (with frequency in Hertz).

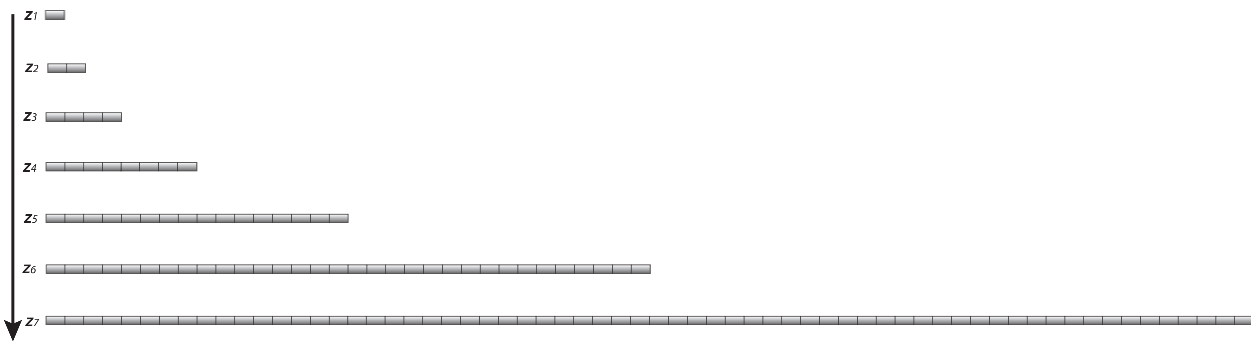

Figure 59: Seven octave sequence represented geometrically. Each horizontal bar from *z1* to *z7* represents the pitches C1 through C7. The cells that comprise each bar represent units of 32.7Hz, being the fundamental of the given sequence.

Western notation belies the geometry of the octave sequence, as the pitches appear equidistant when notated (Figure 58) but logarithmically scaled when graphed as frequencies (Figure 59). These kinds of conflicts arising from dualities and paradoxes are resolved when one tempers judgement by considering that the subjective perception of symmetry in a virtual, musical world is as valid as an objective acoustic one, and do indeed coexist (Csapó 187).

For all the conscious effort that might go into the symmetrical design of a composition's form, pitch and rhythmic material, the question arises whether such symmetry is perceptible to the listener. Toussaint proposes that there is a real limit to the listener's ability to observe relationships of a nonconsecutive nature, and shares the finding that the 'conscious present' lasts for about 3 seconds ("The Geometry of Musical Rhythm" 35). Whilst he suspects that the brain unconsciously recognises such relationships, there is no experimental evidence yet to prove or disprove whether these relationships play a role in the perception of rhythmic similarity, as there has been for nonadjacent pitches (35). Research into the schism between acoustic reality and auditory perception is taken up by Sethares, whose findings focus on rhythm and time (though not symmetry *per se*).

An alternative approach to the question of perceptibility of symmetry is to question the question itself. Does an observer of Polykleitos' sculptures need to read his treatise on aesthetic proportion (*kanon*) to appreciate his art and experience the inherent *symmetria*? I suggest this is not the case. In musical composition, as I believe as in sculpture, intelligibility and aesthetic appreciation occur on conscious and subconscious levels. Even when symmetrical concepts are not consciously applied in composition, symmetry and proportion have far-reaching ramifications in the intuitive choices the composer makes. They are part of innate natural forces. I quoted Voloshinov and Weyl in my introduction to this chapter, who seem to share this opinion.

By imposing limits, patterns, relationships and rules upon my work, ideas are stimulated and I seek to make sounds that resolve the problem set up by those limits, using intuition and subjectivity in-hand with cognitive reasoning. Thus, the work would not come to exist without such limits and my language would not develop. Symmetrical and proportional considerations are a means to self-imposed limits, and not an end in themselves. Furthermore, many of the symmetric patterns found in my work (and those of the great masters such as Bartók) have been discovered post-hoc, apparently the result of more intuitive processes whose causality we may never know. I feel I am justified in employing analysis of symmetric patterns by virtue of the results born by the process, which are at once plentiful and illuminate relationships—integral to objectivity according to Poincaré (350).

This chapter has surveyed symmetry in music as it relates to my folio of compositions. The concept of symmetry inspires a great deal of compositional material, and its retrospective application during analysis can reveal patterns, relationships and correspondences. Techniques that are symmetrical in nature are not to be seen as separate from some other kind of vagrant techniques that maybe responsible for creating asymmetrical material. Rather, the holistic nature of symmetry and asymmetry as oneness, and the metaphoric and symbolic nature of symmetry in such a multidimensional temporally-bound art form as music is duly recognised (Csapó 183, 199). Furthermore, the creation of material is recognized as distinct to the *process* of composition, and it is the transformation of material over time that is the primary concern of my folio and the analysis in the critical commentary (Pearsall 33, 39).

The introduction to this chapter quoted Wilczek, who writes about symmetry in the context of quantum chromodynamics (QCD). QCD concerns quarks and gluons, but has a fascinating resemblance to musical considerations such as the aforementioned dilemma that questioned the need for perceptibility of symmetry. Wilczek discusses the requirement for gluon fields to be included in situations of local symmetry. “An idea—local symmetry—is so powerful and restrictive that it produces a definite set of equations. …Implementing an idea produces a candidate reality... New ingredients—colour gluon fields—are part of the recipe for its candidate world… The candidate reality, hatched from ideas, is reality itself” (72).

Chapter 3
Polyrhythm and Time Cycles

Rhythm is the translational symmetry of time.

Alexander Voloshinov, *Symmetry as a Superprinciple of Science and Art* (1996), p.112

3.1 Background and terminology

The stratification of music into constituent temporal layers has been approached by composers in multiple ways. Nancarrow's *temporal dissonance*, Carter's *rhythmic succession* and *simultaneity*, Coleman's *polyrhythmic signatures*, and Messiaen's *rhythmic pedals* represent some of the diversity of approach in the music of the West during the past century.[1] Schillinger's theory of *interferences of periodicities* applies interference theory through such methods as his *melodic attack groups*, and shows the nature of his unique and complementary approach.[2]

I share with these composers an interest in polyrhythm and the stratification of material into cyclical layers. Polyrhythm and time cycles have acted as an enduring and prominent aspect of my composition, employed for their powerful contribution towards fuelling and foiling expectancy in my music.[3] Polyrhythm allows me to manipulate the sense of how time flows, and provides an opportunity to be strongly directional as well as illusionary—educing "perceptual rivalry and multiplicity" (Pressing, "Black Atlantic Rhythm" 285).

Polyrhythm is a term whose definition and application lacks universal conformity, just as other temporal terms such as *pulse* and *timeline*.[4] Literal translation of the

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1 Refer to Thomas for a study of temporal stratification in the work of Nancarrow, Ives, Cowell, Carter and Ligeti.

2 Refer to *The Schillinger System of Musical Composition* book I chapter 2 and book VI chapter 2.

3 My intention underpins the concepts of auto-correlation and self-similarity. Refer to Meyer, and Huron, for investigations into emotional responses to music that arise from the fulfilment or thwarting of expectation.

4 See Agawu, "Structural Analysis or Cultural Analysis?" for another perspective on terminology in the context of West African rhythm.

constituent words *poly* and *rhythm* correspond to Agawu's interpretation, which befits the West African rhythmic textures that he researches (see Agawu, "Structural Analysis or Cultural Analysis?"). However my definition draws from contemporary drum set practice (for example Chaffee; Magadini), and is consistent with its application in contemporary music practice.

Polyrhythm: The superimposition of two or more layers of regular groupings of pulse.

Given that the essential feature of *beat* is the periodicity of pulse grouping,[5] and *metre* by definition comprises recurring groups of beats, *polyrhythm* could alternatively be defined as the *superimposition of two or more metres*, or termed *polymetre*.

Polyrhythms in my folio and dissertation follow standard convention, being represented by a ratio expressed in lowest common denominator terms. Figures 1 and 2 illustrate the two inversions of the 3:2 polyrhythm with their proportions related to Western notation.[6]

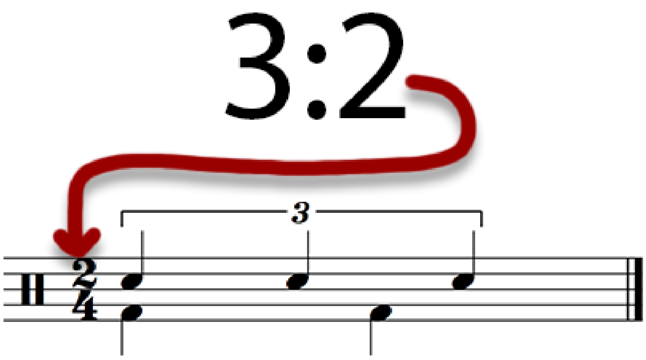

Figure 1: 3:2 polyrhythm showing relationship to metre and constituent layers in Western notation.

Figure 2: 2:3 polyrhythm showing relationship to metre and constituent layers in Western notation.

5 Termed *modular pulsation* by Jay Rahn (66).

6 The denominator of the polyrhythm always translates to the lower layer of the polyrhythm, being the upper figure of the time signature.

Durations are chosen in the knowledge that it is the attack point—not the duration—which communicates the essence of a polyrhythm. Musical interpretation allows for a myriad of choices of attack length, and how they may be realised with further subordinate attacks.

Figure 3 profiles the eleven polyrhythms most prevalent in my music.

Polyrhythm	Cumulative rhythm	Adjacent interval vector
2:3	<2-1-1-2>	<<2,2>>
3:4	<3-1-2-2-1-3>	<<2,2,2>>
2:5	<2-2-1-1-2-2>	<<2,4>>
3:5	<3-2-1-3-1-2-3>	<<2,2,3>>
4:5	<4-1-3-2-2-3-1-4>	<<2,2,2,2>>
5:6	<5-1-4-2-3-3-2-4-1-5>	<<2,2,2,2,2>>
2:7	<2-2-2-1-1-2-2-2>	<<2,6>>
3:7	<3-3-1-2-3-2-1-3-3>	<<2,2,5>>
4:7	<4-3-1-4-2-2-4-1-3-4>	<<2,2,2,4>>
5:7	<5-2-3-4-1-5-1-4-3-2-5>	<<2,2,2,2,3>>
6:7	<6-1-5-2-4-3-3-4-2-5-1-6>	<<2,2,2,2,2,2>>

Figure 3: Eleven polyrhythms that combine integers from 2 through 7.

When two or more rhythmic layers are superimposed, their merged combination creates a resultant rhythm, which I call a *cumulative rhythm*. In polyrhythm, this periodic rhythm is unique and characteristic of the particular ratio (in either of its inversions), and can provide essential orientation and identification information for the performer and listener.[7] The cumulative rhythm is the source of the *rhythmic contour* of a polyrhythm.

Cumulative rhythm: The resultant rhythmic phrase which represents the combination of two or more layers of a polyrhythm.

Figure 4 expresses the two inversions of the 3:2 polyrhythm as cumulative rhythms, taking the <2-1-1-2> inter-onset intervals of Figure 3 and placing them in Western notation. Cumulative rhythms are palindromic, and reveal the bilateral symmetry inherent in all polyrhythm.

7 Many musicians use mnemonic phrases to conjure the sound of specific cumulative rhythms, such as “cold cup of tea” for 2:3. See Palmqvist.

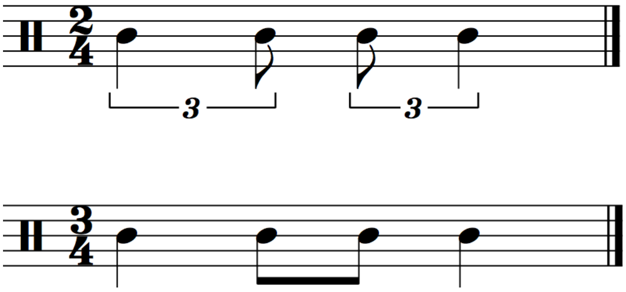

Figure 4: Cumulative rhythm for the 3:2 (top) and 2:3 (bottom) polyrhythm.

The 2:3 cumulative rhythm is known as the *hemiola* polyrhythm, prevalent in genres around the world. Figure 5 identifies four examples, the last of which contains 2:3 and 3:4—or 2:3:4—concurrently at one point.

Artist	Album	Composition
Sting	The Soul Cages	Island of Souls
Richard Bona	Munia	Sona Mama
Chiquinho Timoteo	Brazil Guitar Masters	Bahia Sarava
Nguyên Lê	Purple: Celebrating Jimi Hendrix	1983...(A Merman I Should Turn to Be)

Figure 5: Four examples of the 2:3 polyrhythm in contemporary music.

In the analysis of rhythmic structure, it is the *onset* that forms the point of measurement. Though the mid-point of a duration would correspond to the point of measurement of mathematical symmetry (Csapó 184) and would result in a truly isotropic measurement, its is the primacy of the onset that corresponds with our rhythmic perception. The same sense of proportion is obtained from the rhythms in Figure 4 regardless of note duration.[8] The symmetry of polyrhythm is now interrogated geometrically with this understanding.

8 For the related basic terms *grouping*, *duration*, and *pulse*, refer to the Glossary. Refer also to Dimond 2007 (chapter 5) for more information on polyrhythm theory and its application.

3.2 Rhythmic contour

Melodic contour analysis is used to deduce the essential topography of a melodic line. Changes in pitches are observed in such a way as to disregard duration whilst retaining temporal order. Specific intervallic size between adjacent pitches is typically also ignored. From these abstract contours, conclusions can be made about the nature of a melody, and its likeness to other melodies.[9]

Abstraction of contour seems equally applicable and rewarding in the rhythmic domain. The rhythmic contour of a motif can be found by replacing the specific durations of its series of inter-onset intervals with a mapping of the relative change in its series of onsets. Toussaint defines rhythmic contour as a qualitative measure of change in the durations of each successive pair of inter-onset intervals ("The Geometry of Musical Rhythm" 47). Toussaint's coding system adopts Polansky & Bassein's symbolization, using the pseudo-quantitative measures of +1, -1, or 0 to represent a relative increase, decrease, or repetition of the prior inter-onset interval, respectively.[10]

Melodic contour is thought to make an important contribution to the perception of melody, its commitment to long- and short-term memory, and subsequent recall (Quinn 440). The potential for a corollary role in the perception and recall of rhythm—such as the characteristic cumulative rhythms of polyrhythms—only increases the value of abstracting and studying rhythmic contour.

Due to my interest in rhythmic contour as a compositional concern (such as in *Mod Times*), I have employed two approaches to its measurement. *Generic rhythmic contour* adopts Toussaint's definition, but uses symbols rather than integers to maintain the qualitative measure intended. The arrows ↑ ↓ → respectively represent a relative lengthening, shortening and repetition of inter-onset intervals.

Specific rhythmic contour takes the series of inter-onset intervals and codes the change in duration with a specific measure of difference. In other words, this approach is

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9 See Straus (126) for a discussion of melodic contour in post-tonal music.

10 Toussaint does acknowledge the symbols L, S, and R to represent lengthening, shortening and repetition (respectively) in rhythmic contours proposed by Hutchinson & Knopoff (281), but does not adopt their symbols nor stylistic application. Polansky & Bassein's symbols are used in their *combinatorial contour* approach, which use *ternary symmetrical contour* measures to acknowledge adjacent and nonadjacent relationships (261).

one step less abstract than the *adjacent interval vector*[11] representation of a rhythm, as it retains the chronology of the onsets. This approach also resembles a procedure used in melodic contour measurements, in the sense that steps (adjacencies) are distinguished from leaps. For measures of specific rhythmic contour, +x, -y, or 0 represent quantifiable changes through an increase, decrease, or repetition of the prior inter-onset interval, respectively. Refer to Figures 6 and 7.

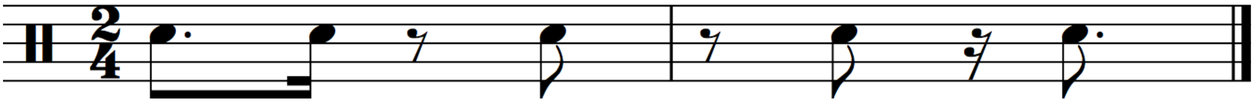

Figure 6: Notation of *bossa-nova* rhythm.

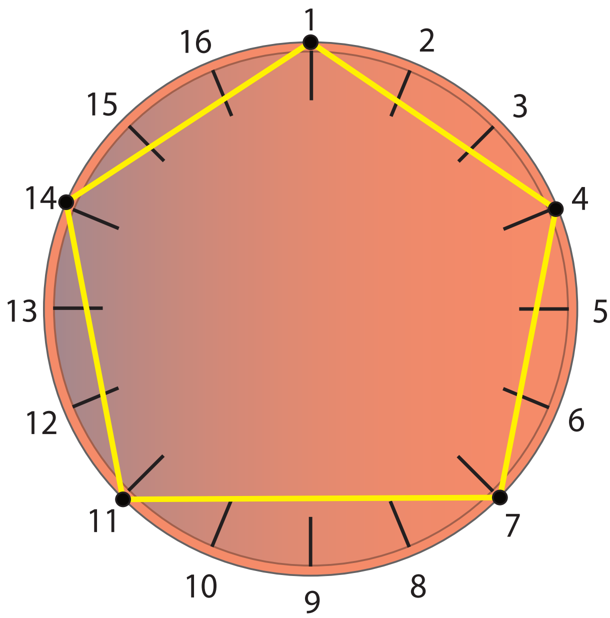

Figure 7: *Bossa-nova* timeline.

Figures 6 and 7 reveal the following about the *bossa-nova*:

- Timeline of 16 pulses
- Inter-onset intervals <3-3-4-3-3>
- Generic rhythmic contour → ↑ ↓ → →
- Specific rhythmic contour 0, +1, -1, 0, 0.

In *Mod Times* I manipulate rhythmic contour with an original technique called *contour modulation*. Essentially, this technique applies augmentation and diminution by unit pulse to the inter-onset intervals of a given rhythm held under metric constancy, applies *modulo* to the value of each duration to constrain the result within a chosen limit (usually a beat), and then applies the profile of the generic rhythmic contour of the original timeline to

11 Interval vectors are discussed in the next section.

revert the relative ordering of onsets to that of the original rhythm. (See Chapter 6.)

Binarization and *ternarization* are examples of the *specific rhythmic contour* concept in the production of self-similar material. A binary timeline contextualized in a ternary metre has an invariant *specific rhythmic contour*, as does the reverse situation. Interesting grooves and musically rewarding effects can be produced by reinterpreting a binary rhythm in a ternary setting, and *vice versa*. Refer to Chapter 7.6 for an example in *Birder*.

3.3 Interval vectors and set theory

In the case of a chord, scale or melody, the identity of a particular sonority is derived from the intervals contained. In post-tonal theory, the constituent pitch-class intervals in a pitch-class set is profiled using an *interval-class vector* that tallies the number of possible instances that each unordered pitch-class interval may be generated from the set.[12] Vectors are always expressed in ascending order from 1. Rhythm may be considered similarly, though its chronological nature benefits from two approaches of abstracting rhythmic interval content.

The *adjacent interval vector* tallies only the chronological intervals in a timeline, whilst the *full interval vector* tallies all *geodesic* intervals in a timeline (Toussaint, "The Geometry of Musical Rhythm" 37).[13] The result of such tallies is expressed either as a string of integers in double-triangular brackets or graphically as histograms. With the interval strings, the places from left to right represent the number of occurrences of pitch-class interval 1, 2, 3, … and so on. Figures 8 and 9 illustrate the full and adjacent interval vectors for the *bossa-nova* timeline illustrated in Figures 6 and 7.

12 See Straus (16) and Rahn, "Basic Atonal Theory" (100) for further explanation.

13 *Geodesic* distances calculate the shortest distance traced around the circle for *every* pair of onsets, considered similarly to unordered pitch-class intervals. Geodesic distances are therefore limited to half the cycle length, which for pitch interval-class vectors in 12-tone pitch space means there are always six positions in an interval-class vector. As the length of timelines is variable, the range of possible geodesic distances varies accordingly, but will always correspond to half the length of the particular cycle.

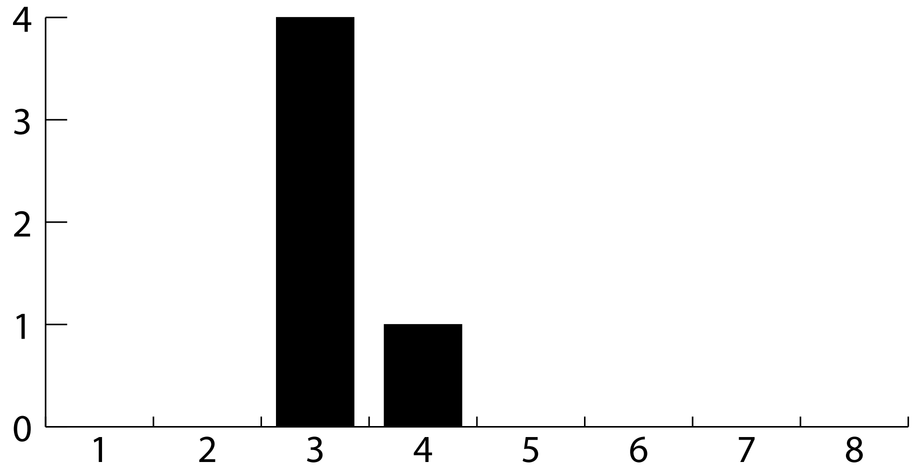

Figure 8: Adjacent interval vector <<0,0,4,1>> for the *bossa-nova* timeline, graphed as a histogram.

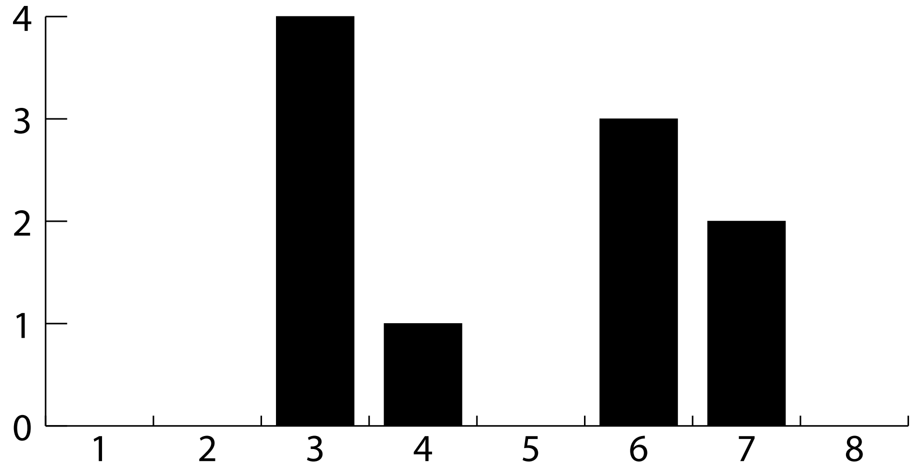

Figure 9: Full interval vector <<0,0,4,1,0,3,2>> for the *bossa-nova* timeline, graphed as a histogram.

The adjacent interval vectors of the polyrhythms featured in this chapter are listed in Figure 3. Full and adjacent interval vectors are a useful way of discriminating between different rhythms, with full interval vectors offering a more abstract (and therefore generalised) perspective as a point of comparison. Toussaint nominates several types of categorization that arise from observation of interval vector, one of which is the category of *deep rhythms* ("The Geometry of Musical Rhythm" 175).

Deep rhythms are featured in *Mod Times*, and feature a distribution of interval content where each inter-onset interval has a unique frequency. In Figure 9 the histogram for the *bossa-nova* has no columns of equal height and thus fulfils this criterion, and so is a deep rhythm.

There is a connection between deep rhythms, self-similarity, polyrhythm, tonality and acoustic consonance. The self-similar *well-formed sets* of Carey & Clampitt possess a single pitch-class interval as their reiterative generator (Carey & Clampitt 63). Of the twelve collections ranging from trichords to septachords tabled in their list of *non-degenerate well-formed sets* (64), eleven have interval vectors that satisfy the criterion of deepness—all except the whole-tone pentachord [02468]. Similarly, Tymoczko identifies twelve of the most common intervals, chords and scales in tonal music that are the most acoustically consonant and also most able to produce effective and harmonically consistent voice-leading (Tymoczko 63). “Highly consonant chords always divide the octave relatively evenly” (63).

The connection to which I refer, instigated by interval vector analysis, relates to the Euclidean property of *maximal evenness*.[14] This property is the result of the procedure for calculating the greatest common divisor between two numbers, and is shown by Toussaint as a creative method for creating enduring and attractive timelines (“The Geometry of Musical Rhythm” 130). Maximally-even rhythms are not perfectly isochronous beat-like timelines, but rather are rhythms whose onsets are distributed as evenly as possible (123). Isochronous beat-like timelines are categorised by Toussaint as *perfectly even* (124), and whilst trivially appease the evenness criteria, they form regular polygons that are *beats* rather than rhythms. A truly maximally-even rhythm sees a rhythm of k onsets distributed on a timeline with n pulses such that $k \div n$ yields a non-integer, such as 3 onsets in a timeline of 8 pulses.[15] Toussaint demonstrates that this example yields <3-3-2>, corresponding to the ubiquitous Cuban *tresillo* rhythm, and is maximally-even (131).

Tymoczko shows via Helmholtz how acoustic consonance is derived from pitch collections that originate from integer multiples of fundamental, ranging from the simple perfect fifth (3:2) through to the major triad (3:4:5), and that these and even larger collections of notes are deemed consonant when they too are “relatively evenly distributed in pitch-class space” (62). Figure 10 provides a geometric illustration of how the self-similar interval generator used to create the property of well-formedness results in an approximately equal distribution of scale degrees in pitch-class space.

14 Euclid invented the algorithm for maximal evenness around 300 B.C.E. and published the finding in book VII of *The Elements* (Toussaint, “The Geometry of Musical Rhythm” 129).

15 Such integers are considered *relative primes*.

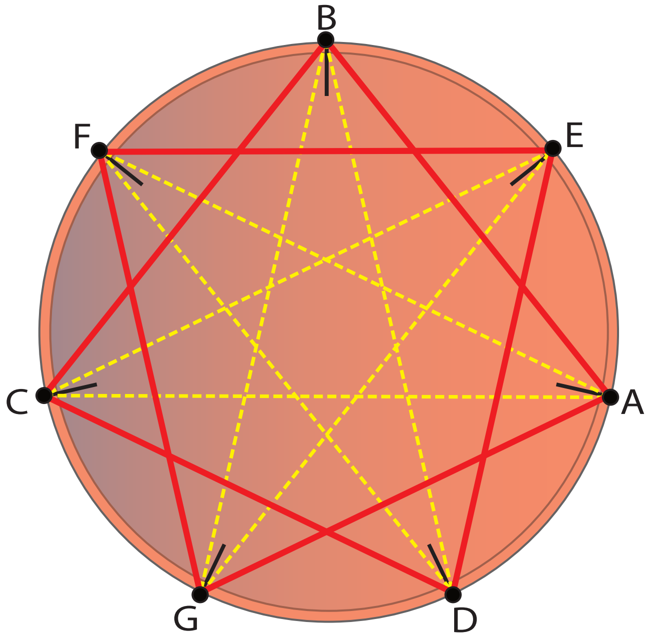

Figure 10: Major scale as self-similar reiteration of interval class 5, plotted on a wheel of seven pitch-classes. Red = diatonic seconds; yellow = diatonic thirds.

It is interesting to reflect on the synecdochic property of self-similarity and symmetry, and its apparent prevalence and preference in music and nature for patterning that is at once homogenous and varied—where parts reflect the organisation of the whole but with subtle, desirable distortions (Carey & Clampitt 66).

Post-tonal theory provides other tools to analyse the principles of polyrhythm via set theory. The 4:5 polyrhythm is used to illustrate these tools.

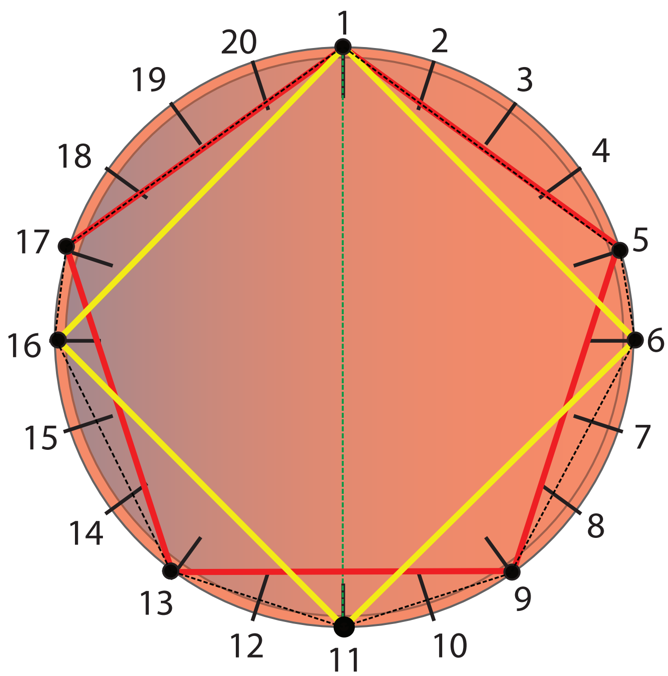

Figure 11: 4:5 polyrhythm timeline.

The rhythmic 'set' for this 4:5 polyrhythm is the series of onsets from the lowest to highest number:

{1, 5, 6, 9, 11, 13, 16, 17}

The formula for *ordered pitch-class intervals* for any two pitch classes *a* and *b* is given by:

b–*a* mod12

(Rahn, "Basic Atonal Theory" 25)

Adopting this formula to timelines, the *inter-onset intervals* of the 4:5 polyrhythm are calculated by:

5–1 = 4

6–5 = 1 and so on.

The calculation of the final inter-onset interval (from 17 to 1) requires a *modulo* adjustment. Given that the series of onsets is in a 'scale' of (4x5) 20 pulses, the final inter-onset interval is calculated by:

1–17 = -16, mod 20 = 4.

The complete inter-onset interval series for the 4:5 polyrhythm illustrated is therefore <4-1-3-2-2-3-1-4>, and is traced by the dotted black line in Figure 11. This is its *cumulative rhythm*, and is the temporal equivalent of *ordered pitch-class intervals* in post-tonal set theory.

3.4 Additive rhythm

In Chapter 1 I cited issues arising from the notation of music that prioritized phrase over beat. These issues are alleviated by an additive rhythmic approach, which provides benefits to the performer, composer, and analyst when approaching music such as my own.[16]

Additive rhythm considers the atomic unit of pulse as the genesis of time rather than the whole bar. Additive rhythm's power and flexibility is derived from its congruency with

16 Divisive rhythm is conventionally taught and employed in Western European art music, and is the basis of its system of notation. It considers a bar as a generative rhythmic unit and progressively divides this into smaller parts—typically powers of 2—to create a 'rhythm tree'.

the *unique-prime-factorization theorem*, which states that every integer greater than 1 either is prime itself or is the product of prime numbers.[17] An outcome of this congruency is that additive rhythm recognises the primacy of 2 and 3 as the first prime numbers,[18] and builds durations by concatenating these small atomic units of time. Jay Rahn refers to these units as “unit intervals of pulsation”, and advocates such an additive approach, claiming it is universally applicable to the analysis of all genres of music (64).

The primacy of 2 and 3 as atomic rhythmic units corresponds to the *metrical feet* of prosody, and could explain some of the universality to which Rahn refers.[19] For example, groups of 3 often correspond in practice to the disyllabic *trochee* ¯ ˘ (stressed-unstressed), with the stressed syllable taking double the length (and weight) of the unstressed syllable, thus totalling 3 units. This approach is distilled in the practice of *Morse code*, which I practiced as an amateur radio operator and incorporated into compositions of my youth. In Morse code, with no option for dynamic or timbral variation to create intelligibility, one is wholly reliant on the rhythm of groups of 1, 2 and 3 to communicate. Thus the code for “C” is —.—. (long-short-long-short), and corresponds to the inter-onset intervals <2-1-2-1>, and must be distinguishable from any other four-onset code, such as “B” which is —… (long-short-short-short) or <2-1-1-1>.

Additive rhythm provides for the development of a more flexible syntax in metres whether they be composite with a regular beat length (as in 12/8), or prime with no option for perfectly even beats (as in 7/8). The atomic rhythmic units of 2 and 3 may group to form repeating patterns of beats of the short and long variety as in prosody, creating identifiable grooves such as the asymmetric metres common in the music of Bulgaria.[20] Take for example the following original Bulgarian-style melody in 11/16 metre.

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17 This theorem originates in Euclid’s *Elements* (circa 300 B.C.E.), discussed subsequently.

18 Since the twentieth century, mathematical consensus is that 1 is not prime, but of its own category. The first seven primes are 2, 3, 5, 7, 11, 13, and 17.

19 The connection between prosody and additive rhythm is supported by Messiaen through his expressed interest in classic prosody, shown through his dedication to Shakespeare and the Holy Bible (Messiaen 8). See also Alcantara chapter 2 for musical application of prosody.

20 Examples from other genres include Elvin Jones *At This Point in Time*, Stravinsky *Sacre du Printemps*, and Prokofiev *Piano Sonata No. 7*.

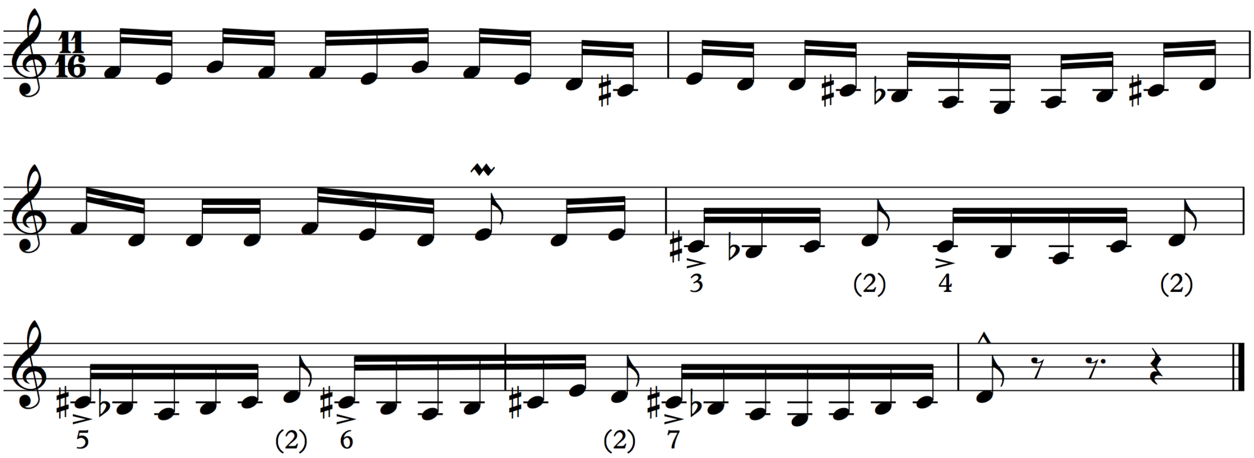

Figure 12: Bulgarian-style melody in 11/16 showing usage of additive rhythm.

This melody reveals beat durations <2-2-3-2-2> in the first 3 bars, corresponding to the *gankino horo* ostinato of Bulgaria. Thereafter, the grouping of semiquavers becomes phrase-centric, opposing the beat structure and the bar-line. The internal logic of the phrasing is indicated by the numbers underneath, and resembles the *yati* patterns discussed in Chapter 2. The performer is much more likely to realise this entire passage via additive rhythm than in any kind of divisive manner that prioritizes beat primacy.

Okazaki produced a set of diagrams called *four beat rhythmic modes* (Okazaki 28). These diagrams systematically list all possible groupings of 2 and 3 that sum to comprise the composites 8, 12, 20 and 28 (equating to total numbers of pulses in a four-beat bar), symbolically represented to facilitate flexible metrical application. By deducing the unique prime factorizations, one can systematically list all the possible groupings of 2 and 3 that may occur in a timeline of any total duration, subsequently realisable by the additive rhythmic approach.[21]

For example the composite 8 has a prime factorization of 2 x 2 x 2. Incorporating the prime 3 into this factorization requires a summation, thus:

8 = (3 x 2) + 2

or

8 = (2 x 3) + 2

Thus the four options of inter-onset intervals for realising a timeline of length 8 with groups of 3 and 2 are:

21 Note that the process moves from factorization, to summation of additive rhythm values. Refer to Toussaint *The Geometry of Musical Rhythm* chapter 36 for more on rhythmic combinatorics.

<2-2-2-2>

<3-3-2>

<3-2-3>

<2-3-3>

These are indeed the options illustrated by Okazaki. The first is the *perfectly even* beat, and the latter three should be recognised as necklaces of the ubiquitous grouping that forms the rhythmic basis of Elvis' *Hound Dog*, the Afro-Cuban *tresillo*, and the *habanera* of Tango.[22]

The process becomes more involved for larger numbers, as there are necessarily more possible combinations. Take for example the ninth prime 23. Being prime, no perfectly even beat division is possible. Its nearest lower composite is 22, whose prime factorization is 2 x 11. The factor 11 can either be grouped <3-3-3-2> or as in a necklace of the beat of Figure 12, as <3-2-2-2-2>. That is, the adjacent interval vectors that utilise 2 and 3 to make 11 are <<0,1,3>> and <<0,4,1>>. Therefore for 22 we could double either of these vectors, but reduce the number of 2s by one so that the deficit 1 (required to total 23) could be added to make an extra 3 to compensate. This logic is outlined as follows:

23 = 22 + 1

23 = (2 x 11) + 1

11 = <<0,1,3>> or <<0,4,1>>

Selecting the first vector option, 23 = 2 x ((1 x 2) + (3 x 3)) + 1

23 = (2 x 2) + (6 x 3) + 1

23 = (1 x 2) + (7 x 3) = <<0,1,7>>

Selecting the second vector option instead, 23 = 2 x ((4 x 2) + (1 x 3)) + 1

23 = (8 x 2) + (2 x 3) + 1

23 = (7 x 2) + (3 x 3) = <<0,7,3>>

---

22 The <3-3-2> is the most popular necklace as it is maximally-even, less off-beat than <3-2-3>, and sets up a pleasing element of surprise due to the repetition of the initial two groupings of 3 that are then foiled by the final group of 2 (Toussaint, "The Geometry of Musical Rhythm" 105, 131).

It is empowering to calculate that the only two ways to use additive rhythmic units of 2 and 3 to create such a large time cycle as one with 23 pulses is limited to these two adjacent interval vectors.

The ordering of onsets is the next consideration. Typically, the factorial equation (symbolised *!*) is used to determine the number of ways a set of elements may be ordered.[23] However, given that we're seeking combinations from a *multiset*, we need to account for the fact that there can exist multiples of a specific interval class in the rhythm. The formula for finding the number of permutations of an interval set that contains repetitions of elements follows:[24]

$$\frac{k!}{m!\, n!\, o!\, \ldots}$$

where:

k equals the number of onsets in the rhythm; and

m, n, o, ... equals the number of elements in each interval class (only for classes with one or more element). It doesn't matter what order these factorials are listed in the product.

Applying this formula to the timeline of 23 pulses with the eight-onset interval vector <<0,1,7>> reveals there are 8*!* ÷ 7*!* = 8 permutations. The second vector option <<0,7,3>> has 10 onsets and so there are 10*!* ÷ 7*!* x 3*!* = 120 permutations. In conclusion, it can be said that there are 128 ways of rendering a timeline of 23 pulses using additive rhythm groupings of 2 and 3.

I have shown how issues arising from divisive rhythm are alleviated by adoption of an additive rhythmic approach, which is far more suited to the realisation of music such as my own that contains frequent changes of metre, metres with an asymmetric beat, polyrhythm, melody based on speech, passages that are phrase-centric and those that incorporate numerical procedures.

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23 Schillinger called these types of operations *general permutations*, distinguishing them from *circular permutations* which retain order, as in necklaces (46).

24 Toussaint, "The Geometry of Musical Rhythm" 276.

3.5 Bilateral symmetry of polyrhythm

Figure 11 reveals a symmetrical arrangement around a vertical axis of reflection. If the timeline illustration was folded along the axis indicated by the dotted green line, all the onsets on one half would line up with the other half. The cumulative rhythm also reveals the inherent bilateral symmetry of any polyrhythm. Timelines with bilateral symmetry create mathematical palindromes from their inter-onset intervals, and therefore by definition are inversionally symmetrical.[25] The sequence of inter-onset intervals can effectively be inverted by reversing their order around the axis of reflection. For example, the 4:5 polyrhythm's inter-onset interval series has an axis between the fourth and fifth onset in the following series:

<4-1-3-2-2-3-1-4>

When the series is reversed around this axis of reflection the series becomes:

<2-3-1-4-4-1-3-2>

This inversional symmetry can be seen in the timeline of Figure 13, which appears as a mirror reflection of Figure 11. The two polygons now intersect at pulse 11—diametrically opposite pulse 1.

25 Rahn states that "an inversionally-symmetrical set always has a canonical ordering whose interval series is its own retrograde…" ("Basic Atonal Theory" 91). By "canonical ordering", Rahn means "an ascending ordering such that the adjacency-interval series of both sets related by TnI are mutually retrograde" (88). Inversion symmetry is therefore also called *retrograde symmetry*. Inversionally-symmetric sets have consecutive interval patterns that are palindromic. The major and minor triad is a common example of an inversionally-symmetric pitch-class set, represented by [037].

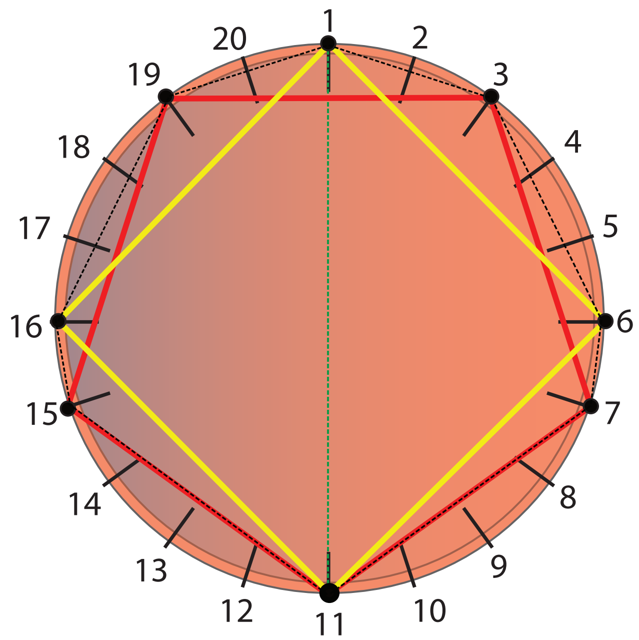

Figure 13: 4:5 polyrhythm timeline bracelet (inverted form of Figure 11).

The bilateral symmetry of polyrhythm can be further inspected via the internal design of cumulative rhythm. Logically, any polyrhythm of ratio x:y requires that any member or pair of adjacent members in its inter-onset interval series <a-b-c-d-…> must equal either *x* or *y*. For example, the 4:5 polyrhythm's inter-onset interval series <4-1-3-2-2-3-1-4> contains 5 groups of 4 pulses and 4 groups of 5 pulses. Figure 14 illustrates the four binomial pairings required to render 4 groups of 5 pulses, superimposed on the monomial and overlapping binomial pairings required to render 5 groups of 4.

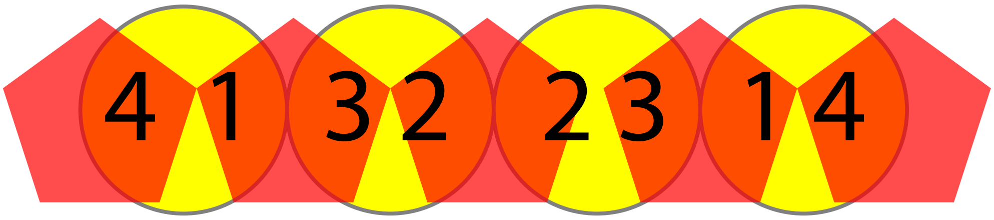

Figure 14: 4:5 polyrhythm inter-onset interval series partitioned into 4 groups of 5 pulses (yellow circles) and 5 groups of 4 pulses (red pentagons).

This process can also be visualised using bar chart diagrams. The following figures illustrate the 4:5 and 5:7 polyrhythms using this alternative method to reveal bilateral symmetry in cumulative rhythm.[26]

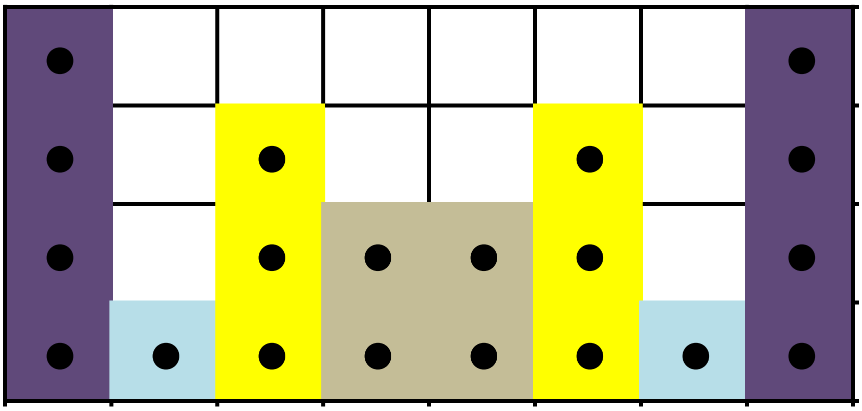

Figure 15: 4:5 polyrhythm inter-onset interval series as a bar chart.

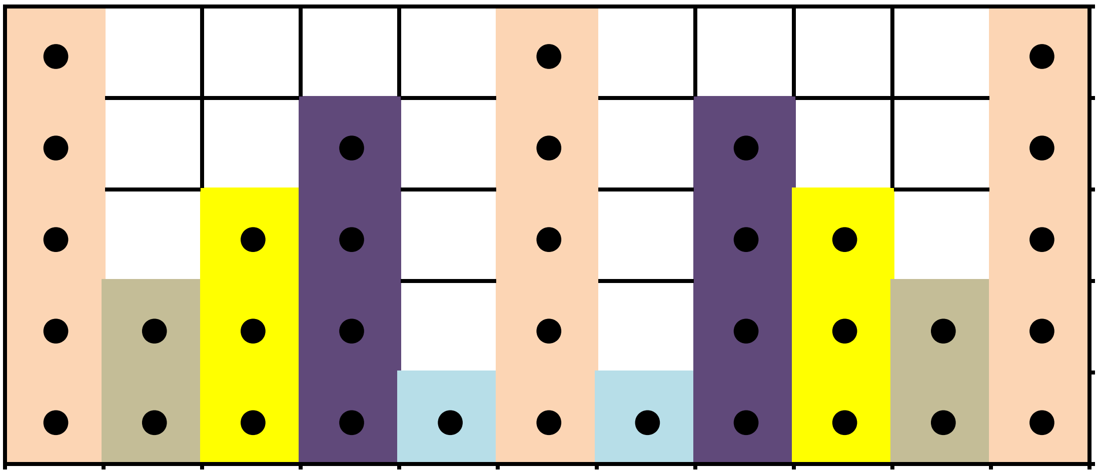

Figure 16: 5:7 polyrhythm inter-onset interval series as a bar chart.

26 The *y* axis represents the duration of each inter-onset interval (colour-coded), whilst the *x* axis tracks the chronology of onsets from left-to-right.

3.6 Coincident hit point and apsis

All the polyrhythms in this chapter have a pulse on the timeline where the two constituent layers coincide.[27] This point is called a *conjunction* in orbital mechanics, and in this dissertation is termed the *coincident hit point.*

COINCIDENT HIT POINT: THE PULSE IN A POLYRHYTHM WHERE THE CONSTITUENT LAYERS SHARE AN ONSET.

All polyrhythms also have a point where their constituent layers are most distant from each other on their periodic trajectory. I call this point the *apsis* (plural *apsides*). This point will always be diametrically opposed to (or 180° from) the coincident hit point.

APSIS: THE PULSE OR PLACE IN THE TIMELINE DIAMETRICALLY OPPOSED TO THE COINCIDENT HIT POINT.

3.7 Bracelets and necklaces

Figures 11 and 13 are considered the same *bracelet*, which is a mathematical term which considers two number sequences of the same bracelet if they are mirror image reflections of each other. The two timelines contain the same sequence of onsets, but in exact inversion.

IF TWO RHYTHMS ARE MIRROR IMAGE REFLECTIONS OF EACH OTHER,

THEY ARE CONSIDERED BRACELETS.

Musicians habitually associate beat 1 (the "down beat") with the coincident hit point in every ostinato or repeating pattern. My compositions explore other possibilities for rhythmic alignment in time cycles to create interesting effects.[28] In the process of phase-shifting the coincident hit point of polyrhythms around timelines I create a variable experience of "temporal gravity". The mathematical term for such cyclic permutations is *necklace*.

---

27 For information on non-hitting polyrhythms and polyrhythm mathematics, refer to Dimond 2007.

28 In South Indian classical music the *eduppu* or starting point of a composition or line may differ from beat 1 (*samam*), creating a certain tension especially when it is in close proximity, such as 1 or 1.5 beats away from beat 1. This tension is not dissimilar to necklace rotations.

If two rhythms are rotations of each other, they are considered necklaces.

There are as many necklaces for a polyrhythm as there are pulses in the timeline.[29] In the case of the 4:5 polyrhythm, values for *n* from 1 to 19 (mod 20) can be added to each of the members of the set {1, 5, 6, 9, 11, 13, 16, 17} to create a unique set of onsets. There are consequently 20 (or 4x5) possible necklaces (rotations) of this polyrhythm.

Figure 17 illustrates the 4:5 polyrhythm timeline for Figure 11 in another of its 20 necklace arrangements.

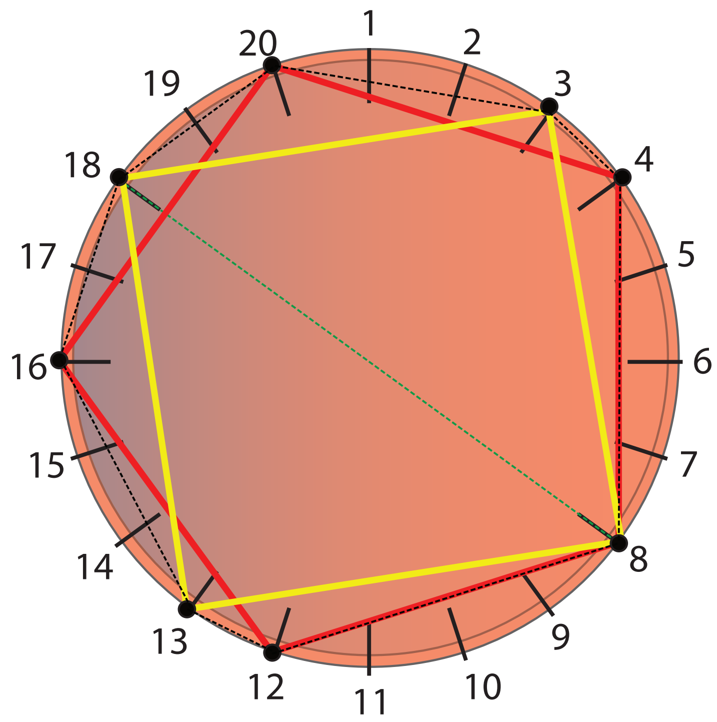

Figure 17: 4:5 polyrhythm necklace with coincident hit point on pulse 8.

29 For any transpositionally-symmetrical rhythm that is perfectly even such as an isochronous 'beat' timeline, the number of necklaces is limited by the degree of symmetry the timeline possesses. A pitch class set is considered transpositionally symmetrical if some value of *n* in the transposition Tn maps the set into itself. The set [0, 3, 6, 9] for example is a tetrachord that maps into itself when *n* is 3, 6, or 9 semitones. This set could also represent the dotted crotchet beat in 12/8 metre. It is thus transpositionally symmetrical, and is considered to have 4 degrees of symmetry in total (including *n* = 0).

Inversionally-symmetric timelines posses a special property which limits the number of times an individual layer in the polyrhythm can be rotated until another necklace of the timeline is recreated. As the cumulative rhythm of polyrhythms is palindromic, necklaces will always create bracelets, and bracelets will create a necklace as long as there an even number of total pulses on the timeline.

Inversionally-symmetric timelines with an odd number of pulses cannot be flipped 180° along the axis of reflection to create a bracelet as there are no diametrically-opposed pulse positions for the onsets. Figure 18 shows the 3:5 polyrhythm timeline with 15 pulses. Only one end of the green bilateral symmetry line falls on an onset, and the apsis end of the line has no corresponding pulse. In such cases, there are only *necklaces*.[30]

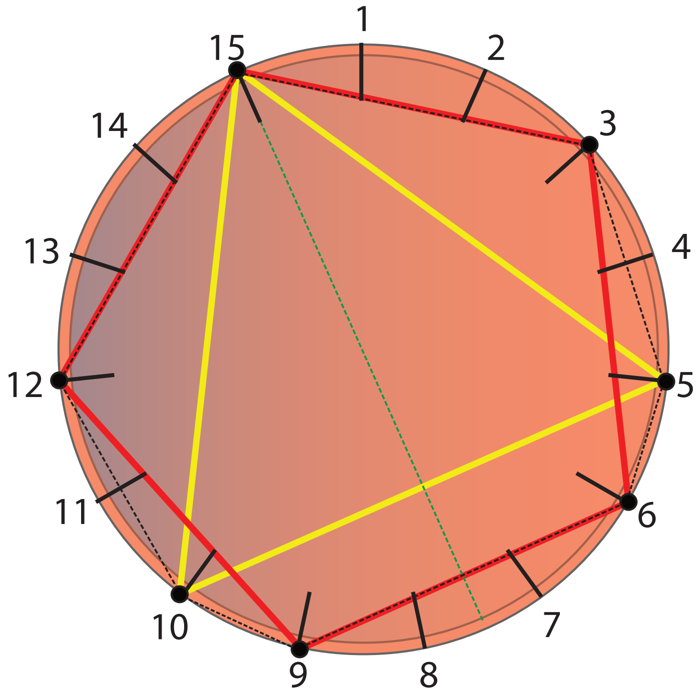

Figure 18: 3:5 polyrhythm timeline necklace showing no option for bracelet (mirror image reflection).

30 The only mathematical solution is to double the number of pulses. This is relates to the procedure for creating *non-hitting polyrhythms*. Refer to Dimond 2007.

3.8 Polyrhythm and its wave-like nature

Orbital mechanics has already shown useful alignment with polyrhythm theory. Beats are inherently periodic, and therefore have a frequency. Polyrhythm therefore also lends itself to analysis in terms of wave motion. There are immediate and useful correspondences between rhythm and pitch when analysed from the mutual perspective of their wave-like nature.[31]

The following examples use the 3:4 polyrhythm to analyse the wave-like nature of polyrhythm. The same colour-coding is used for all examples that follow.

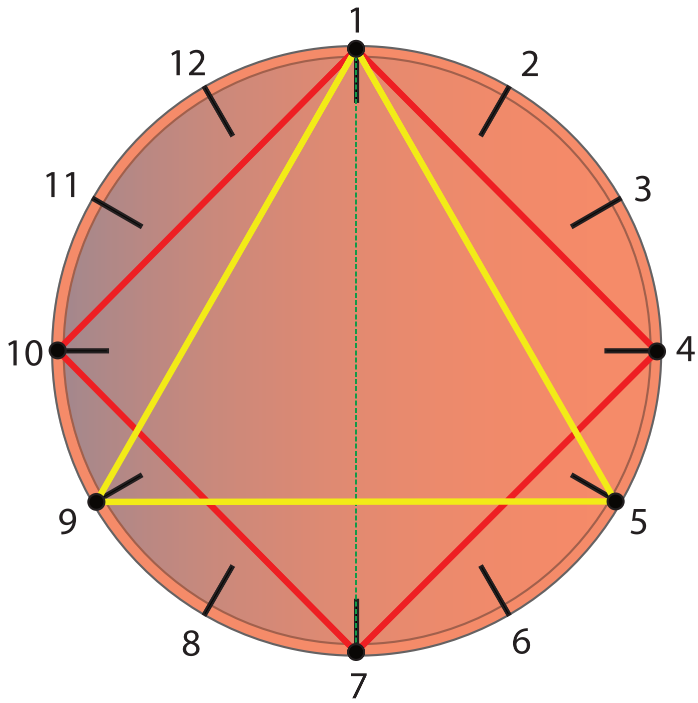

Figure 19: 3:4 polyrhythm timeline.

Figure 20 takes the circular representation of this timeline and stretches the circumference out as a line segment. The constituent beats of each side of the polyrhythm are represented by semi-circles on this linear strand of time.[32]

31 Cowell pioneered pitch-rhythm correlations, and used the harmonic series as the basis of many of his ideas, including *scales of rhythm*. Refer to Cowell (part II).

32 This diagram is a fitting abstraction of how the 3:4 polyrhythm might be represented as a conducting pattern, if conducted with two interdependent limbs.

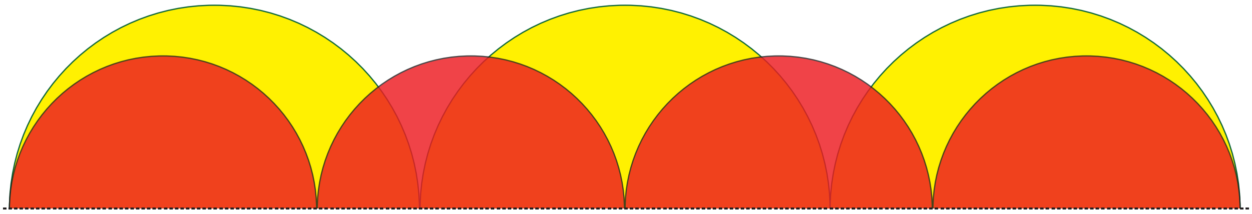

Figure 20: 3:4 polyrhythm as semi-circles on a line segment.

The self-similar nature of polyrhythm is revealed by the fact that the reiterative beats in each layer are scaled versions of the other layer—naturally in the ratio of 3:4.

Figure 20 reminded me of transverse waves and the conventional sinusoidal representation of sound waves, due to the direction of travel (left-to-right) that is perpendicular to the direction of movement of the semicircular plot. Since the waves of polyrhythmic layers combine to create a distinctive cumulative rhythm, I was curious to see what could be learned from expressing this feature from the perspective of wave motion. Benson outlines the trigonometry required to calculate the relationship between the phase of acoustic beats and the phase of the respective generative sine waves (18). I opted for a practical experiment, and generated two pitches at the ratio 3:4 as separate sine waves, and overlaid the graphed result in Figure 21. The yellow line plots G4 (392Hz) and the red line C5 (523.25Hz), a perfect fourth above.

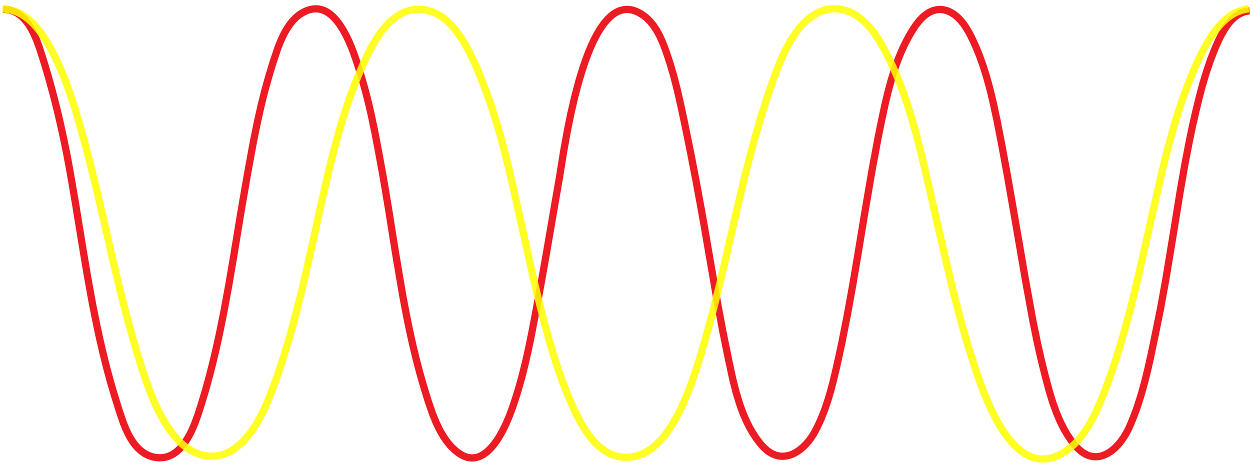

Figure 21: Superimposition of the graphs of two independently-generated sine waves at ratio 3:4.

This superimposition shares the characteristics of the timeline in Figure 19, including bilateral symmetry, and the sinusoidal equivalent of the conjunction (at the extreme left and right ends) and apsis (in the centre). The inherent proportions of the polyrhythm (3:4) can be observed by counting the crests or troughs of each waveform.

As a graphic superimposition of separately-generated waves, Figure 21 maintains the independence of the constituent layers. I next wanted to experimentally observe the wave profile of the interactive *superposition* of the layers upon each other (per Benson's calculations), in order to draw parallels with cumulative rhythm of polyrhythm theory.[33] How does the inherent bilateral symmetry of cumulative rhythm manifest in a wave model of polyrhythm?

Figure 22 traces the composite result of the waveforms in Figure 21, capturing their constructive and destructive interference patterns.

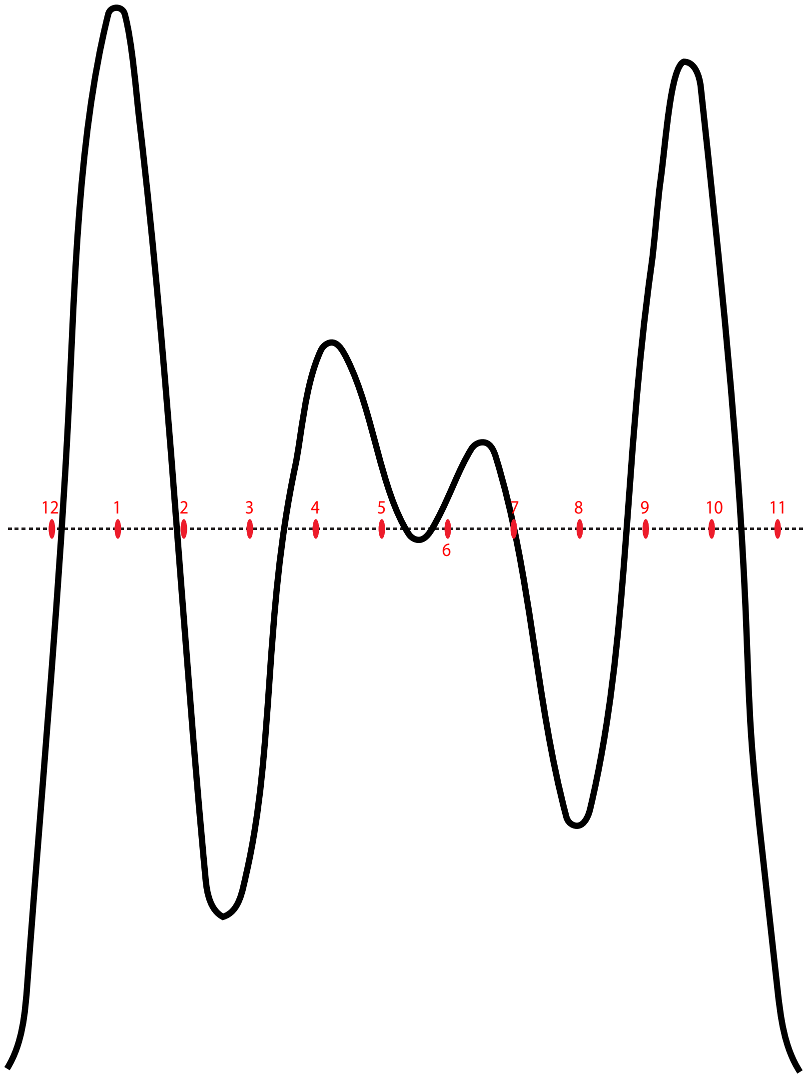

Figure 22: Superposition of two sine waves at ratio 3:4. The equilibrium (dotted line) shows the 12 equal divisions of the corresponding polyrhythm timeline of Figure 19.

33 The principle of superposition of pitches and their resulting *combinational tones* is examined by Helmholtz in chapter VII, and the theoretical nature of dissonance examined experimentally by Herrero, and Plomp & Levelt.

The resulting waveform shows the result of constructive and destructive interference.[34] I added a metric of twelve equidistant pulses at the equilibrium position to draw parallels to the timeline of Figure 19. The most prominent crest at pulse 1 corresponds to the left and right extremes of Figure 21 where constructive interference is highest. This corresponds to the coincident hit point pulse 1 of the timeline in Figure 19. The near-zero wave height around pulse 7 of Figure 22 corresponds with the central apsis of Figure 21 where almost-total phase cancellation would be expected. This pulse is seen in the timeline Figure 19 as diametrically-opposed to pulse 1. The two medium peaks correspond to the twin consecutive peaks in Figure 21 either side of its centre. These correspond to the onset pairs 4 & 5, and 9 & 10 in the timeline of Figure 19. Viewing the superimposition as a series of crests and troughs reveals 4 positive crests and 3 negative troughs—the essence of the 3:4 polyrhythm.

This waveform experiment emphasizes the holistic aspect of polyrhythm, with its corresponding sense of direction or trajectory over time. Waves make concrete the concept of apsis and conjunction, providing a linear chronology that complements the circular perspective of structure provided by timelines. The waveform analogy enhances the interdependent nature of the constituent layers, which contrasts the perspective of

34 Whenever two frequencies are combined, a frequency of their difference results. Musicians commonly experience this as *beating* when tuning two closely-matched pitches. When the two pitches are further apart, this low frequency oscillation experienced during tuning is replaced by beating of a rate sufficient to be heard as pitch. Helmholtz calls such pitches *combinational tones*, also known as *Tartini tones* after the eighteenth century Italian violinist (Helmholtz 152). Since the initial sine waves of my experiment were G4 (392Hz) and C5 (523.25Hz), a differential tone would be expected at a slightly sharp C3 (131.25Hz). Indeed, this low C was audibly present in my experiment, heard as a perfect twelfth beneath the lowest note, and could be considered one cause of some of the distortions of Figure 22. It is also notable that the equally-tempered perfect fourth used in this experiment differs slightly from the just intoned 4:3 ratio, which places C5 at 522.66Hz and would likely produce a more uniform waveform.

polyrhythm being a result of the stratification of independent, opposing layers.[35] Viewing polyrhythm ratios as composites of waves also facilitates an isomorphic relationship with pitch as demonstrated by my experiment, with the table of low-order integer ratios in Figure 3 contributors of much of the musical material that is the basis of tonality, discovered by Pythagorus some 2700 years ago (Tymoczko 62).

35 My experimental findings align with those of Helmholtz (154). More sophisticated tools that grant control over bandwidth and display parameters with plots are employed in the experiments by Herrero, and Plomp & Levelt. A powerful mathematical tool for analysing sounds and resolving them into their constituent simple sine waves is the *Fourier transform*. Fourier transform is able to offer insight into the vibrations of both harmonic and more complex aperiodic sounds. This tool was used by Voss & Clarke in their experiments into fractal noise.

3.9 Plane figurate numbers and polyrhythm

Figurate numbers are numbers that can be represented by a "regular and discrete geometric pattern of equally spaced points" (Deza & Deza 9). Figurate numbers have as their origin a desire by the Pythagoreans to connect the mathematical fields of arithmetic and geometry in the sixth century B.C.E. (Deza & Deza 9). I discovered experimentally certain relationships between polyrhythm and figurate numbers that offer insight into the structure of polyrhythm and its potential application.

Plane figurate numbers are represented in the two-dimensional plane. The first five members of triangular, square, and rectangular figurate numbers are illustrated in Figures 23 to 25. Further polygonal numbers and their respective sequences are derived from series of pentagons, hexagons, and so on.[36]

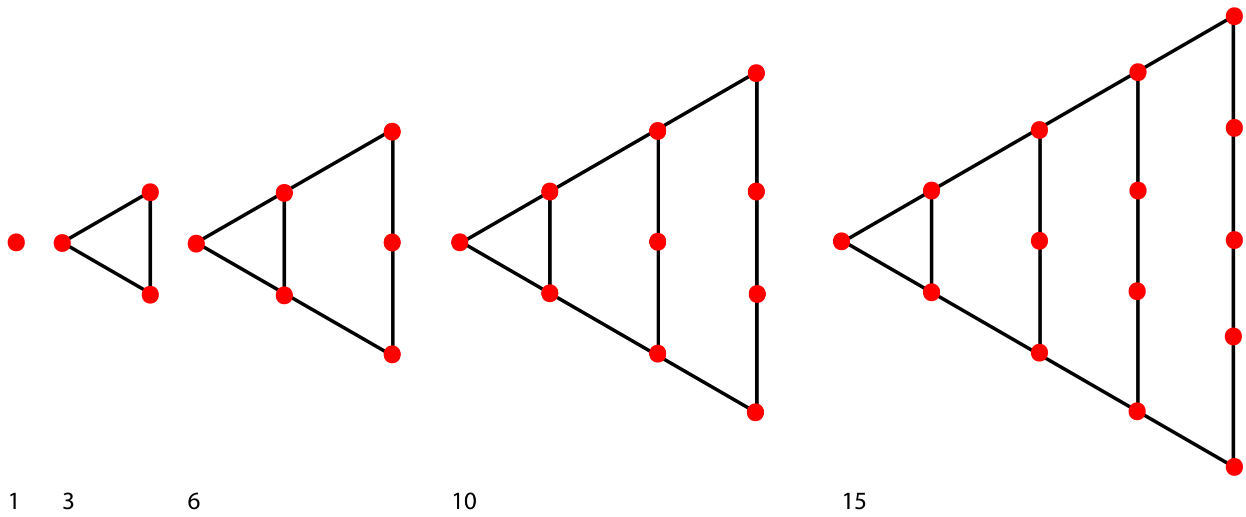

Figure 23: The first five members of triangular figurate number sequence (T1–T5). The values of triangular numbers are given by the formula Tn = n(n+1)÷2.

36 Refer to Sloane's *On-Line Encyclopedia of Integer Sequences* for more information on these and other mathematical sequences <https://oeis.org>.

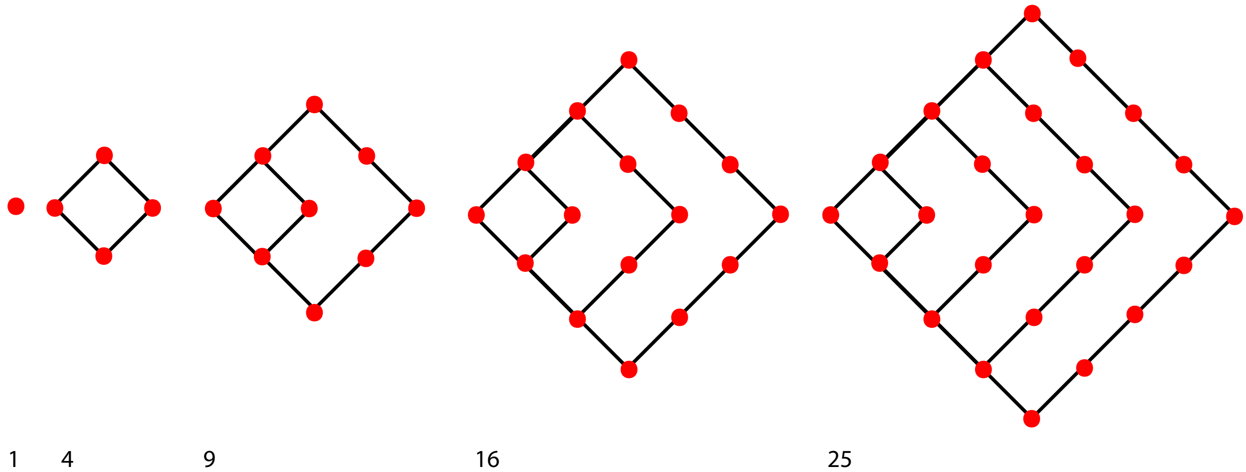

Figure 24: The first five members of the square figurate number sequence (S1–S5). The values of square numbers are given by the formula Sn = n^2.

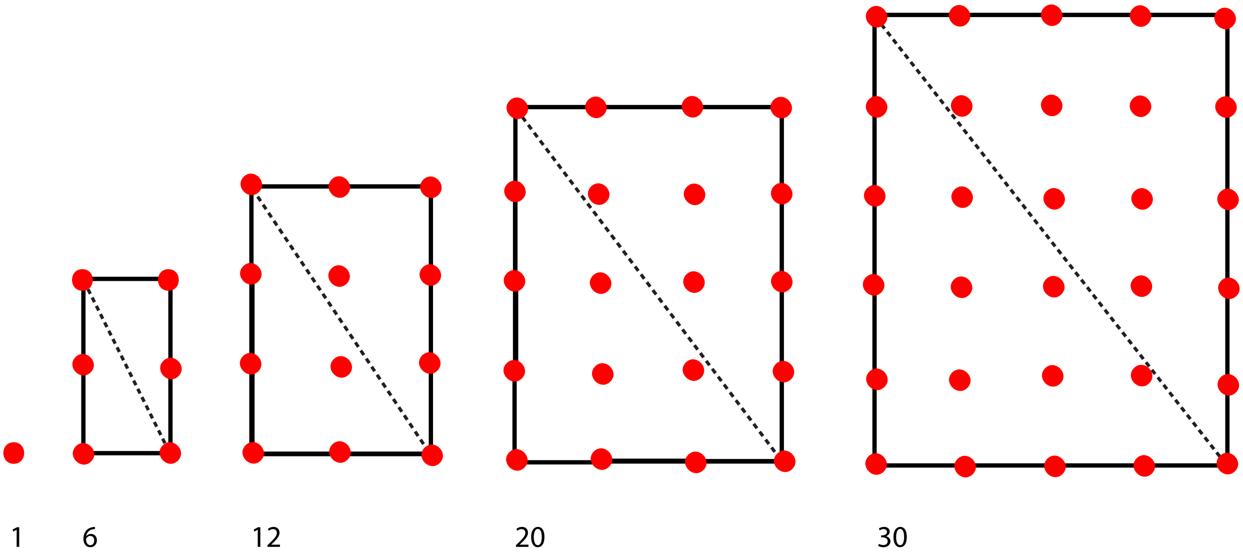

Figure 25: The first five members of the rectangular figurate number sequence (R1–R5). The values of rectangular numbers are given by the formula Rn = n(n+1). The dotted line shows the relationship to triangular numbers, being half the size as shown by comparing their two formulas.

The aforementioned definition of figurate numbers, and their geometric representation in these figures prompts an immediate connection back to the symmetrical formations introduced in Chapter 2. Furthermore a self-similar property emerges—the objects scale in a recursive manner where the parent object is contained within the offspring, thus resembling fractals. Further observation of the counting sequence reveals a progression that is arithmetic (not geometric), using a *gnomon*.[37]

37 The proofs of triangular and square figurate number sequences were not established until the late eighteenth century (Deza & Deza 10). The *n*-th figurate number in a sequence of any polygon with *m* sides can be found by S*m*(*n*) = *n*((*m*−2)*n*−*m*+4)÷2 (Deza & Deza 7).

A *gnomon* is a piece added to a figurate number to transform it to the next in the series, being a larger version of the original. The gnomon of triangular numbers is the positive integer of the form:

$n + 1$, $n = 1, 2, 3,\ldots$

where the n-th triangular number is formed by adding a gnomon of $n + 1$ to the prior member of the series (Deza & Deza 3). The first five members of triangular figurate number sequence in Figure 22 could be written as follows, with the respective gnomon appearing in brackets:

0+(0+1)
1+(1+1)
3+(2+1)
6+(3+1)
10+(4+1)

This self-similar gnomon concept is further illustrated by the following triangle of gnomons, based on these five triangular figurate numbers (Deza & Deza 3):

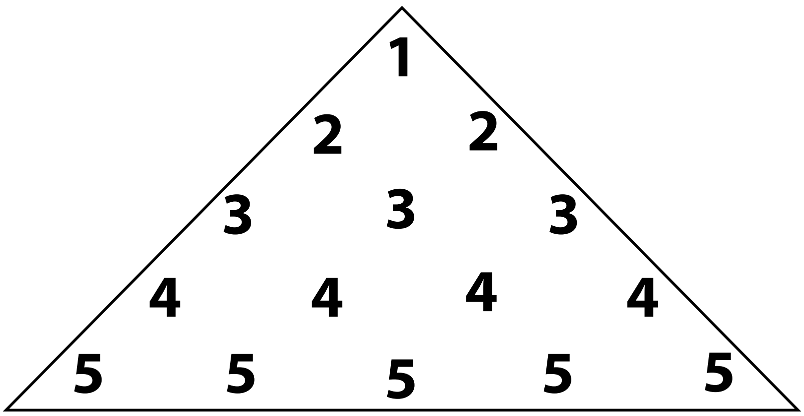

Figure 26: Triangle of gnomons, based on the first five triangular figurate numbers (T1–T5).

My observations of polyrhythm through the lens of post-tonal set theory has been introduced in section 3 of this chapter. Further experiments in this field now entwine with the use of figurate numbers. Firstly, I applied the rules of *normal form* to the rhythmic 'set' of polyrhythm.[38] Continuing with the example of a 4:5 polyrhythm, its set is {1, 5, 6, 9, 11, 13, 16, 17} with the inter-onset interval series <4-1-3-2-2-3-1-4> (see Figure 11). Normal form requires that the set be rotated to place the largest interval on the outside and the smallest on the left. This results in set {5, 6, 9, 11, 13, 16, 17, 1} with the inter-onset interval series <1-3-2-2-3-1-4-4>, which satisfies criteria of normal form. Transposing this to pulse 1 (the equivalent of zero if it were a pitch-class set) requires T–4 mod 20 for each member, resulting in the new set {1, 2, 5, 7, 9, 12, 13, 17}. I assumed the result would be one of the 20 necklaces of the original polyrhythm. Indeed it is this one:

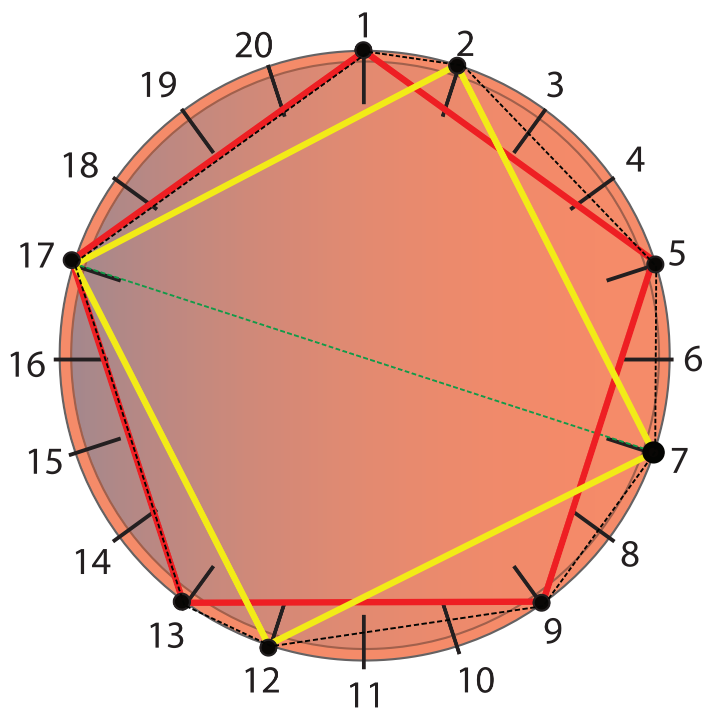

Figure 27: 4:5 polyrhythm timeline necklace with coincident hit point on pulse 17, as the result of *normal form* operation.

38 Refer to Rahn, "Basic Atonal Theory" 31.

The question that prompted this procedure was whether normal form is useful for organising or categorising timelines as it is for pitch-class sets. I concluded the only useful characteristic is that it is sure to create a representative necklace that commences on pulse 1. The 'most packed to the left' criterion of normal form however showed little value to the performing musician as an intuitive class of rhythmic categorization. This conclusion did give me a further idea about figurate number representation, however. I returned to the original inter-onset interval series <4-1-3-2-2-3-1-4> and reordered it in an increasing fashion to create (1, 1, 2, 2, 3, 3, 4, 4),[39] thus ignoring the order of the original set. I termed this the *unordered inter-onset interval series*,[40] and plotted the result as a figurate number arrangement.

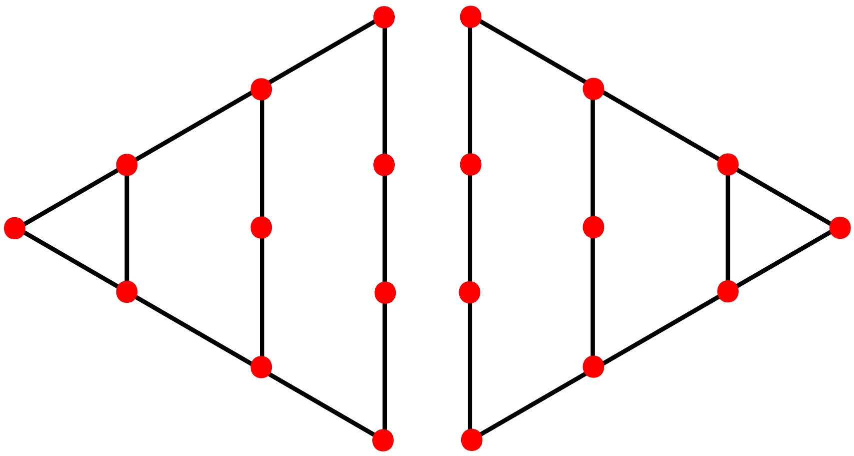

Figure 28: Double-triangle formation from the unordered inter-onset interval series of the 4:5 polyrhythm.

39 Note the particular use of brackets indicates the category of set membership, with triangular brackets for inter-onset interval series, curly brackets for the set of onsets, and now round brackets for the unordered inter-onset interval series.

40 This diversion in terminology from post-tonal theory is justified. In the pitch realm, pitches can occur above and below each other during an ordered sequence. In rhythm, because it is temporarily bound, onsets (the pitch class equivalence in rhythm) are always going to occur in increasing order over time. (If this were the case with pitch, it would mean that pitch class always sounds in increasing order within an octave, i.e. ascending.)

My goal was to find a figurate number geometry that represented the geometry of polyrhythm with its inherent mirror reflection made explicit by this new unordered inter-onset interval series. Since in polyrhythmic timelines, two constituent regular polygons combine to plot a new composite polygon, my rationale was that some kind of plane figurate number should be produced from a single two-dimensional geometry that demonstrated the symmetry of the internal structure. The mirrored arrangement of two triangular numbers in Figure 28 seemed unsatisfactory, however. Of the eleven polyrhythms listed in Figure 3, many conformed with either triangular, square, rectangular or hexagonal figurate numbers. The existence of exceptions (such as 2:7 and 5:7) prompted further research for a unified type of plane figurate number.

I created other bilaterally-symmetrical folded geometrical forms in the manner rectangular numbers may be derived from two triangles. My final discovery was the *isosceles trapezoid*—a form of convex quadrilateral with one pair of parallel sides, capable of bilateral symmetry.[41] A trapezoid (also known as a *trapezium*) neatly resolves instances of all symmetrical timelines whose total value is not a rectangular number. My experiments with figurate numbers arrived at this final thesis.

All polyrhythms can be expressed as trapezoids.

Figure 29 shows how the 5:7 polyrhythm—one of the polyrhythms that heretofore evaded figurate number representation—is constructed as a plane figurate number. The trapezoid is constructed based upon the interval vector, where the number of onset intervals of each size starting from the largest interval to the smallest is plotted from the middle outward in horizontal sequences. For 5:7 with its interval vector <<2,2,2,2,3>> I plotted the three 5s in a horizontal stack above each other, proceeding with the 4s, 3s, 2s, and lastly 1s, so that each vector straddles the initial core of 5s. The arrows in Figure 29 illustrate how the trapezoid is constructed in this manner, by progressively plotting the values from the interval vector reading right-to-left (largest to smallest inter-onset intervals).

41 A trapezoid is a four-sided polygon that comprises one pair of parallel opposite sides, called the *bases,* and two other sides, called the *legs*. An isosceles trapezoid is formed when both angles coming from the bases are equal, and the legs aren’t parallel but are equal in length.

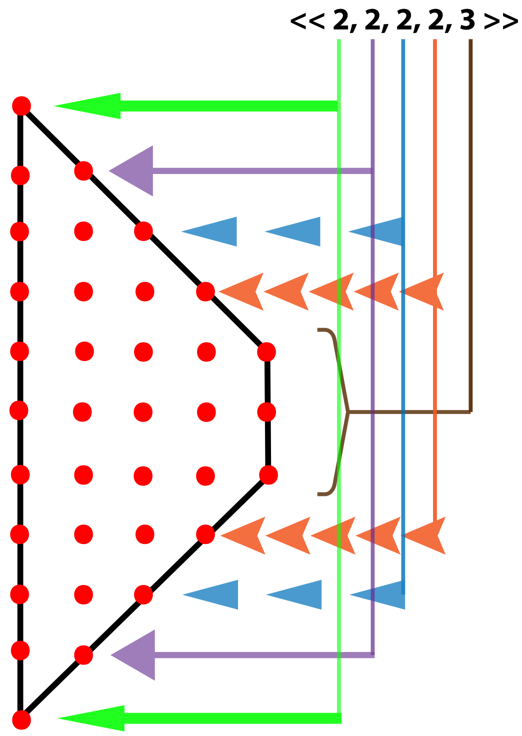

Figure 29: 5:7 polyrhythm as plane figurate number showing trapezoid construction.

Figure 30 shows the 4:5 polyrhythm as a plane figurate number. Compare this trapezoid with the alternative diagrams of the same polyrhythm in Figures 27 and 28.

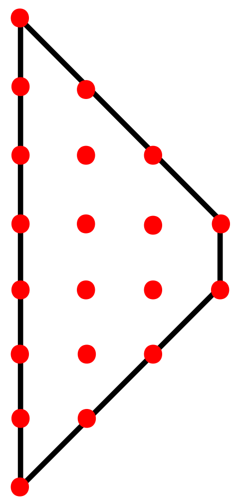

Figure 30: 4:5 polyrhythm as a plane figurate number (trapezoid).

The following observations can be made about trapezoids as useful representatives of any polyrhythm (*x:y*), such as those of Figures 29 and 30. Figure 31 presents all the polyrhythms tabulated in Figure 3 as trapezoids, and reinforces these observations:

- The number of trapezoidal columns (*c*) equals the smaller of *x* or *y*, which in turn equals the string length of the interval vector <<o, p, q, r, ...>>.
- Trapezoidal numbers are odd when their factors *x* and *y* are odd, and even otherwise.
- For odd trapezoidal numbers the sequence of column heights (*c*) will be based on an odd number sequence (e.g. 3, 5, 7, 9, …), whereas an even total will yield an even sequence of column heights (e.g. 2, 4, 6, 8, …).
- The smaller of *x* or *y* equals the largest member of the cumulative rhythm <j-k-l-m-...>.
- All trapezoids will have single points (1) at the extremes of the larger base, requiring that the polyrhythm represented is expressed in terms of lowest common denominator (and therefore will have 1 as its corresponding smallest inter-onset interval).
- Interval vectors for polyrhythms with even trapezoidal numbers will always contain a multiple of 2 of any existent inter-onset interval.
- Interval vectors for polyrhythms with odd trapezoidal numbers will always contain an odd number of rows in the trapezoid central ‘core’ (horizontal to the intersection of the legs and smaller base).
- Trapezoidal numbers are the difference between any two nonconsecutive triangular numbers (Gamer, Roeder & Watkins 108). See Figure 23.
- If *x:y* are adjacent counting numbers (such as 2:3), their factorization creates a *pronic* number, with adjacent factors in the form *x*:(*x*+1). The polyrhythm can be consequently represented by a rectangular number such as those illustrated in Figure 25. The following rectangular number sequence appears in the top row of Figure 31:

2:3 <<2,2>> R2

3:4 <<2,2,2>> R3

4:5 <<2,2,2,2>> R4

5:6 <<2,2,2,2,2>> R5

6:7 <<2,2,2,2,2,2>> R6

- If *x* times *y* equals half a pronic number factorization, the polyrhythm can also be represented by a triangular number, such as the middle row of Figure 31. (See also Figure 23.)

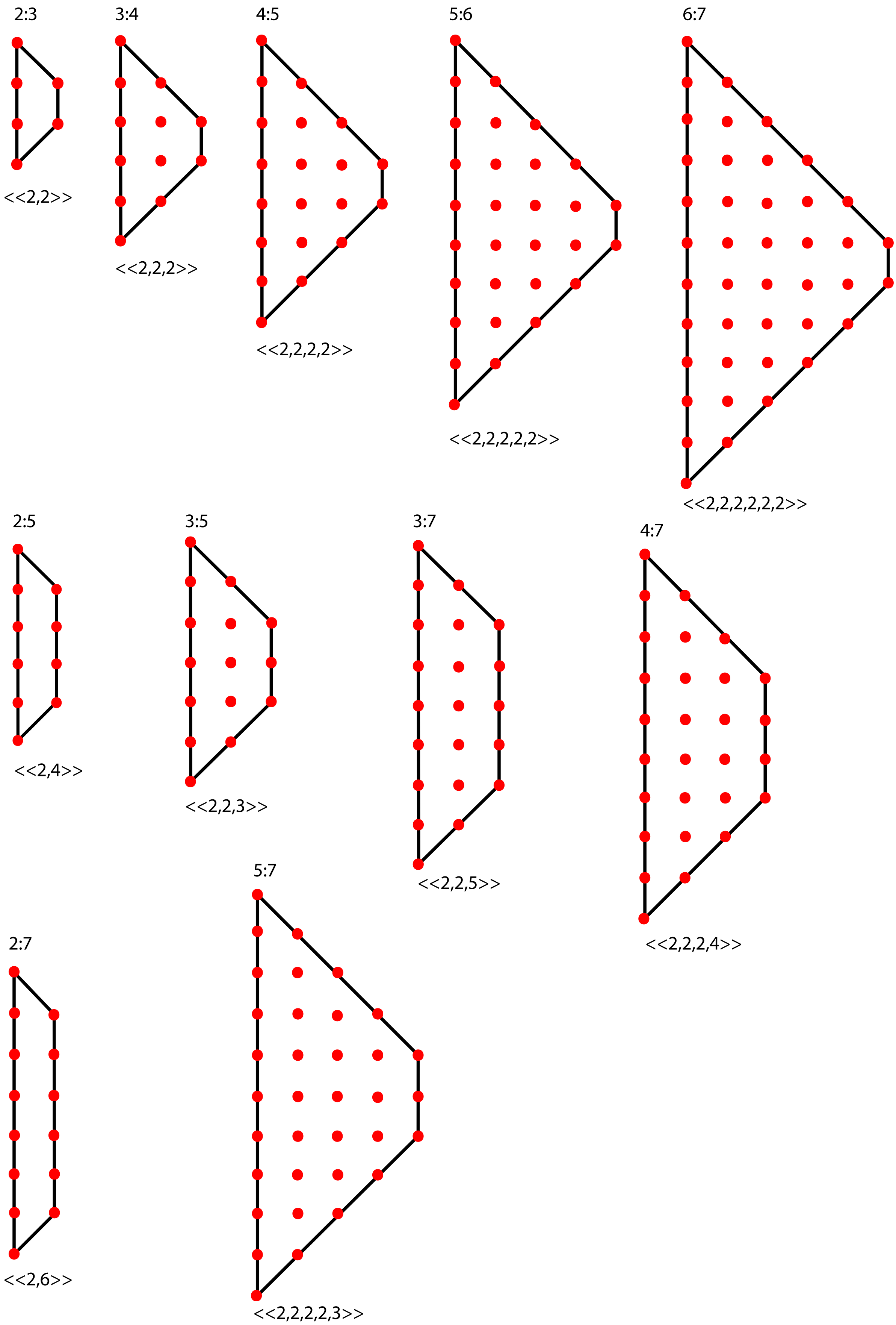

Figure 31: Eleven polyrhythms that combine integers from 2 through 7, as trapezoids.

Gamer and others discovered that the only numbers that cannot be written as sums of two or more consecutive integers are the powers of 2 (Gamer, Roeder & Watkins 108). These numbers (1, 2, 4, 8, 16, 32, 64, …) form the octave sequence illustrated in Chapter 2.9. Gamer and others makes the observation that trapezoidal numbers denote those partials of the overtone series that are not octave equivalents of the fundamental, and also makes the corollary that the sums of two or more consecutive frequencies in the overtone series create a partial not an octave equivalent of the fundamental (110).[42]

I have outlined my experiments that led to a unified geometric expression of polyrhythm as figurate numbers. Trapezoids are not merely neat illustrations, but connect polyrhythm theory to Golden Section,[43] the overtone series, and other aspects of symmetry and self-similarity that are the subject of much of this dissertation.

42 The partitioning of material into consecutive chunks is a technique employed in twelve-tone serial composition, as discussed by Schoenberg regarding his *Variations* opus 31 ("Style and Idea" 236), and theorised by John Rahn ("Basic Atonal Theory" 74–96). Madden looks at the self-similarity of such musical choices (95), and Gamer and others makes the connection to trapezoidal numbers (108).

43 Refer to Kaplan et al., "A Simple Construction of the Golden Ratio," *World Applied Sciences Journal* 7 (2009): 833.

3.10 Metric modulation

Metric modulation is defined in various ways by composers and theorists, but in my practice, polyrhythm is considered its salient feature. The Oxford Dictionary defines *metric modulation* as a “term and technique introduced by American composer Elliott Carter for changing the rhythm (not necessarily the metre) from one section to another.”[44] The Oxford Companion to Music defines it as “a technique introduced by Elliott Carter, by which changing time signatures effect a transition from one metre to another, just as a series of chords can effect a harmonic modulation from one key to another.”[45]

Tempo is primarily a function of the periodicity of beat, upon which metre depends, which is one reason the former definition is so unsatisfactory. It lacks a distinction between *rhythm* and *beat*. The latter definition is more consistent with my metric approach, and the naming of Carter as an exemplar is justified,[46] since his linear approach is in such stark contrast to the *moment form* of Stockhausen in the 1960s (Kramer, “The Time of Music” 204).

Better still is Starr’s definition, being “the achievement of a perceptible and precisely executed change of pulse duration through notational means only” (Starr 36). This definition fits with modern performance practice as it does with music from the Romantic era. The reference to notation is debatable however, though arguably present to eliminate confusion over conducted, improvisatory and other uncontrolled or indeterminate performative influences. I advance my own definition, consistent with other terminology in this chapter.

Metric Modulation: The redefinition of beat resulting in a change of perceived tempo and possibly metric structure.

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44 Kennedy, Michael. ”Metric Modulation.” The Oxford Dictionary of Music Online. 2nd ed. 1 Mar. 2017 <http://www.oxfordmusiconline.com.ezproxy.library.uq.edu.au/subscriber/article/opr/t237/e6759>.

45 Latham, Alison. “Metric Modulation.” The Oxford Companion to Music Online. Accessed 1 Mar. 2017 <http://www.oxfordmusiconline.com.ezproxy.library.uq.edu.au/subscriber/article/opr/t114/e4389>.

46 This definition is notably Euro-centric however. The creation of the illusion of different tempi within a set *laya* is found in South Indian *gati bedam* technique. Refer to Reina for a Western interpretation.

A *l'istesso* change from 12/8 to 4/4 (with the dotted crotchet equalling the new crotchet) is rejected under my definition of metric modulation, as the result is merely a change of subdivision. If the crotchet was held steady in this situation however, the beat would be scaled by 2/3 of the rate, resulting in metric modulation and a requisite change in tempo. Figure 32 illustrates these scenarios.

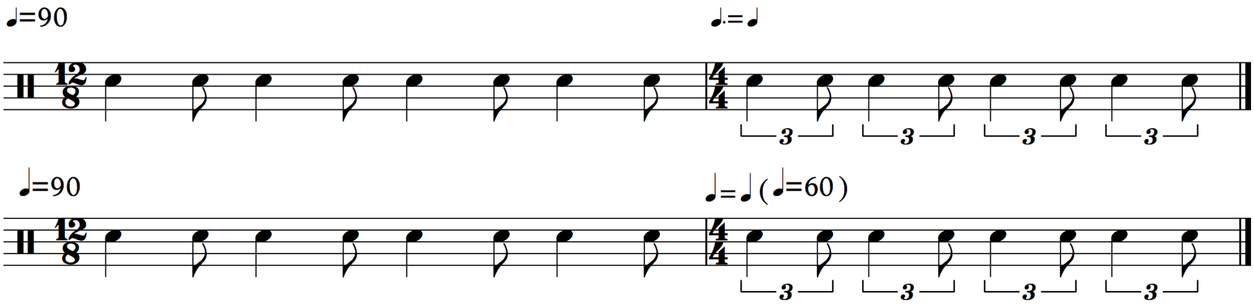


Figure 32: Metric modulation is present in the bottom line but not in the top line, as it does not involve a scaling of beat/perceived tempo.

Metric modulation is a natural outcome of polyrhythm, and it may be encouraged (through presentation of multiple beat-like layers with corresponding harmonic or melodic material), implied (as in traditional *hemiola*) or enforced by the composer through establishment of specific modulatory formulae in notation. The establishment of recurring polyrhythmic layers in stable time cycles may result in a perception of *ad hoc* metric modulation by the listener even when it is not intended by the composer. In establishing the ambiguity and paradox that polyrhythm entails, the listener is free to flit between opposing layers in terms of beat autonomy, hierarchy and frame of reference. All of these approaches are contained in *Mod Times* (see Chapter 6).

Chapter 4
Diagrammatic survey of symmetry in folio of works

This chapter surveys occurrences of symmetry present in the following folio compositions: *Locked-In, Paco, Binary Times, A Kayak,* and *Perfect Storm.* Symmetrical relationships are illustrated on microscopic levels (such as short rhythmic timelines), moderate levels (such as extended themes), and macroscopic levels (such as seven-layer canons, lyric patterning, and deep-structure symmetrical procedures responsible for generating the mode of an entire work). Bilateral, translational and rotational symmetries are all represented.

4.1 Locked-in

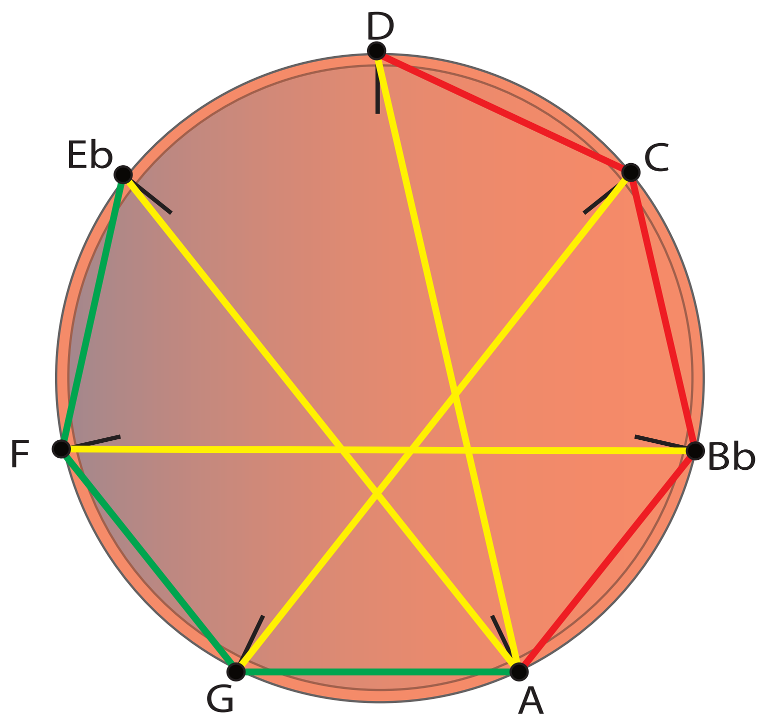


Figure 1: Translational and bilateral symmetry of the introduction (bar 1–4) arising from relationship of the top line (red), inner line (green), and sequence of the dyadic motive in the accordion (yellow).

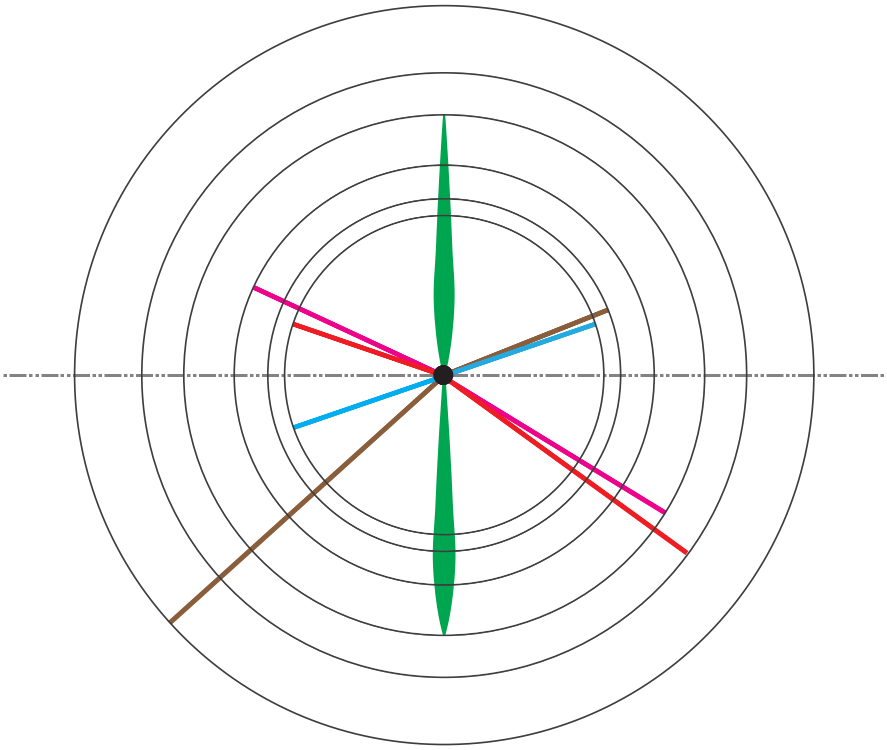


Figure 2: Symmetry of syllabic patterning of lyric. The number of syllables per section are represented as lines radiating above the horizontal for the first half of the composition, and below the horizontal for the second half. Colour-coding matches the score sections as follows: red (A & H), pink (B & I), blue (E & G), brown (D & F), and green (C & J).

4.2 Paco

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
L		•				•				•			•			•			•					
R	•			•			•							•				•				•		

Figure 3: Symmetry arising from the reflection rhythm interpretation of the cumulative rhythm for the polyrhythm 3:4 coincident on 10 <1-2-2-1-3-3>, orchestrated over two cycles (24 pulses). This rhythm appears early on in the work (see section C) and is fully developed in the penultimate section (see section D).

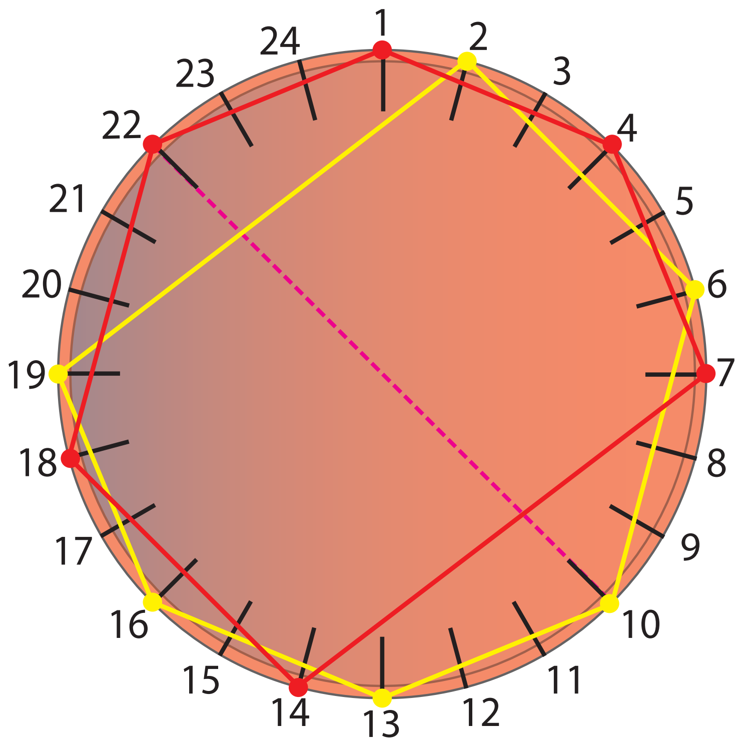


Figure 4: Timeline illustration of the box diagram of Figure 3 revealing bilateral symmetry about the axis of reflection, and the orchestration of the left (yellow) and right (red) hands. The timeline illustrates how, as a reflection rhythm interpretation of the 3:4 coincident rhythm, the polyrhythm layers swap sides (hands) about the coincident hit point (pulse 10 and 22).

Figure 5: Translational symmetry arising from the seven-part canonic interpretation of the reflection rhythm <4-4-3-3-3-7> in section D. This rhythm results from the pattern formed by each hand (see Figure 4). This section, reminiscent of Flamenco *palmas*, is played by hand-claps, foot-triggered percussion and drum set, with the following orchestration of the seven canonic voices:

L1 Bass–claps

L2 Drums–claps (moving to snare from bar 76)

L3 Bass–foot taps

L4 Trombone–foot taps

L9 Drums–tom-tom

L10 Drums–ride cymbal

L11 Trombone–claps

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
L1	•				•				•			•			•			•						
L2		•				•				•			•			•			•					
L3			•				•				•			•			•			•				
L4				•				•				•			•			•			•			
L9		•							•				•				•			•			•	
L10			•							•				•				•			•			•
L11	•			•							•				•				•			•		

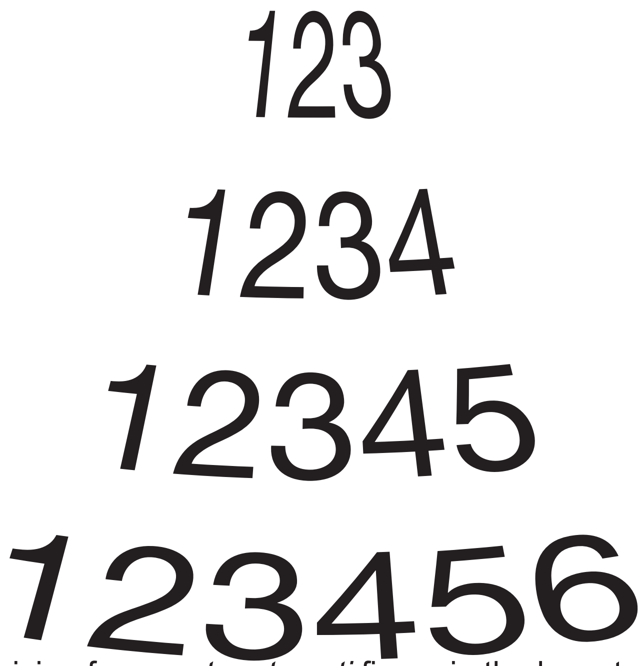


Figure 6: Symmetry arising from *srotogata yati* figure in the bass trombone as the cadence of section D, from bar 82. The cadence concludes with an extension of values 7 and 8 played with the drum set and bass guitar.

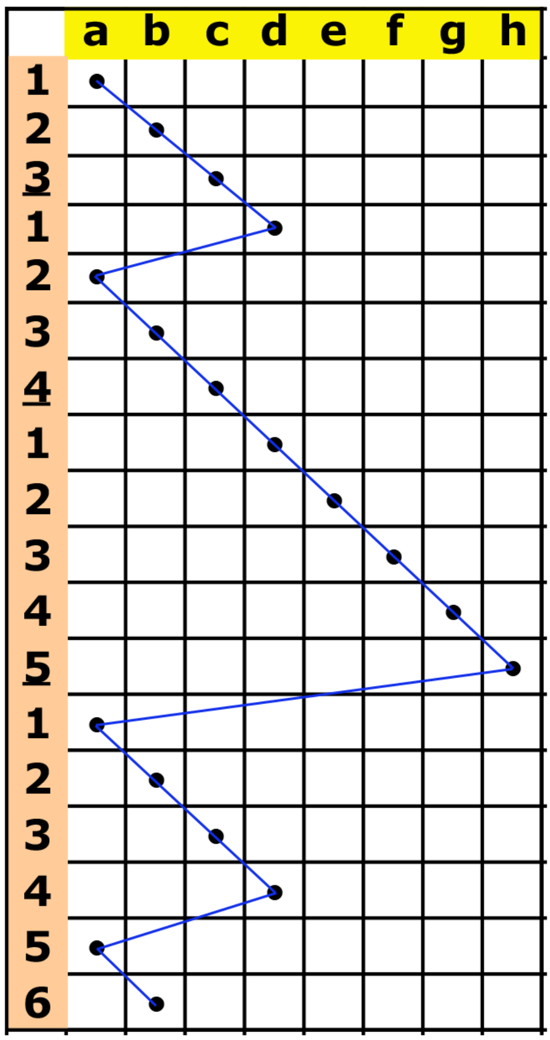


Figure 7: [Previous page] Rotational symmetry arising from *isorhythmic* procedure combining the 8 pitches (*colour*—horizontal axis) of the bass trombone theme from bar 42–43 with the 18 onsets (*talea*—vertical axis) of the *srotogata yati* figure illustrated in the previous diagram, bars 82–86. The rotational effect is enhanced by the circular nature of the pitch collection (*colour*), which features three iterations of a 4-note motive *a–b–c–d* during the passage from bars 42–44.

4.3 Binary Times

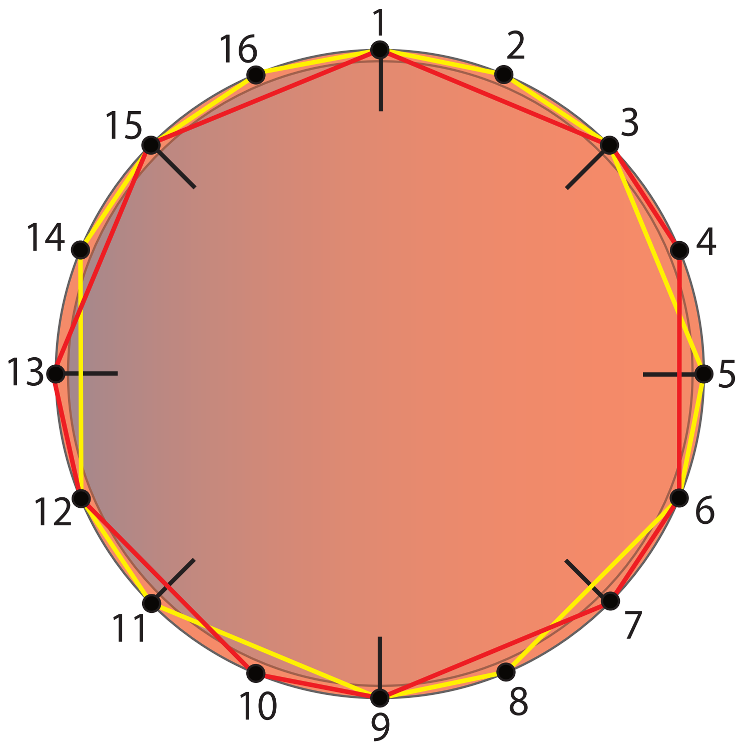


Figure 8: Symmetry arising from *kotekan* figurations during the trombone solo, bars 104–105.[1]

1 As this section is improvised the diagram is only indicative of the tiling that is occurring spontaneously between the instruments.

4.4 A Kayak

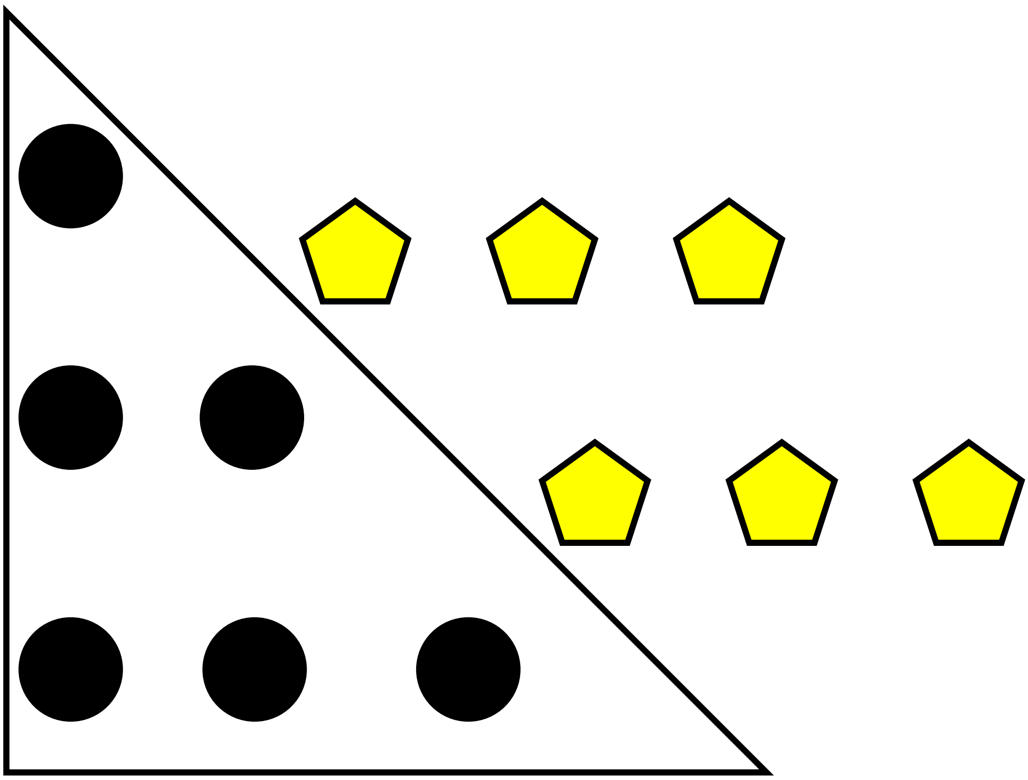


Figure 9: Symmetry arising from the *srotogata yati* figure in the introduction from bar 8. The timeline has 36 pulses, divided into three stages, and thus resembles a *tihai* or *arudi*. The black dots represent the bass line and the yellow pentagons the trombone *karvai* between these stages. This figure is recapitulated for the conclusion of the work (from bar 131).

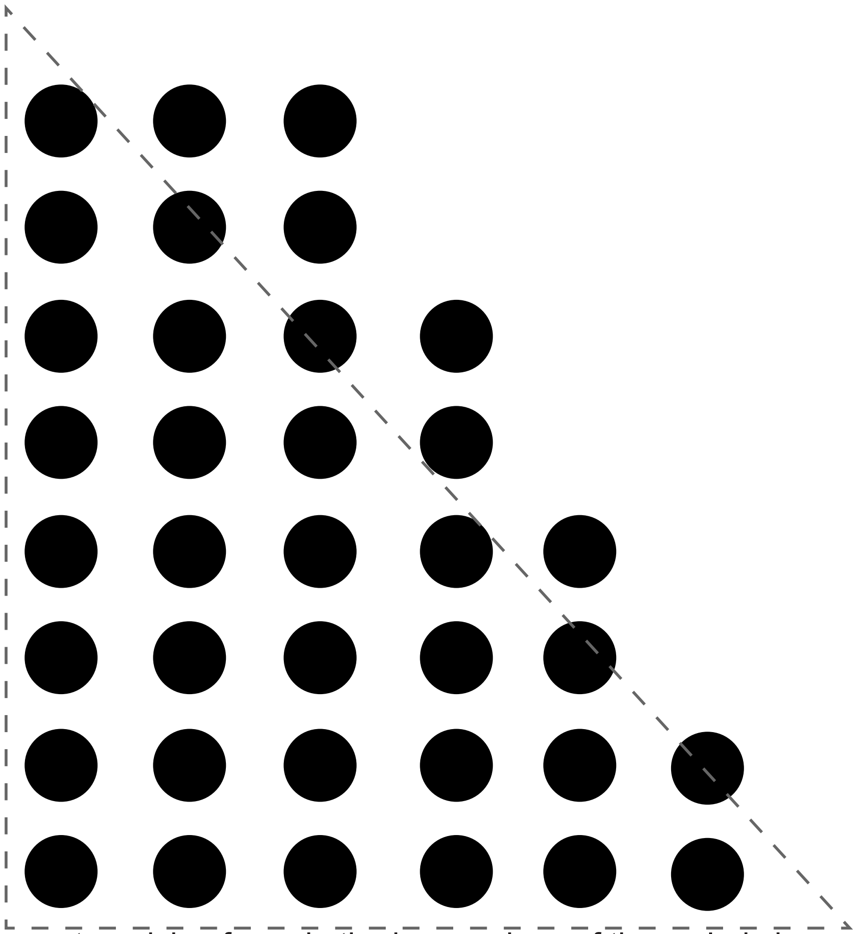


Figure 10: Symmetry arising from rhythmic groupings of the melody in section C (from bar 30). The timeline has 36 pulses, realised here as a figurate number resembling a *srotogata yati* figure.[2]

2 36 is the eighth triangular number, being the accumulated sum of 1+2+…+8. The terraced representation in this diagrammatic representation arises from line doubling from 3 to 6, and still takes on a triangular form.

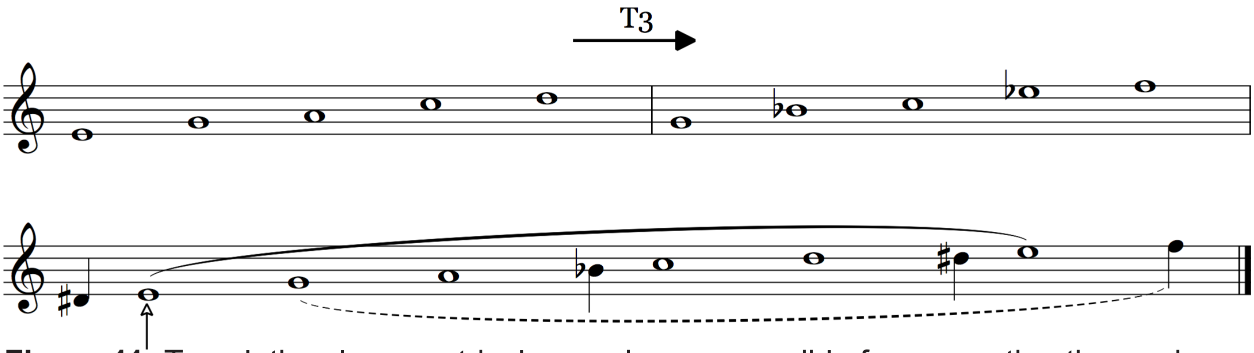


Figure 11: Translational symmetrical procedure responsible for generating the mode utilised for the work. Pentatonic mode three from E is transposed up 3 semitones (top line) and then the two forms combined (bottom line). The two discrete keys are sometimes exposed thematically (e.g. sections C versus section D), and other times the two are juxtaposed as a composite mode (e.g. section I, J). The two common pitches of the modes (G and C) are featured in the trombone and bass parts during "percussive" roles (e.g. bass introduction and sections B and D, trombone bar 8–10 and 23–29).

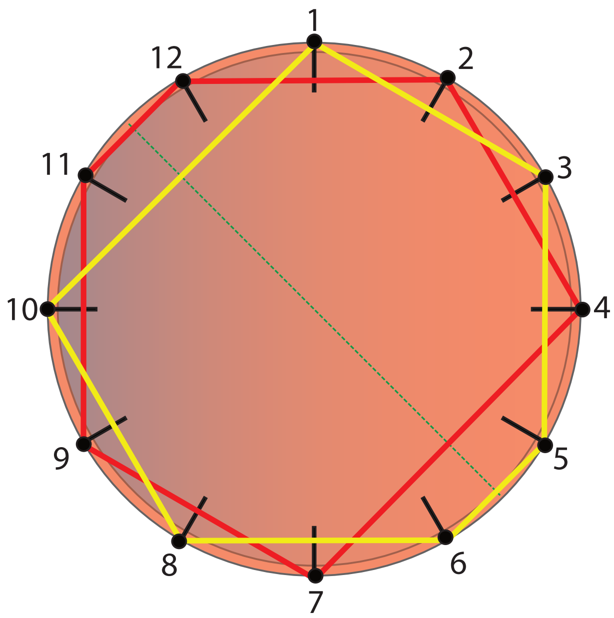


Figure 12: Tiling of 12-pulse timeline via the reflection rhythm <2-2-1-2-2-3>.[3] This rhythm commences the work (bass from bar 1) and is the basis of the bass line and drum pattern featured with the theme (e.g. sections B, C, E).

3 A mirror-symmetric image of a rhythm is the rhythm's complement. This particular *bell pattern* is studied extensively as the 'standard pattern' of sub-Saharan Africa and the Caribbean. Refer to Toussaint; Agawu.

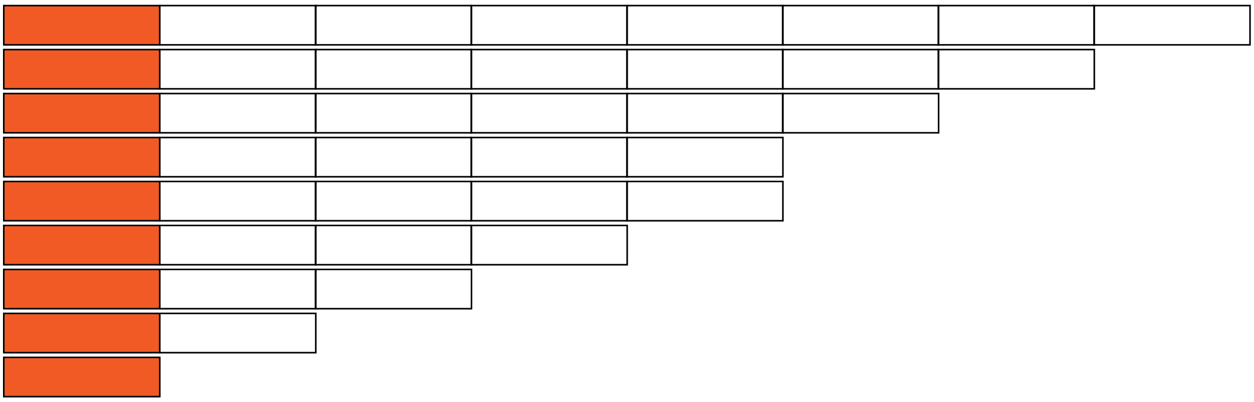


Figure 13: Symmetry arising from rhythmic groupings of the transition from the trombone to bass solos section J (from bar 107). The timeline has 36 pulses, realised here as a figurate number resembling a *gopuccha yati* figure—an inversion and reshaping of the *srotogata yati* of Figure 10.

4.5 Perfect Storm

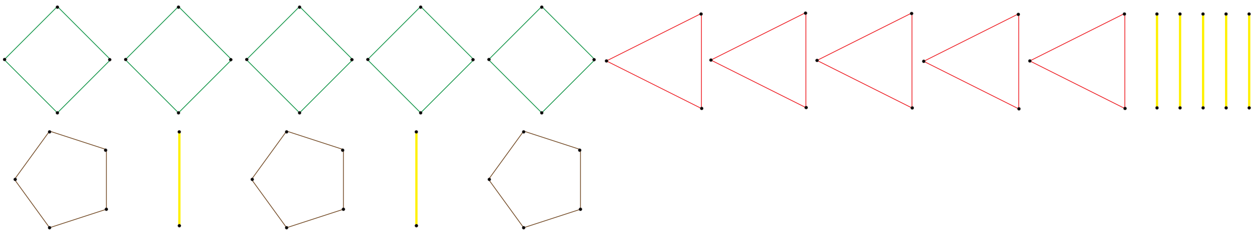


Figure 14: Symmetry arising from final *korvai* (section F). Each polygon represents the duration of an onset. The first line shows motivic scaling (and thus self-similarity) as well as translational symmetry. The second line shows a typical tripartite *arudi* patterning. The entire structure is sounded three times, encompassing 576 pulses, and so shows the translational symmetry of a *chakradar tihai* on a macroscopic scale.

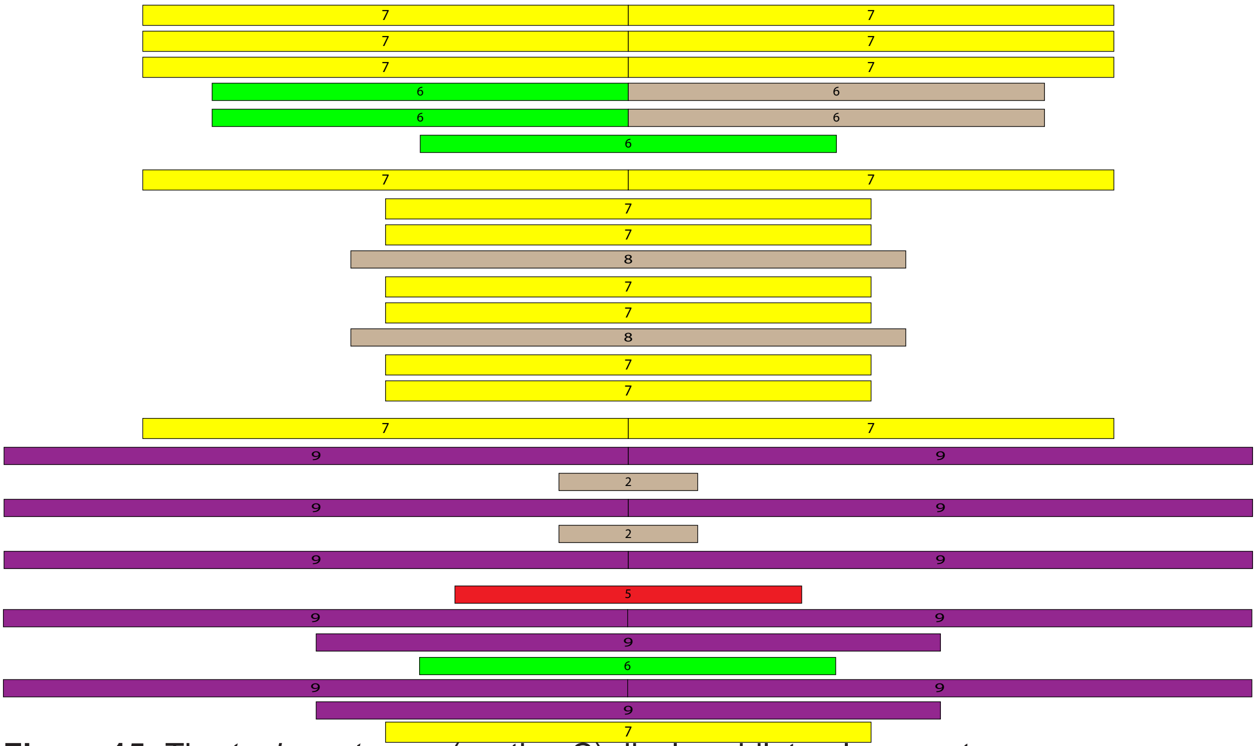


Figure 15: The *tani avartanam* (section C) displays bilateral symmetry on a macroscopic level and within phrase structure. Parts one through four of the leading *mridangam* drum are illustrated, with *karvai* portions in brown and the constituent phrases colour-coded. Each part totals 72 pulses.

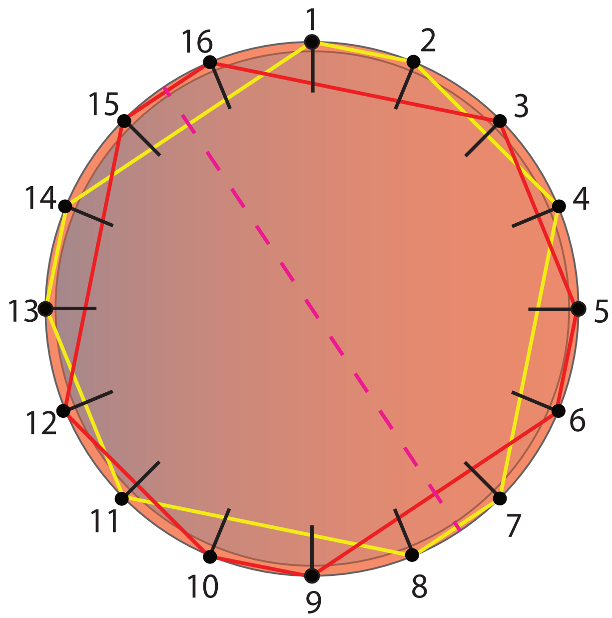


Figure 16: The *kendang* feature (section D) comprises rhythmic tiling canons. This timeline illustrates the bilateral symmetry of the first bar (118) of the *kendang* theme (yellow polygon) with inter-onset intervals <1-2-3-1-3-2-1-3>, which is symmetrically translated in the *tabla* and *mridangam* parts (red polygon bracelet) in such a way that their combination not only tiles the complete 16-pulse timeline without hole or overlap, but also provides mirror symmetry overall.

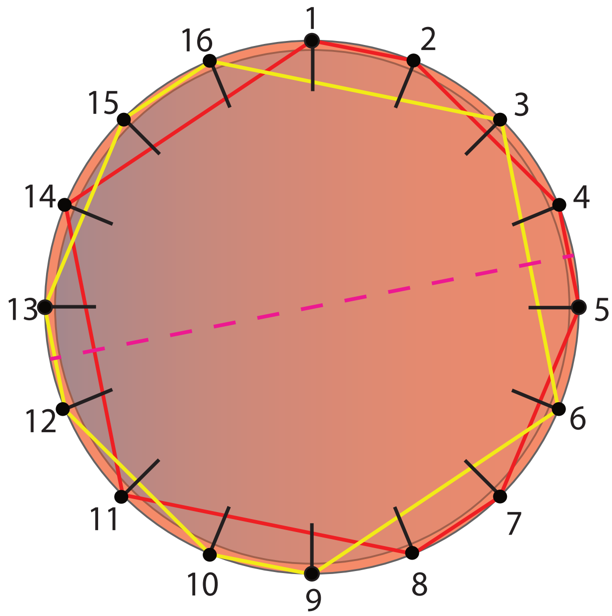


Figure 17: Timeline illustrating the bilateral symmetry of the second bar of the *kendang* theme of section D (bar 119). The *tabla* and *mridangam* parts lead, in response to the *kendang* in the prior bar. Their inter-onset interval pattern (red polygon) <1-2-1-2-1-3-3-3> is symmetrically translated in the *kendang* part (yellow polygon) in such a way that their combination not only tiles the complete 16-pulse timeline without gap or overlap, but also provides mirror symmetry overall. The red and yellow polygons are bracelets of each other that create a perfect open tiling.

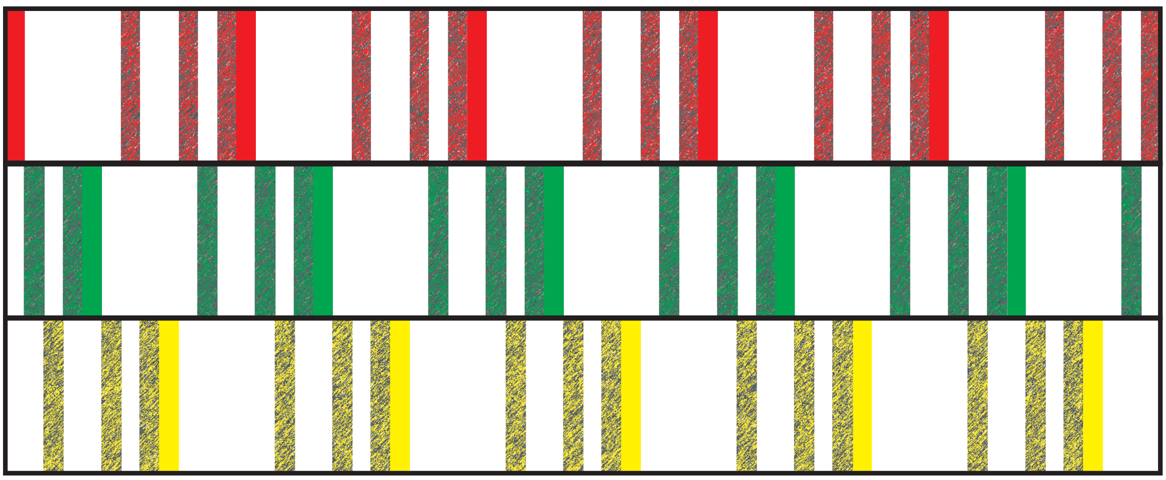


Figure 18: Perfect open rhythmic tiling canon design of the cadence of the *kendang* theme of section D (commencing bar 126 beat 2). The *kendang* (red), *tabla* (green), and *mridangam* (yellow) all sound translations of the same motif <6-3-2-1>, symmetrically translated in such a way as their combination tiles the complete 60-pulse line without gap or overlap.

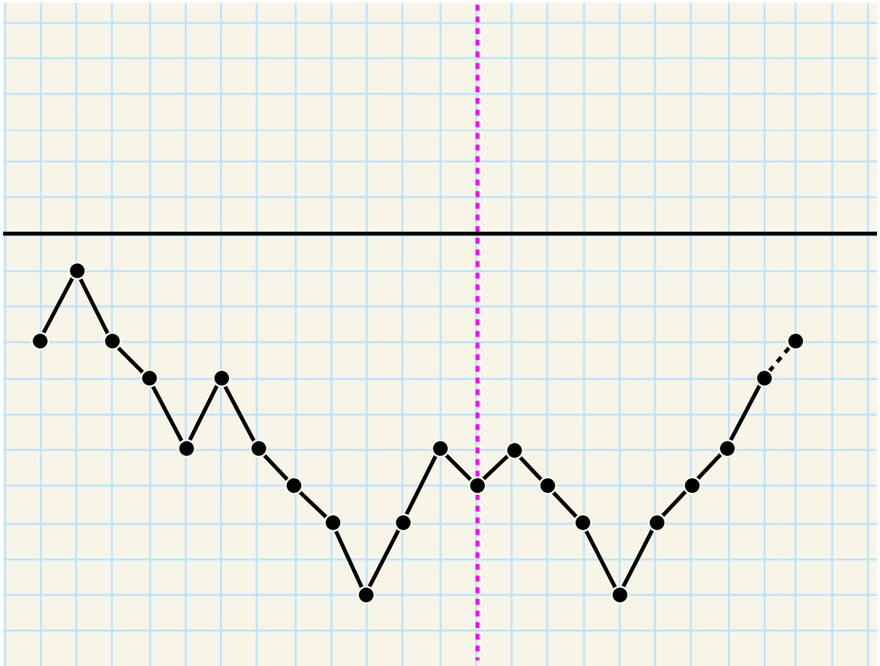


Figure 19: Approximate bilateral symmetry arising from the pitch contour of the *asthayi* line 1 (introduced as the first theme, bars 6–9).[4]

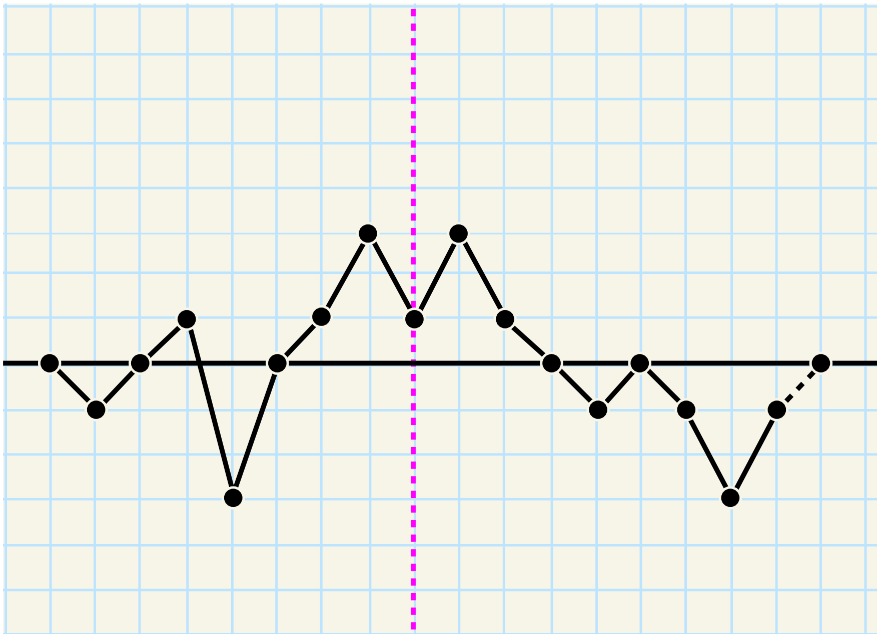


Figure 20: Approximate bilateral symmetry arising from the pitch contour of the *antara* line 1 (introduced as the second theme, bars 37–40).[4]

4 The black horizontal line represents pitch D4, whilst the pink dotted vertical line represents the axis of reflection. The dotted black line on the right indicates the link back to the first pitch, as the theme repeats (translational symmetry).

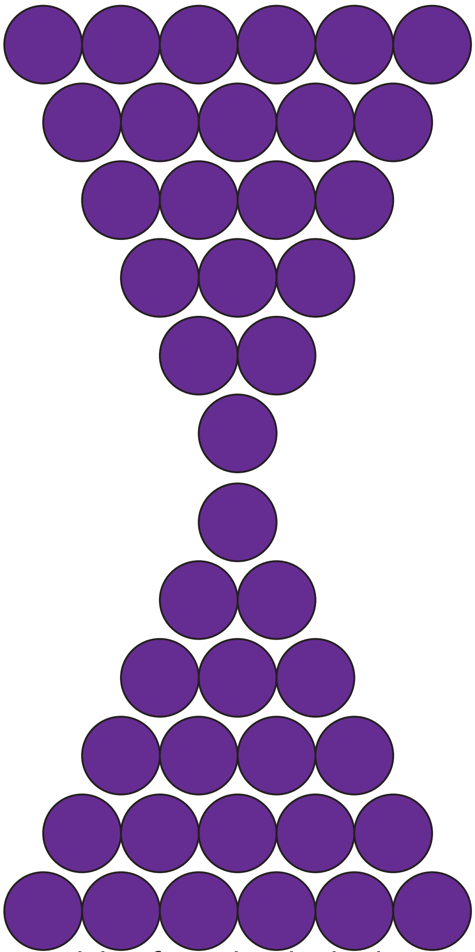


Figure 21: Bilateral symmetry arising from the rhythmic patterns in the drum break, bars 49–52. The palindromic sequence is a *damaru yati*. The 72 pulses comprise two triangular numbers of value 21 each (T6), with *karvai* (gaps) of 3 between each line (except the central join).

Chapter 5
Diagrammatic survey of polyrhythm in folio of works

This chapter surveys occurrences of symmetry present in the following folio compositions: *Locked-In, Paco, Binary Times, A Kayak,* and *Perfect Storm.* Timelines are most useful for revealing rhythmic stratification, and so feature in this chapter. Whilst *ad hoc* occurrences of polyrhythm saturate much of my music, only special occurrences of persistent motivic usage of polyrhythm, or large-scale polyrhythmic ideas in my compositions are surveyed here.

5.1 Locked-in

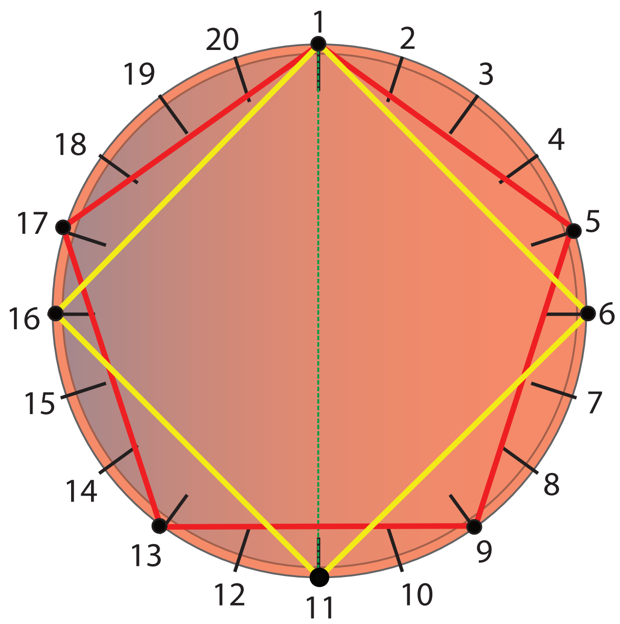


Figure 1: The 5:4 polyrhythm is used directly and incompletely from different starting positions and in different subdivisions/scalings. Refer to trumpet bar 6, bass trombone bar 77, bass guitar bar 69, accordion bars 57 and 67, and the ensemble bar 55.

5.2 Paco

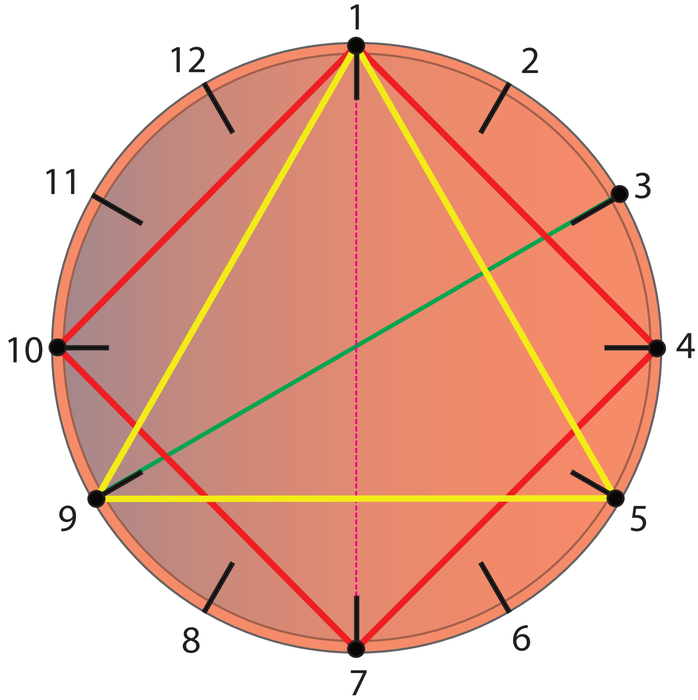


Figure 2: Timeline for sections A and B, consisting of 3:4 coincident on 1, overlaid with displaced 2:12 coincident with the ninth pulse (third beat of the 3 side of the polyrhythm), to fortify the opposition of the harmonic rhythm to the down-beat. 4:3 (the inversion of the same polyrhythm) is also featured in the final section E.

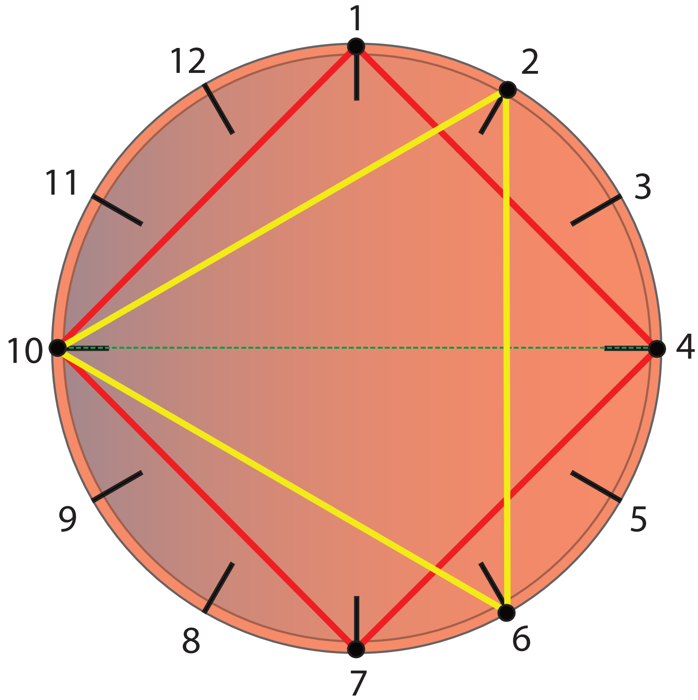


Figure 3: Timeline for section C, consisting of 3:4 coincident on 10.

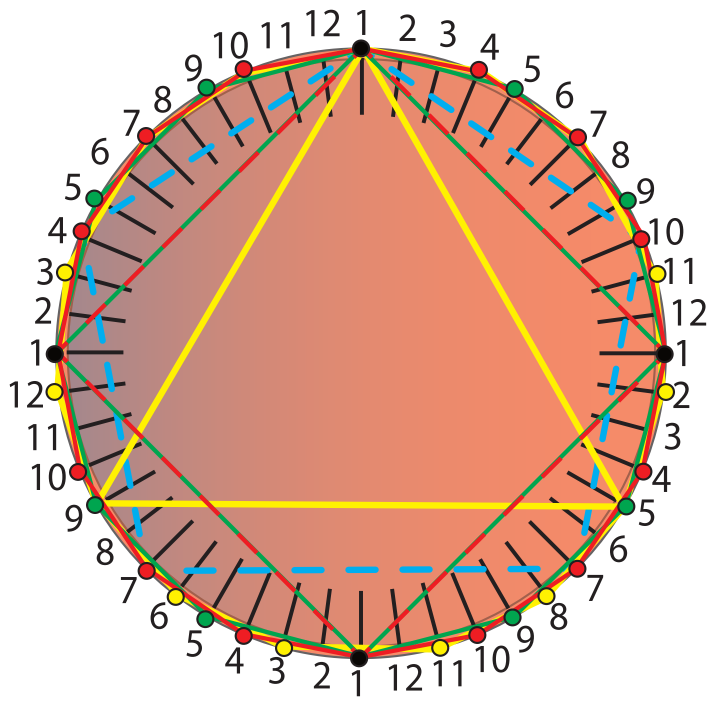


Figure 4: Timeline for section E (from bar 97), consisting of 3:4 polyrhythm in macroscopic scale arising from 16-pulse *bossa-nova* pattern (yellow) overlaid thrice during four bars of 3/4 metre. Colour-coding indicates the micro-level 4:3 polyrhythm (green and red, respectively), and the implied over-arching triple-scale *bossa-nova* pattern (dashed blue). The typical *surdo* drum pattern is captured by the bass guitar and floor tom, in an altered 3/4 metrical pattern. This coda thus combines symmetry, polyrhythm and self-similarity.

5.3 A Kayak

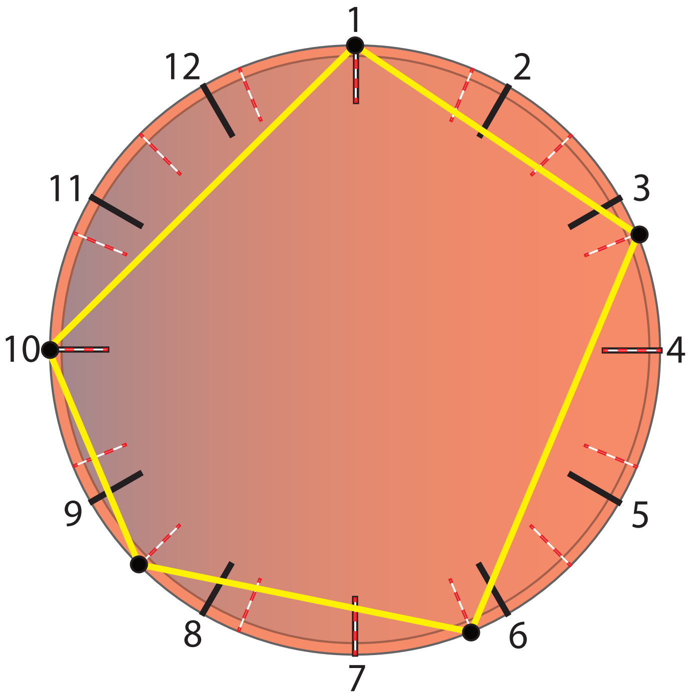


Figure 5: Timeline showing the *ternarization* of the 3-2 *rumba clavé* at modulatory points (e.g. bars 52–53). The 16 dashed red/white metrics indicate the inference of the 2/4 metre to which the *rumba clavé* belongs.

5.4 Binary Times

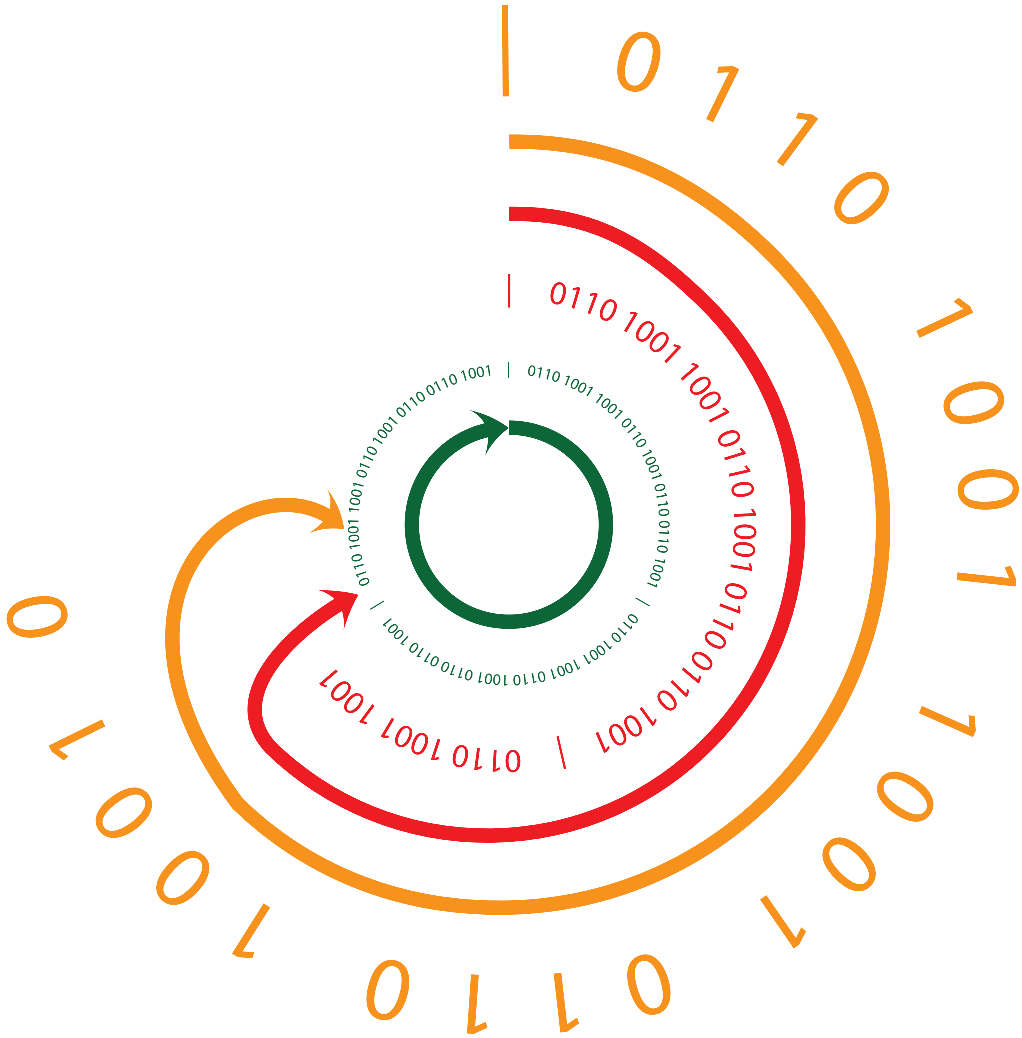


Figure 6: Large-scale polyrhythm arising from the proportion canon in the coda, from bar 109 to the end. The three concentric circles proportionally trace the 32-bit *Thue-morse* sequence in three converging tempo levels (scalings) in the bass trombone (red, 1:1), drum set (green, 3:2) and bass guitar (orange, 1:2). Zeros represent note onsets, and the sequence begins at twelve o'clock. Arrows indicate the trajectory of the three lines as the bass trombone and then bass guitar merge to join the drum set.

5.5 Perfect Storm

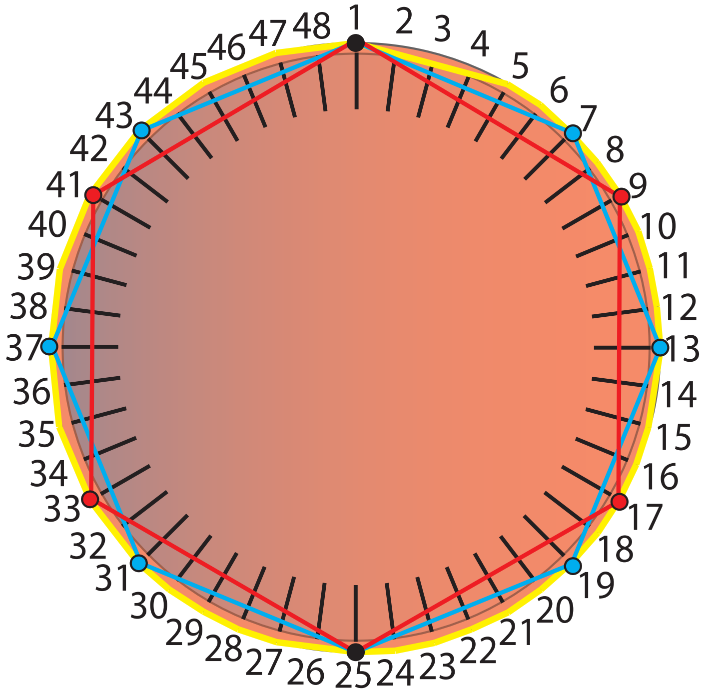


Figure 7: Timeline showing the *ternarization* of the *tabla kaida* in section E. Originally presented in 6/4 metre (red hexagon), the theme and variations are modulated into 4/4 (blue octagon). The initial theme is shown as the yellow polygon.

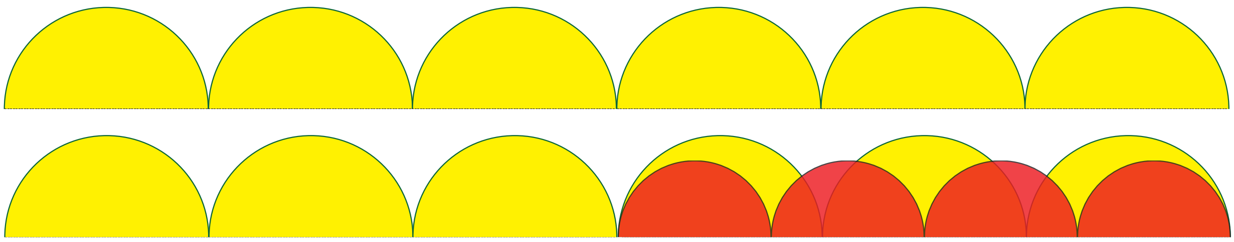


Figure 8: Geometric representation of the 4:3 polyrhythm intrinsic to 6-beat *dadra tala* featured as the rhythmic cycle throughout the piece. Refer to the drum set, bars 5–8.[1]

1 Like the *savari talas, dadra tala* often includes 'fractional' beats at this cadential position. The four 3/4-length durations (shown here as red semicircular overlays) are also intrinsic to the 11-beat *tala chartal ki savari*, for example.

Chapter 6
Analysis of work 1—*Mod Times*

6.1 Background

“Mod” in the title refers to the terms *modulation* and *modulo*—two terms that relate to processes featured in the essential rhythmic development of this composition.

Although the principal developmental technique in *Mod Times* is geometric in origin, the majority of the geometric diagrams appearing in this chapter were created retrospectively. These diagrams are offered as useful analytical tools that may also be applied pro-actively for the creation of new musical ideas.

The primary organizing feature of *Mod Times* is the *Pythagorean triple*, which is a theorem that is used to determine features of the right-angled triangle. Represented algebraically as $a^2 + b^2 = c^2$, it is the fundamental geometric law for determining the relationship of the length, width and hypotenuse of a right-angled triangle. The smallest numeric representation of the Pythagorean triple is $3^2 + 4^2 = 5^2$ and is represented geometrically in Figures 1 and 2.

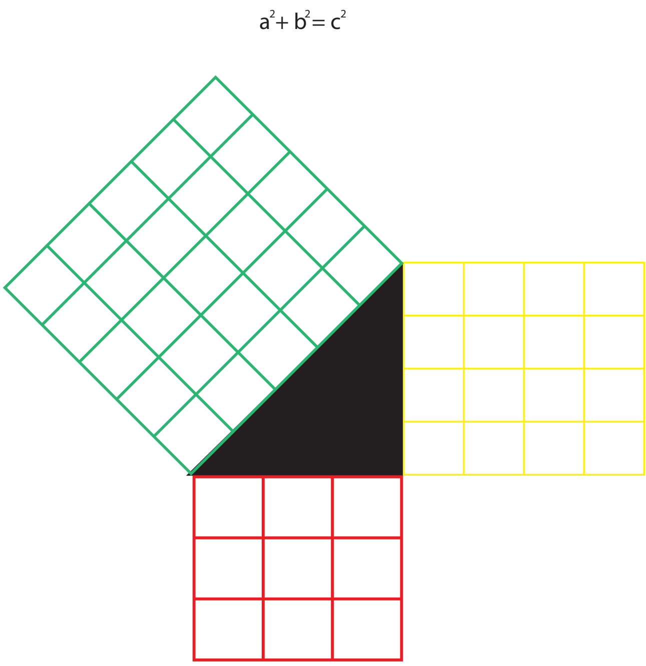


Figure 1: The *Pythagorean triple* – 3:4:5 triangle showing $3^2 + 4^2 = 5^2$ relationship.

The diatessaron pentagon of Figure 2 shows the Pythagorean triple relationship and generates appropriate proportions in the ten right-angles triangles enclosed forming the pentagon and the containing circle.

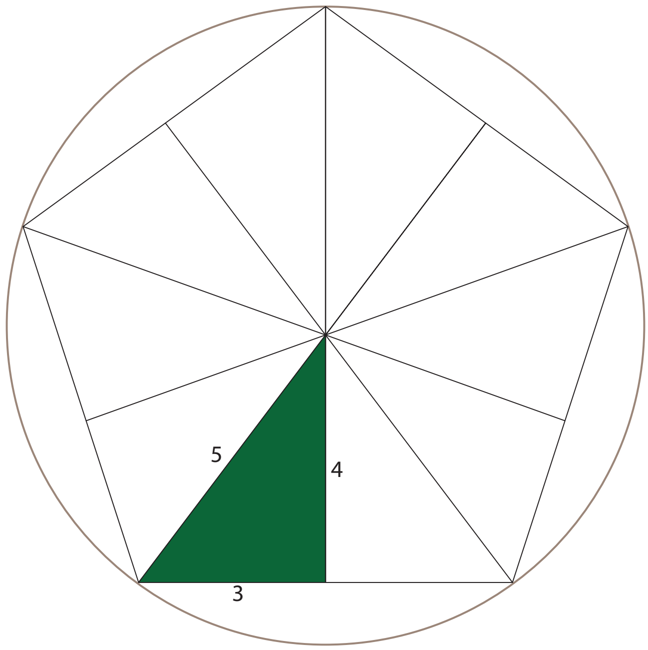


Figure 2: Diatessaron pentagon displaying relationship to a 3:4:5 triangle.

These proportional relationships are employed to create self-similar numeric sequences, which are then applied to create pitch collections (modes), harmonic relationships (intervals), rhythmic motifs and form, on both a microscopic and deep structural level. Many of these essential features are represented geometrically.

6.2 Modulation of rhythmic contour under augmentation and diminution

The experiments that preceded and inspired the composition of *Mod Times* involved the application of rhythmic augmentation and diminution by pulse to polyrhythms. In Chapter 2 rhythmic augmentation and diminution were mentioned in the context of *translation* and *shear* symmetry, as well as Indian counting sequences. My experiments verified the *invariance of specific rhythmic contour* that resulted when rhythmic augmentation and diminution was applied exactly (uniformly), per Messiaen's *non-retrogradable rhythms* (Messiaen 20).[1] I proved that rhythmic augmentation and diminution would predictably maintain a particular polyrhythm's inherent palindromic sequence of onsets, just as it does in the non-retrogradable phrases of Messiaen's *Quatour pour la Fin du Temps*.

Now that geometric abstraction can be allowed to precede the Western notation of Messiaen, the question arises of what happens when cycle length (rather than pulse duration) is held at a constant temporal unity during augmentation and diminution, instead of allowing it to expand and contract (per the Messiaen example). Figure 3 tabulates what happens when a 4:3 polyrhythm is augmented uniformly and incrementally by pulse. The increasing total timeline duration requires a corresponding increase in subdivision to accommodate the augmentation procedure. (Note that this is a limitless procedure, and I cease after three additions of pulse, revealing the resulting pattern.)

No. of pulses added to each interval	Resulting series of inter-onset intervals	No. of pulses in time-line	Interval vector
0	<3-1-2-2-1-3>	12	<<2,2,2,0>>
1	<4-2-3-3-2-4>	18	<<0,2,2,2>>
2	<5-3-4-4-3-5>	24	<<0,0,2,2,2>>
3	<6-4-5-5-4-6>	30	<<0,0,0,2,2,2>>

Figure 3: 4:3 polyrhythm with uniform augmentation of inter-onset intervals.

Bars 114–117 feature the first non-augmented version of the 4:3 polyrhythm. This motive, subjected to the three augmentations in Figure 3 would be notated as follows.

1 Examples occur in Messiaen's *Quatour pour la Fin du Temps* "Danse de la fureur, pour les sept trompettes" (Movement VI).

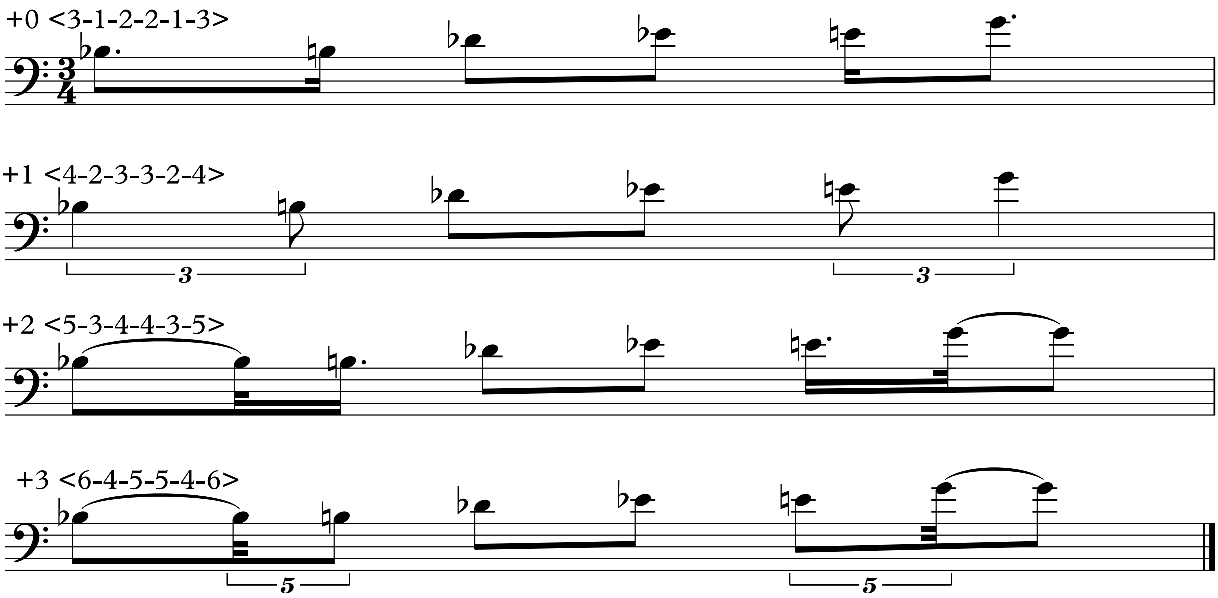


Figure 4: Notation of 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals.

Note that this interpretation maintains cycle length (3/4 metre), resulting in a modulation of phrase, rather than effectively a modulation of tempo. The bilateral symmetry of the polyrhythm's inter-onset intervals is as clear here in the notation as in the table of Figure 3.

Figures 5 to 8 plot the same series of timelines on composite pitch-time wheels. The yellow equilateral triangle clearly illustrates the constancy of the beat's role in the polyrhythm, whilst the overlaid red polygon modulates from an equilateral polygon (square) to three forms of kite.

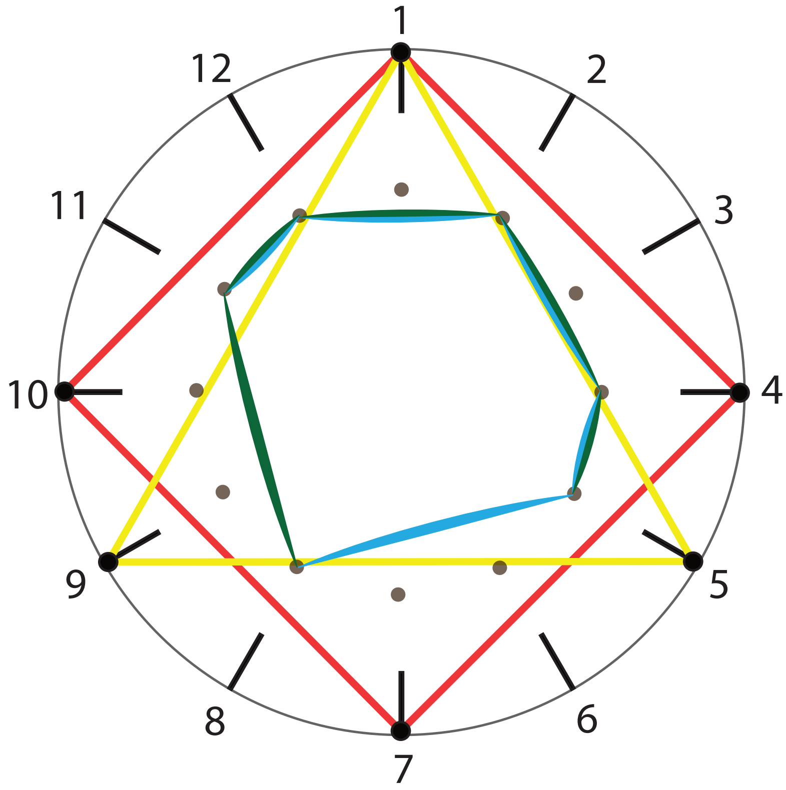


Figure 5: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – zero pulses added.

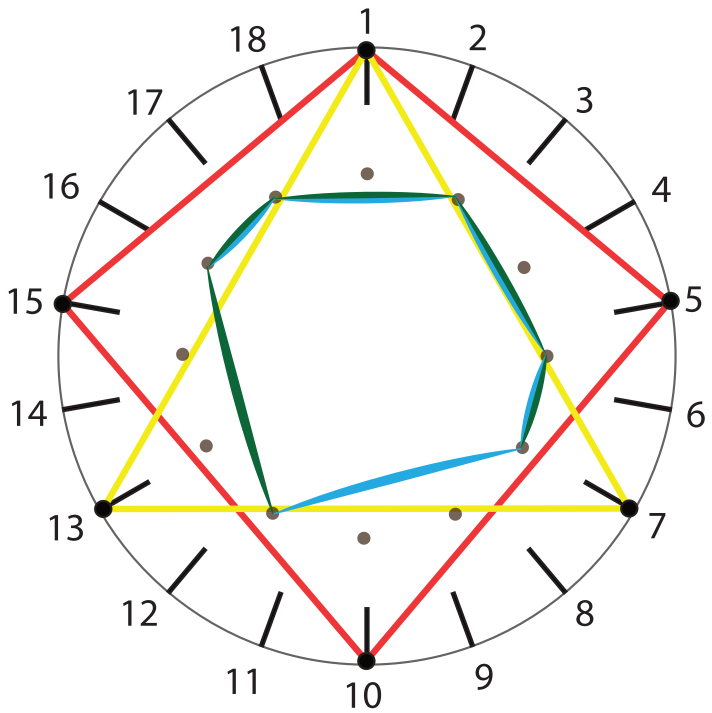


Figure 6: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 1 pulse added.

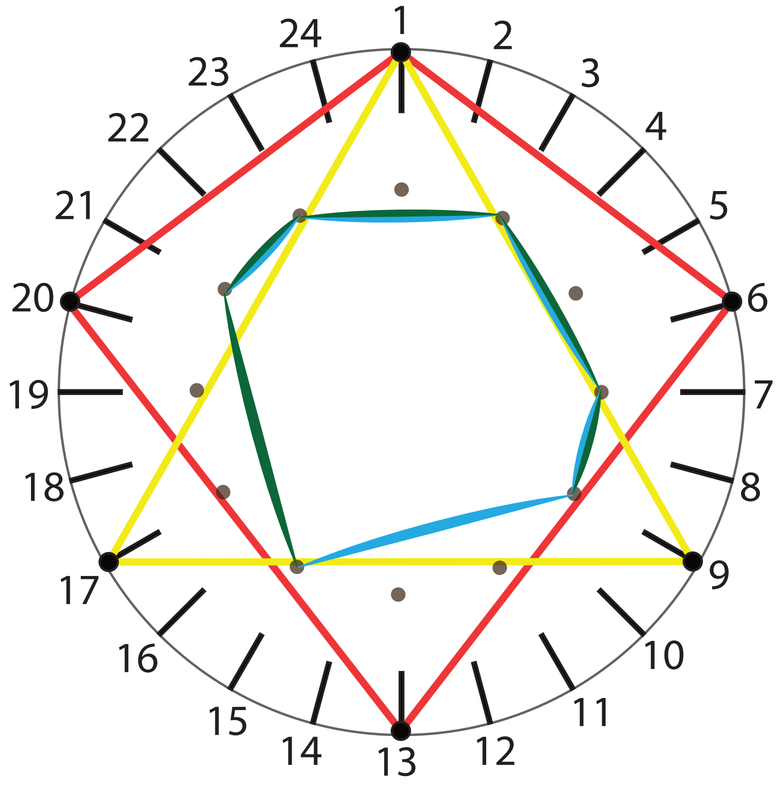


Figure 7: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 2 pulses added.

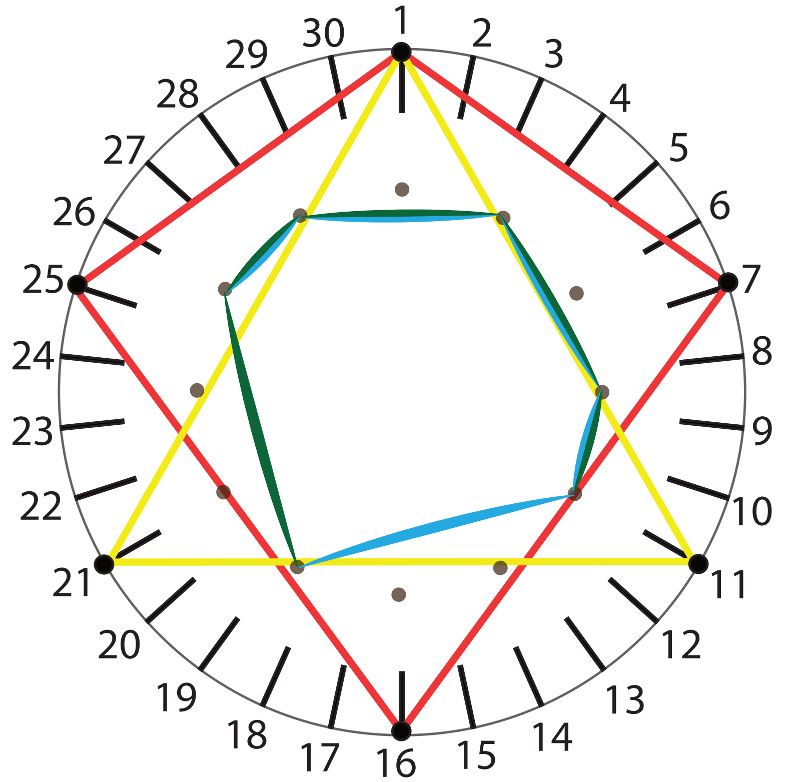


Figure 8: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 3 pulses added.

It can be observed that this approach maintains the original motive's generic rhythmic contour (being its long-short-medium-medium-short-long distribution of durations), but commits one to an ever-increasing number of pulses per cycle. This is paralleled by the interval vector sequence's incremental rightward movement to smaller durations. It is very interesting that the palindromic cumulative rhythm for the 4:3 polyrhythm remains as a distinct composite of two beat-like layers even when subjected to the distortions of augmentation under the conditions of metric constancy. Augmentation and diminution of inter-onset intervals by unit pulse does not destroy the independence of the constituent layers of polyrhythm. The polygons on the timelines clearly show how these strata remain intact, despite the *modulation of rhythmic contour* resulting from progressive augmentation.

6.3 Modulo as a rhythmic concept

I have now demonstrated the effect of translation in relation to rhythmic augmentation and diminution, including the *invariance of rhythmic contour* under conditions of pulse unity, and also the *modulation of rhythmic contour* under conditions of metric constancy. I have proven that both approaches are valuable as realisable techniques of motivic development, perceivable by the listener due to inherent likeness, and also honour symmetry as an over-arching principle. My next challenge—and its solution—arose from a practical concern in the rendering of rhythmic contours distorted by augmentation within fixed rhythmic cycles: How does one meet the challenge of accurately rendering smaller and smaller subdivisions when the bar is kept at a constant length?

I subsequently experimented with modular arithmetic, and observed the effect when applied to rhythm. *Modulo* (or *mod* for short) is used in post-tonal theory in order to convert pitches into pitch classes (amongst other purposes). By converting pitches of any integer to 'measures' of 12, modulo constrains pitch classes within the range of 0 to 11, much in the way that 24-hour time can be converted to 12-hour time by taking away lots of 12. This seemed attractive because I was interested in rhythmic development (namely augmentation and diminution) that worked within consistent time cycles. One of the reasons for this was my desire to create stratified rhythmic layers in my music so that multiple instruments could pursue different trajectories within the same overall rhythmic cycle.

Applying modulo during a summation operation on a timeline, with the *mod* value equalling the cycle length results in necklace rhythms (displacements) of the original rhythm, just as transposition constrains pitches classes to the octave. The cumulative rhythm for the original 4:3 polyrhythm in Figure 5 has onsets {1, 4, 5, 7, 9, 10}. Adding 1 to each results in {2, 5, 6, 8, 10, 11}, illustrated in Figure 9.

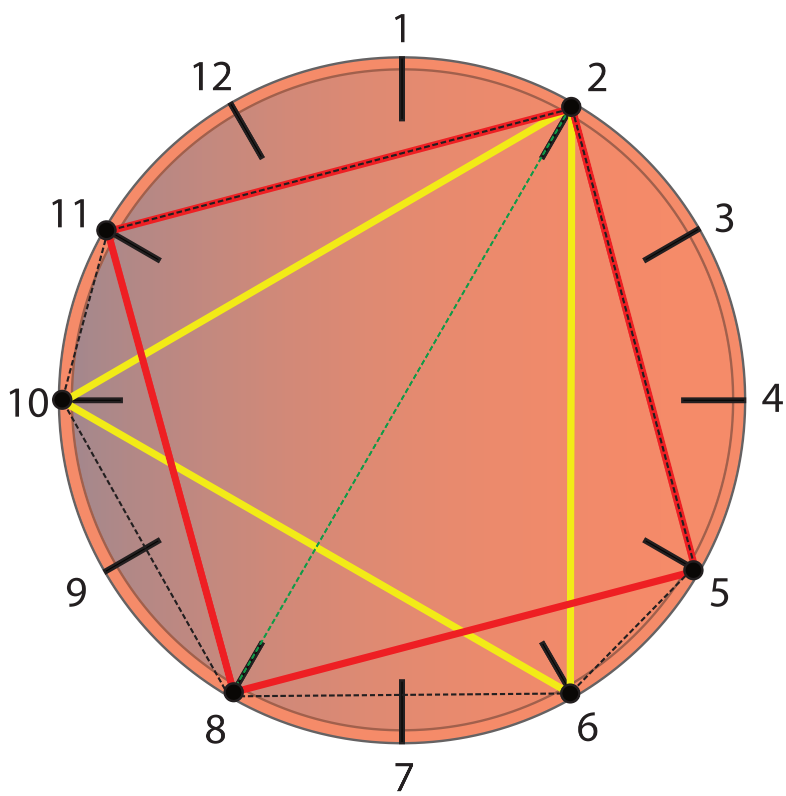


Figure 9: 4:3 polyrhythm necklace with coincident hit point on 2.

Progressive additions of pulse would soon require mod 12 to contain onsets within the cycle length. The original onsets {1, 4, 5, 7, 9, 10} with an addition of 5 for example would result in {6, 9, 10, 12, 14, 15}. Applying mod 12 to the last two onsets yields the correct necklace of {6, 9, 10, 12, 2, 3}, illustrated in Figure 10.

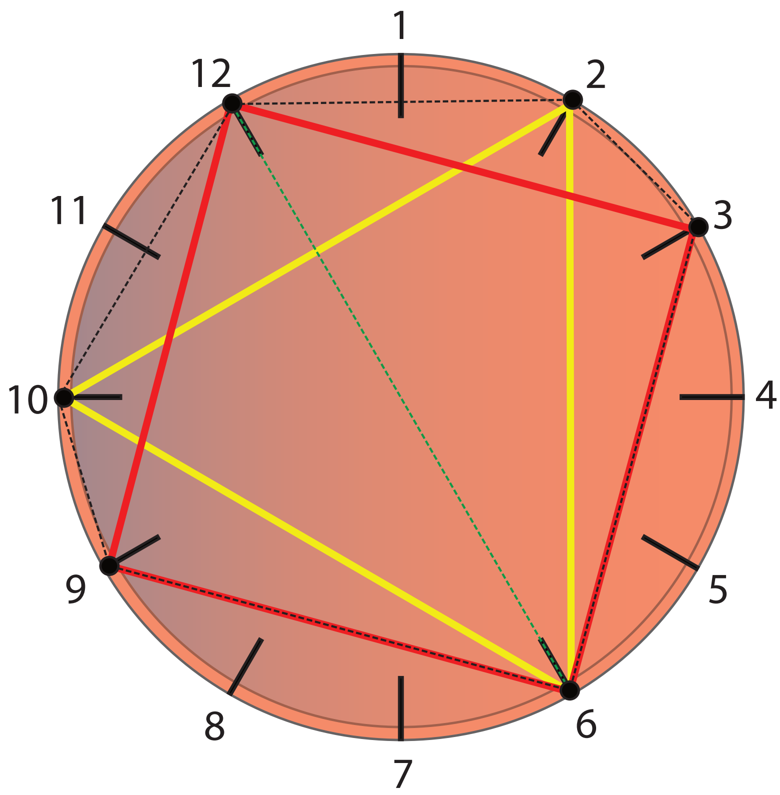


Figure 10: 4:3 polyrhythm necklace with coincident hit point on 6.

This experiment showed one application of modulo to rhythm, and proved that polyrhythms are not transpositionally symmetrical, as they possess as many necklaces as pulses in the timeline.[2] However a solution has not yet been provided to my challenge concerning augmentation and diminution under consistent cycle length.

2 Only isochronous 'beat' timelines possess transpositional symmetry by mapping onto themselves within a cycle.

6.4 Modulo applied to beat

I next wanted to see what would happen when the ‘measure’ for modulo was not of the timeline’s total cycle length, but rather the total value of the subdivisions of each beat. When calculating ordered pitch class intervals in a line of pitches, the modulo operation (mod 12) is used to keep intervals ascending and between 0 and 11. This seemed to relate to the fact that with rhythm, inter-onset intervals need to progress chronologically, in a clockwise direction as illustrated my timeline diagrams. I postulated that modulo used in a beat-centric manner may constrain the timeline within the same total number of pulses, thereby offering a solution to the issue of metric constancy under rhythmic augmentation with finite minimum subdivision length. (This contrasts with the prior technique illustrated where the cycle length for the 4:3 polyrhythm kept increasing, with corresponding ever-decreasing size of subdivisions required.)

Figure 11 tabulates what happens when a 4:3 polyrhythm is augmented uniformly and incrementally by pulse, but mod 4 per inter-onset interval. The value of 4 was chosen because if expressed in 3/4 metre, 4:3 would require semiquaver subdivisions.

No. of pulses added to each interval	Resulting series of inter-onset intervals	No. of pulses in time-line	Interval vector
0 (mod 4)	<3-1-2-2-1-3>	12	<<2,2,2,0>>
1 (mod 4)	<4-2-3-3-2-4>	18	<<0,2,2,2>>
2 (mod 4)	<1-3-4-4-3-1>	16	<<2,0,2,2>>
3 (mod 4)	<2-4-1-1-4-2>	14	<<2,2,0,2>>

Figure 11: 4:3 polyrhythm with uniform augmentation of inter-onset intervals, mod 4.

The first two results are identical to the previous version because modulo did not come into effect as the onsets were below 5. (Refer to the tabulated profile in Figure 3, and the timelines Figures 5 and 6.) However the results of augmentation by 2 and 3 pulses mod 4 show promise to a solution of constraining subdivision, and are illustrated in the timelines of Figures 12 and 13.

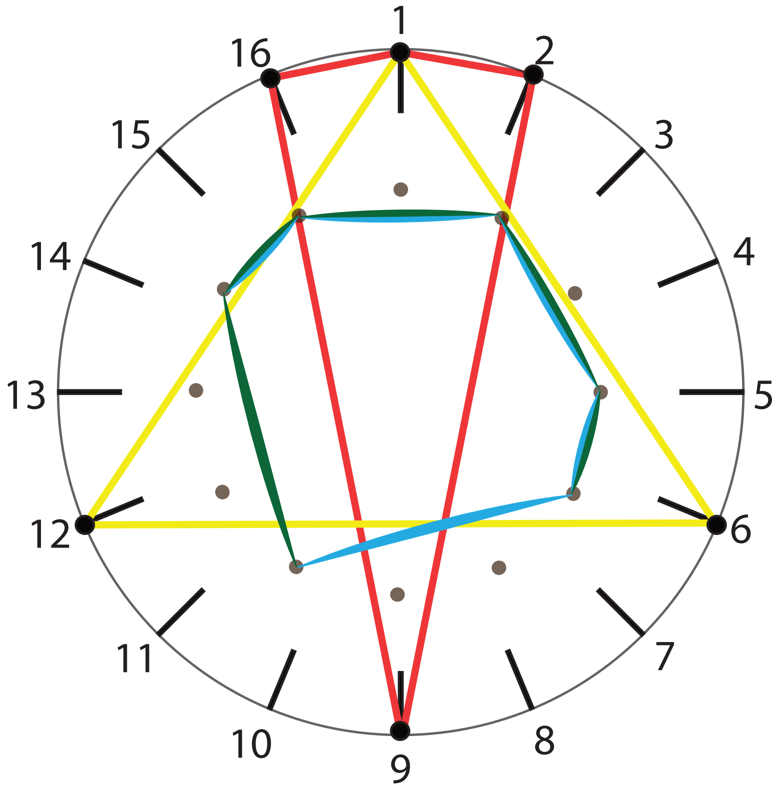


Figure 12: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 2 pulses added, mod 4.

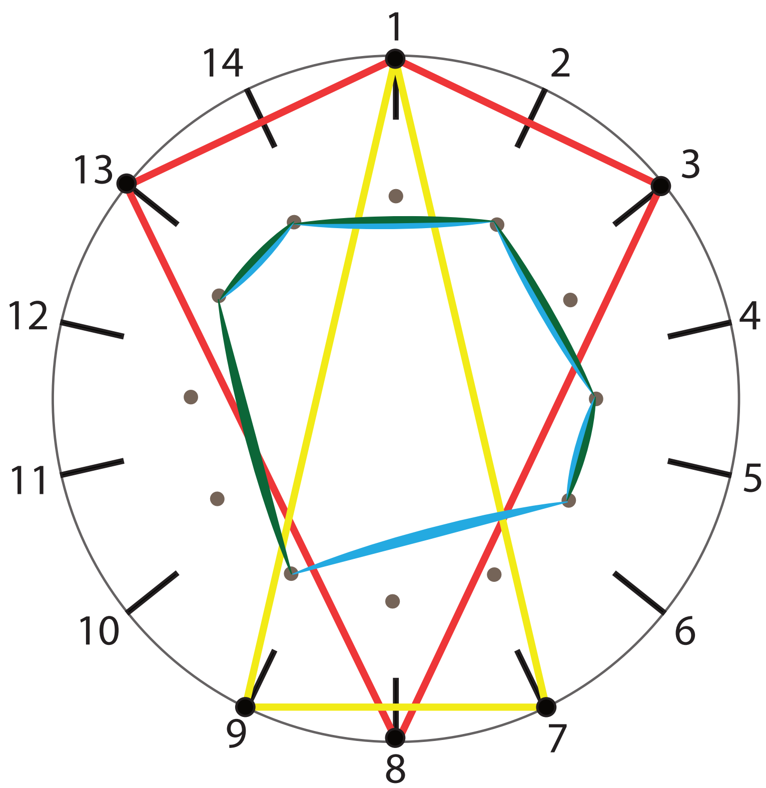


Figure 13: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 3 pulses added, mod 4.

Figure 14 notates these four versions of the 4:3 polyrhythm motive under progressive augmentation with modulo applied to each beat. (Refer to the tabulated profile in Figure 11, and the timelines illustrated in Figures 5, 6, 12 and 13.) The second bar of each line features an alternative equivalent approach to notation, where applicable.

Figure 14: Notation of 4:3 polyrhythmic motive with uniform augmentation of inter-onset intervals, mod 4.

A summary of the differences between the modulo and prior non-modulo version of these augmentation operations follows:

- Modulo constrains the total timeline duration to $t + n$ pulses, where t is the original timeline length and n is the modulo value.
- When confined in the original timeline's metre, the modulo calculation constrains the size of the required subdivisions.
- After exceeding the mid-point of the value of n (the modulo value), the modulo technique redistributes the original timeline's distribution of durations (whereas the long and short durations of the timeline would remain in their original relative ordering without modulo).
- The interval vector for timelines modulated with the modulo technique show a retention of the equivalent balance of inter-onset intervals, redistributed amongst n (the modulo value) intervals. Without the modulo technique, the original timeline's interval vector is simply pushed to increasingly high values, back-filling with zeros.

In terms of the third point, above, I refined what was to ultimately become my *contour modulation technique* by reinstating the relative ordering of the inter-onset interval durations—the *generic rhythmic contour*.[3] With polyrhythm and accumulative rhythm, because augmentation by pulse retains the palindromic series of inter-onset intervals regardless of the presence of the modulo operation, rearranging the rhythmic contour in this manner also retains the palindrome and therefore the cumulative rhythm's symmetry. As a consequence, applying the technique became aurally perceptible as motivic development, just as in classic augmentation and diminution, despite distortions in the proportions of the rhythmic contour. In computational geometry, the process of restructuring a polygon in this manner is called a *flip-turn* (Toussaint, "The Geometry of Musical Rhythm" 192).

3 Refer to Chapter 3.2.

6.5 Contour modulation technique

With the solution in sight, all that is required to arrive at my ultimate *contour modulation* technique is to rearrange the inter-onset intervals to match the generic rhythmic contour of the original motive, after the modulo process is applied.

Figure 15 tabulates what happens when a 4:3 polyrhythm is augmented uniformly and incrementally by pulse, mod 4 per inter-onset interval, and then reordered in this manner.

No. of pulses added to each interval	Resulting series of inter-onset intervals	No. of pulses in time-line	Interval vector
0 (mod 4)	<3-1-2-2-1-3>	12	<<2,2,2,0>>
1 (mod 4)	<4-2-3-3-2-4>	18	<<0,2,2,2>>
2 (mod 4)	<4-1-3-3-1-4>	16	<<2,0,2,2>>
3 (mod 4)	<4-1-2-2-1-4>	14	<<2,2,0,2>>

Figure 15: 4:3 polyrhythm with uniform augmentation of inter-onset intervals, mod 4, reordered to retain the same generic rhythmic contour.

The first two results are identical to the previous version because modulo did not come into effect as the onsets were below 5. (Refer to the tabulated profile in Figure 11, and the timelines Figures 5 and 6.) However the results of augmentation by 2 and 3 pulses mod 4 required re-ordering to maintain the *generic rhythmic contour* of the original motive, and are illustrated in the timelines of Figures 16 and 17.

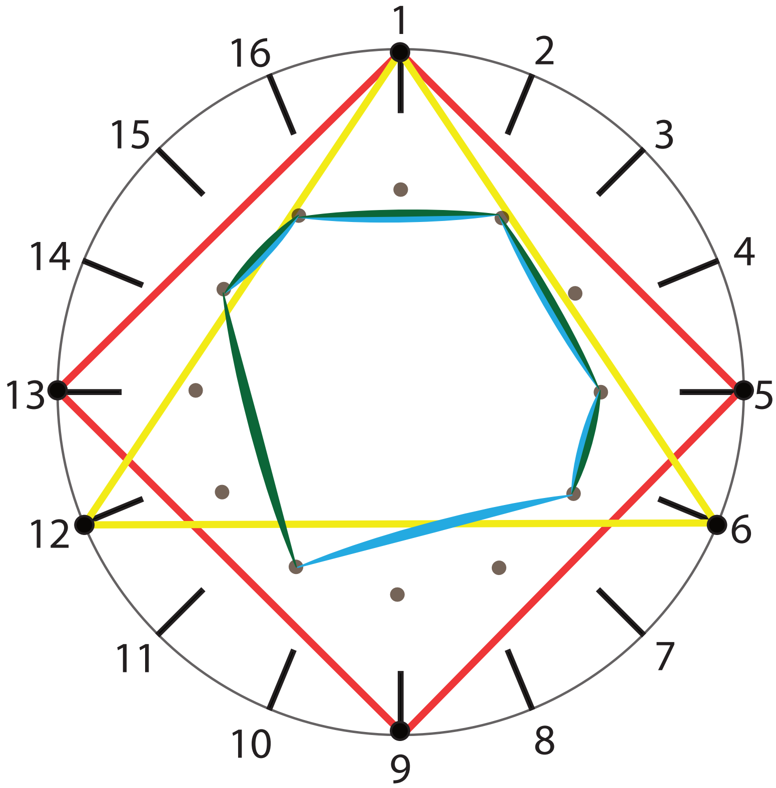


Figure 16: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 2 pulses added, mod 4, reordered.

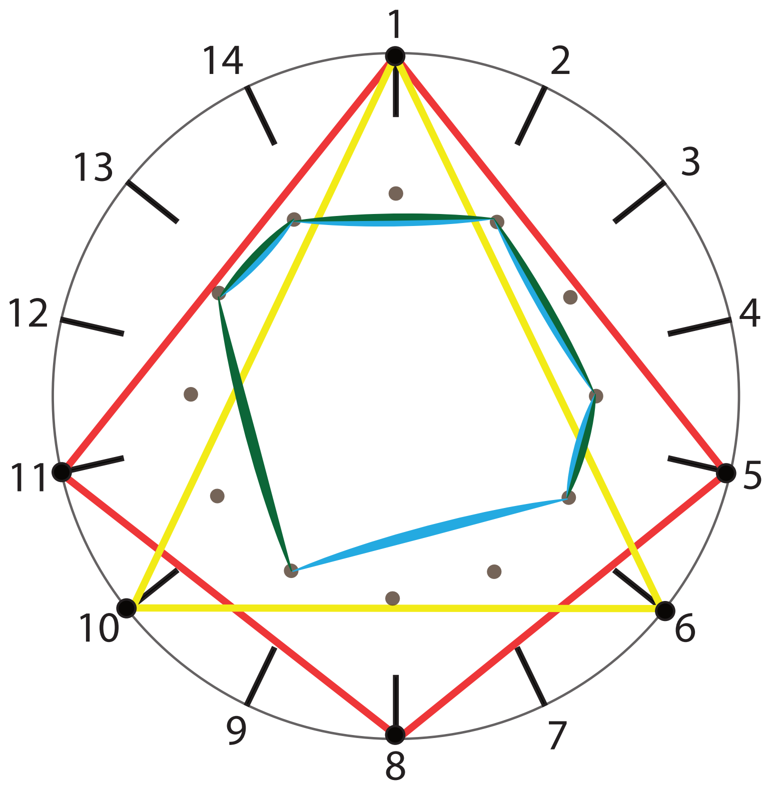


Figure 17: 4:3 polyrhythmic motive from bars 114–117 with uniform augmentation of inter-onset intervals – 3 pulses added, mod 4, reordered.

Figure 18 notates these four versions of the 4:3 polyrhythm motive under progressive augmentation with modulo applied to each beat. (Refer to the tabulated profile in Figure 15, and the timelines illustrated in Figures 5, 6, 16 and 17.) The second bar of each line features an alternative equivalent approach to notation where applicable.

Figure 18: Notation of 4:3 polyrhythmic motive (cumulative rhythm) with uniform augmentation of inter-onset intervals, mod 4, reordered where necessary.

This is precisely the sequence found in *Mod Times*, bars 114–129.

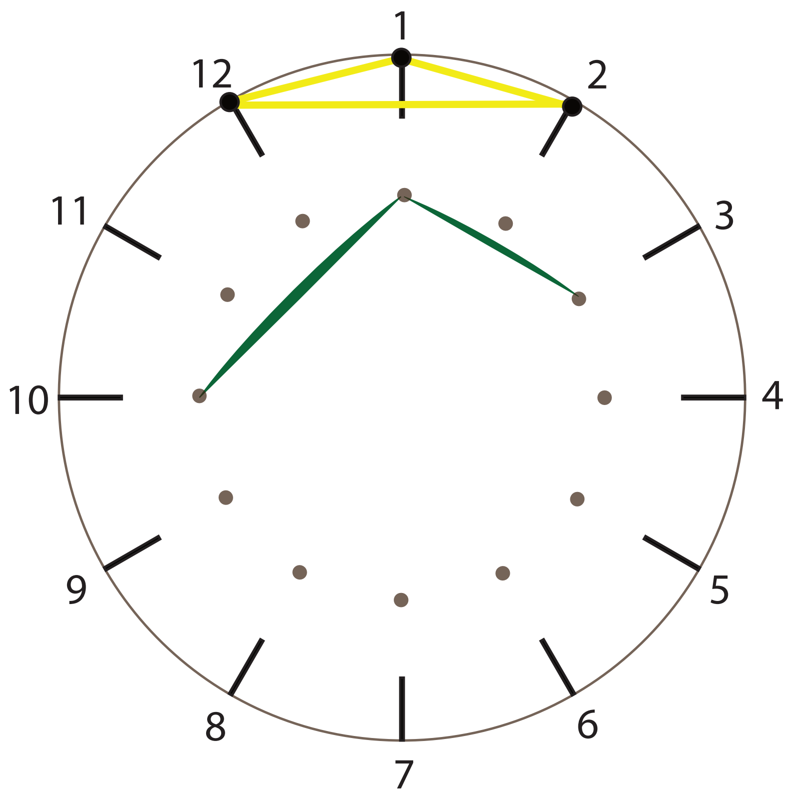
I have now demonstrated how my *contour modulation* technique is ultimately derived from classic rhythmic augmentation and diminution, yet falls outside of known sub-categories, borrowing from concepts found in post-tonal theory. The results of my approach would likely be loosely described by Messiaen as “rhythmic variants” (Messiaen 19). The beauty of these variants is that they maintain a perceptible connection to the original motive developmentally, they retain the rhythmic contour of the original motive’s cumulative rhythm, retain symmetry where it is present, and constrain the subdivisions required to render the rhythms to a finite smallest duration, thus facilitating their performance.

6.6 Deep rhythms

Mod Times commences with an African *bell* pattern by the drum set, after which the initial motive is sounded by the bass trombone and bass guitar (bar 4–6). This three-note motive and its subsequent variations form a series of four isosceles triangles illustrated in Figures 19 to 22, which I call *rhythmic scaling triangles*. Their inter-onset intervals and interval vectors are listed in the table below.[4]

Figure no.	Inter-onset intervals	Interval vector
19	<1-10-1>	<<2,1>>
20	<2-8-2>	<<0,2,0,1>>
21	<3-6-3>	<<0,0,2,0,0,1>>
22	<5-2-5>	<<0,1,0,0,2>>

Figure 19: Opening motive (bars 3–6).



4 Refer to Chapter 3.3 for an explanation of interval vectors and the property of *deepness*.

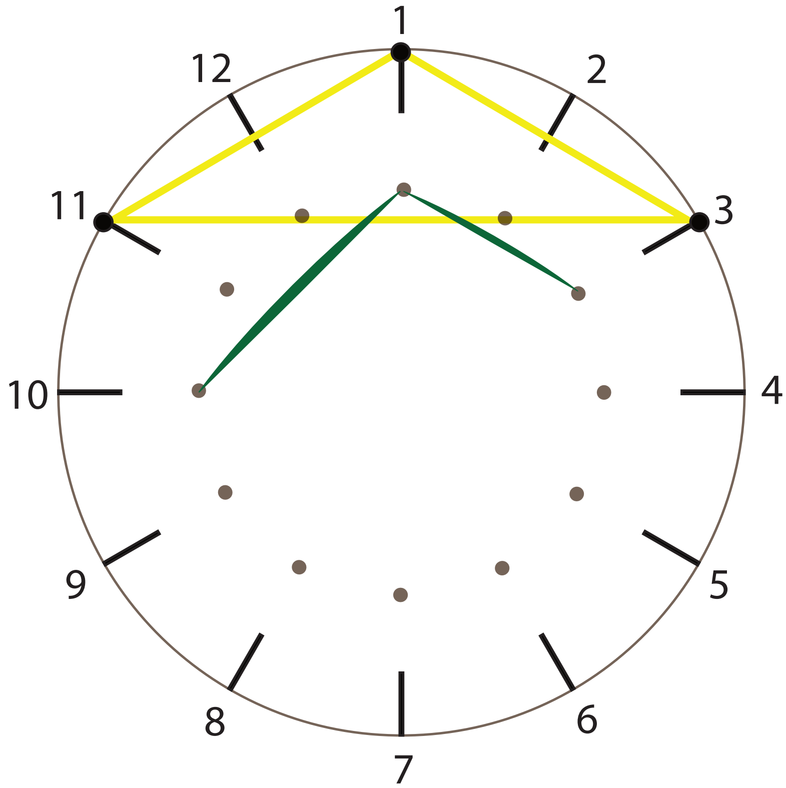


Figure 20: Opening motive (bars 7–9).

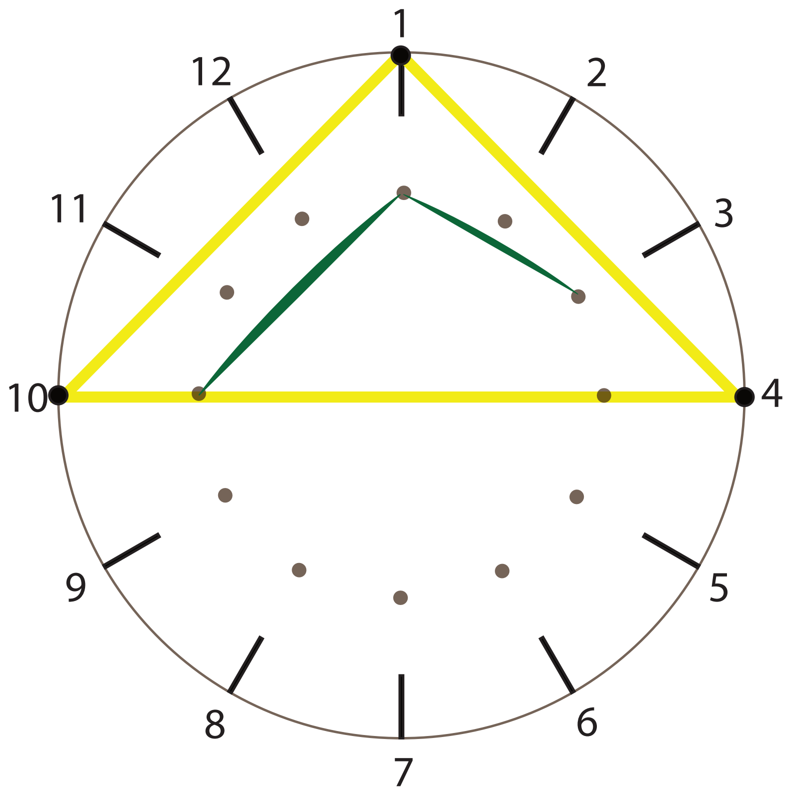


Figure 21: Opening motive (bars 10–12).

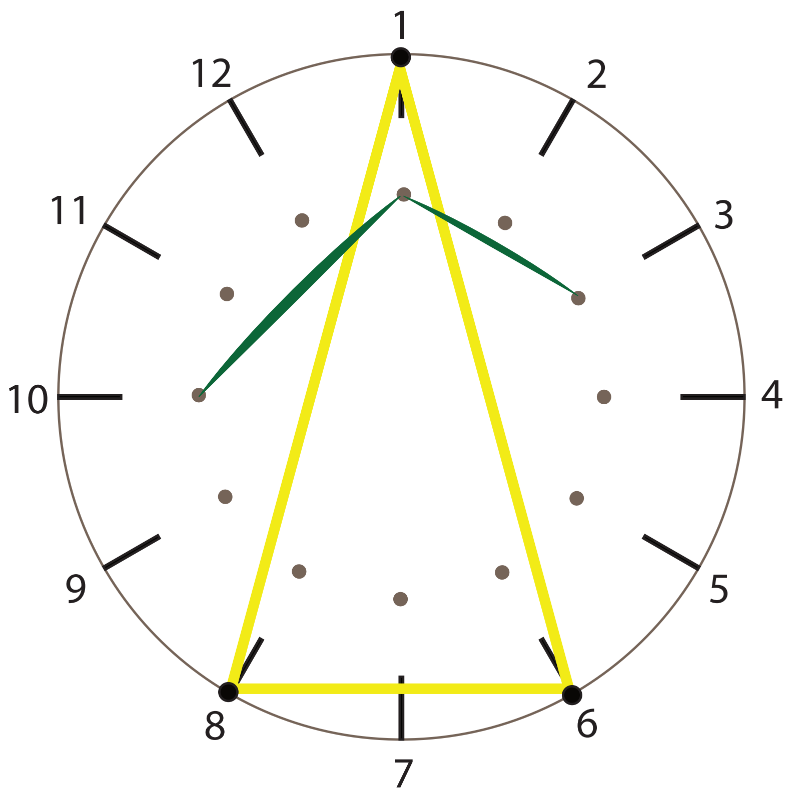


Figure 22: Opening motive (bars 13–15).

These are examples of *deep rhythms*, which are rhythms where the inter-onset interval vectors have a unique value for each interval. Toussaint adopts the term from *deep scales* (attributed to Winograd and Gamer) to categorise the property of unique multiplicity of pitch intervals ("The Geometry of Musical Rhythm" 183). Deep rhythms occur in *Mod Times*. The histograms for these four simple rhythms appear in Figures 23 to 26, illustrating the distribution of full interval vectors and thus illuminating the property of deepness.[5]

5 The connection between *deep rhythm* and the *well-formed* property of self-similar pitch collections is discussed in Chapter 3.3.

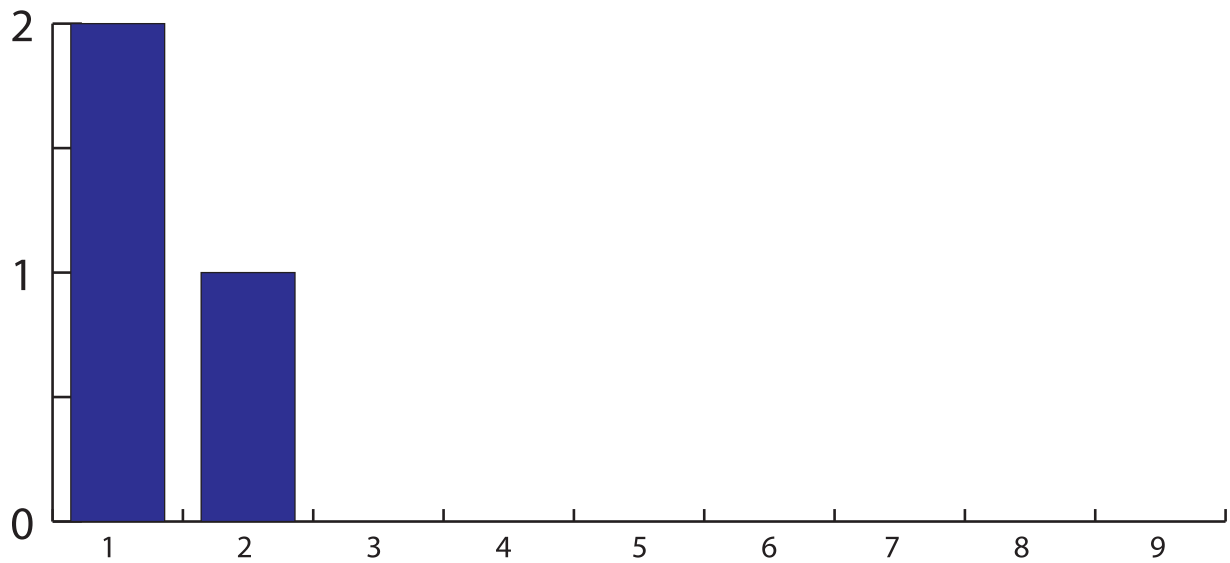


Figure 23: Rhythmic inter-onset interval histogram illustrating property of deepness in opening motive, bars 3–6.

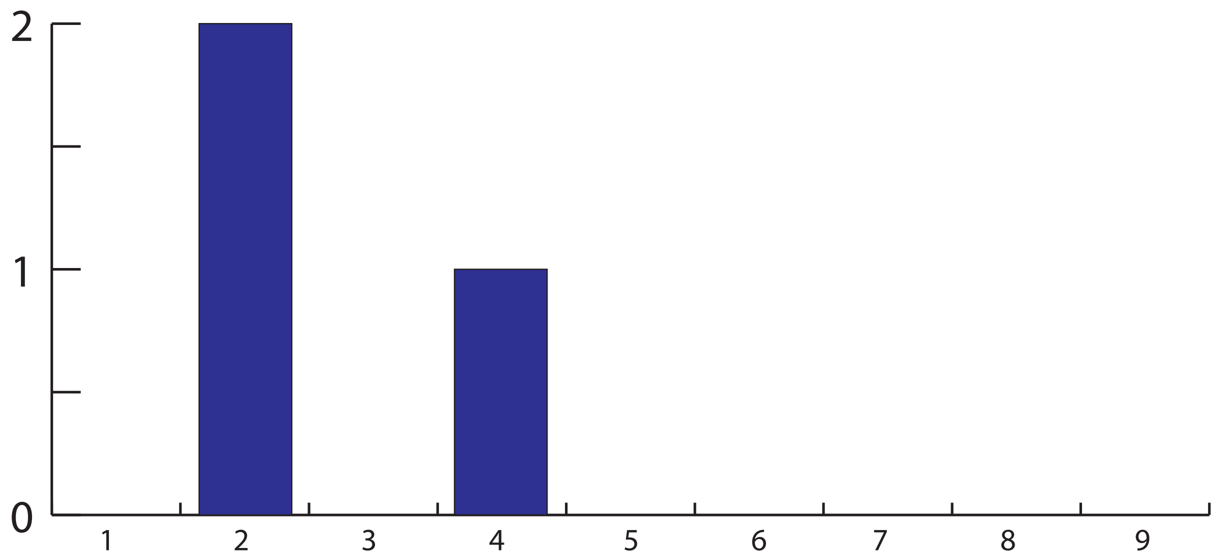


Figure 24: Rhythmic inter-onset interval histogram illustrating property of deepness in opening motive, bars 7–9.

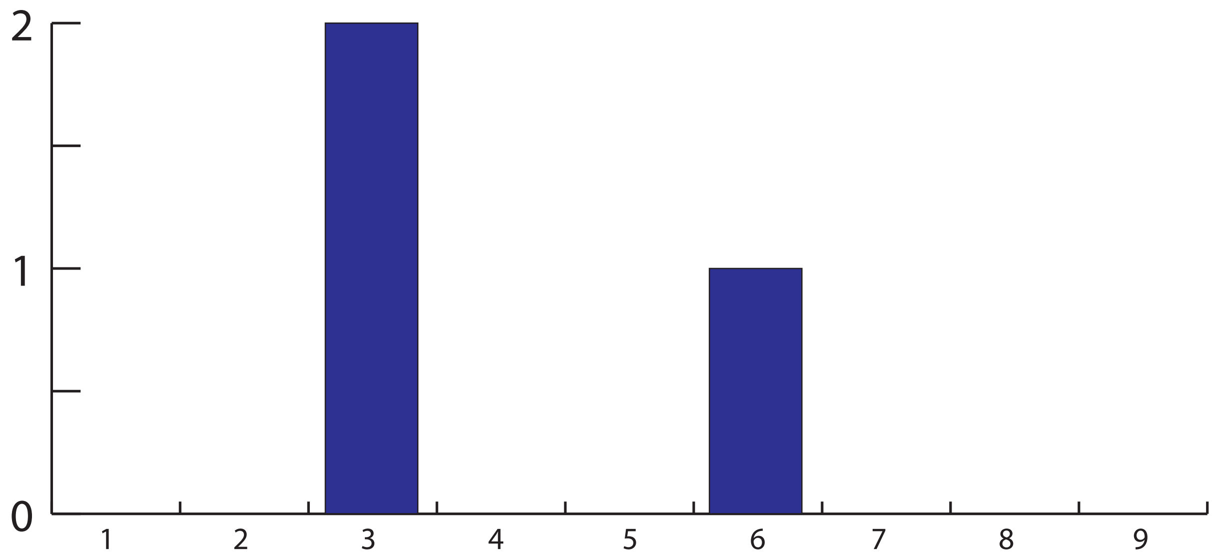


Figure 25: Rhythmic inter-onset interval histogram illustrating property of deepness in opening motive, bars 10–12.

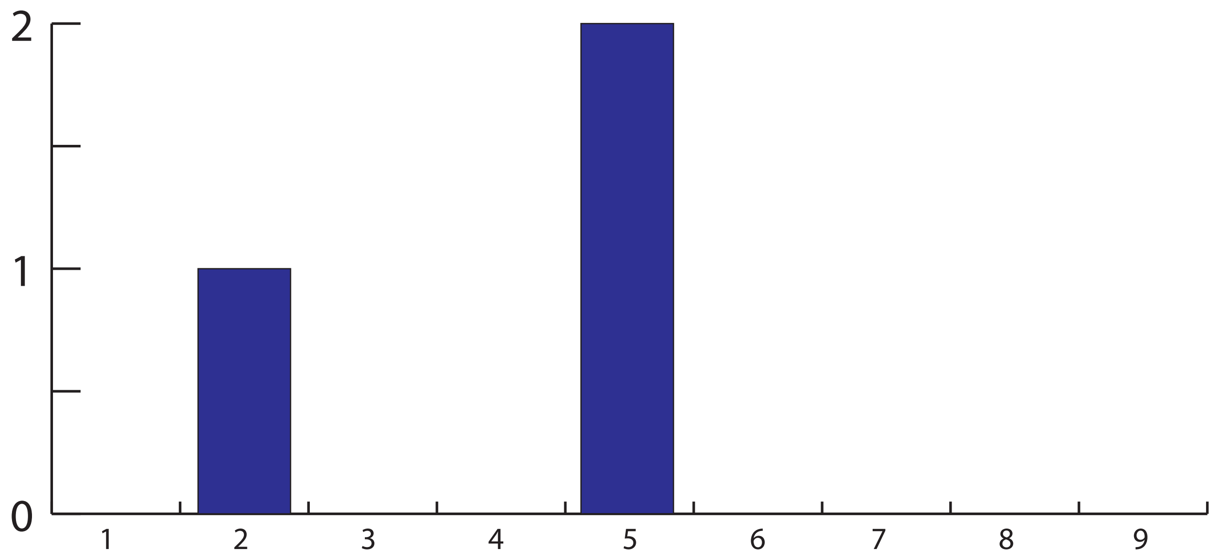


Figure 26: Rhythmic inter-onset interval histogram illustrating property of deepness in opening motive, bars 13–15.

6.7 Self-similarity in structure, polyrhythm and tempo

Mod Times features the three polyrhythms 2:3, 3:4, and 4:5. Being low-order (comprising small integers), their variants more easily rendered and aurally recognisable than more complex polyrhythms. 3:4:5 is also intrinsic to the Pythagorean triple. These three polyrhythms share the proportions with the rectangular numbers R2 (2:3), R3 (3:4) and R4 (4:5).[6] The numbers 2, 3, 4, 5 are also a simple arithmetic progression and are easily realized as sums of 2 and 3 as the first prime numbers.[7]

The following diagram is a chronological map of the series of tempi employed in the different sections of *Mod Times*.

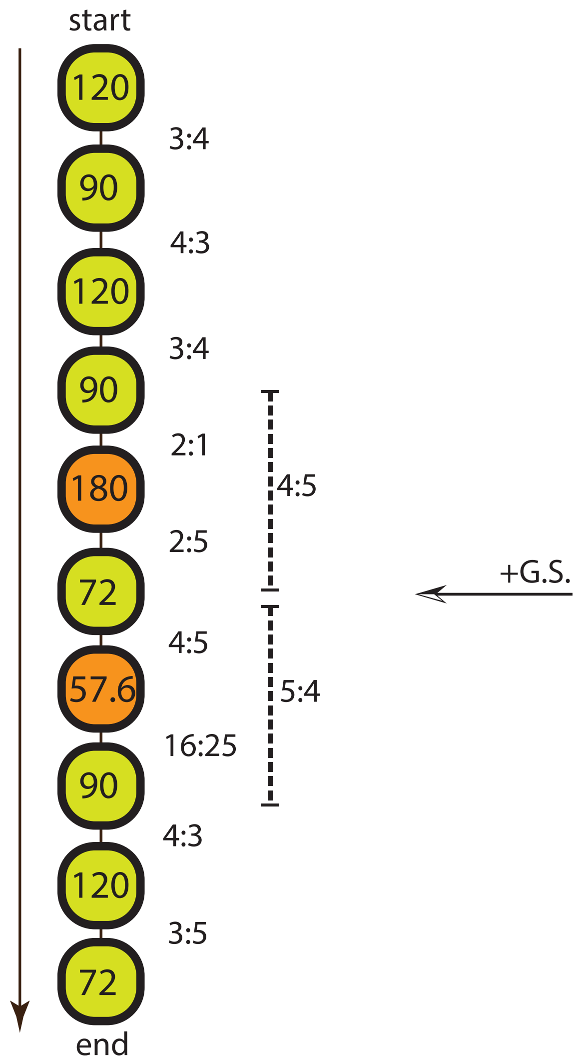


Figure 27: Tempo map showing relationships of tempi and positive Golden Section.

6 Refer to Chapter 3.9, Figure 25.

7 Refer to Chapter 3.4 for a discussion of unique prime factorization and the primacy of 2 and 3 in additive rhythm.

The sections coloured in orange are implicit in the composition (appearing as a superimposed layer, for example). The sections in green are explicit tempo markings in the score.

Some observations about this tempo series map:

1. Golden Section.

The placement of the 72 b.p.m. section appears at the corresponding positive golden section point of the entire composition, considering the total number of bars (including the D.S.). 215 bars x 0.618 = 132.87. It is precisely at bar 132 that the 72 b.p.m. section appears.

The placement of this section, and its appearance in the middle of the palindromic 4:5 and 5:4 tempo relationships were intuitive decisions revealed by subsequent analysis, and not predetermined.

2. Proportions.

The integers 1-2-3-4-5 comprise the proportional relationships of all tempi. My conscious intention in the planning of the composition was to feature the Pythagorean triple 3:4:5, but in the process of composing I felt like extending this arithmetic progression to also include 1 and 2, as the longer series offered a logical and more extended series of relationships. The frequency of occurrence of chronological tempo relationships are illustrated in Figure 28.

TEMPO RATIO	FREQUENCY	IMPLICIT FREQUENCY
1:2		1
2:3		1
3:4	4	
4:5	2	1
2:5		1
3:5	1	

Figure 28: Tempo ratio frequency table.

This table reinforces the primacy of the 3:4:5 relationship integral to the Pythagorean triple ratio, with 6 of the 7 explicit tempo ratios (and 6 of the total 11 tempo ratios) falling between the ratios of 3:4 and 4:5.

In Lewin's 'Generalized Interval Systems', pitches are used to offer insight into tempo ratios existing in the rich mensural layers of Carter's *String Quartet no.1* (Lewin 68). The following figure adopts Lewin's approach to express the tempo map of Figure 27 as pitch, using frequency ratios to determine successive directed intervals. This figure reveals the role of the structural Golden Section point on the sequence of tempi. The proportional relationships are neatly contained as a B-flat minor collection in the treble staff prior, but then diverge into a descending sequence into the bass staff thereafter.

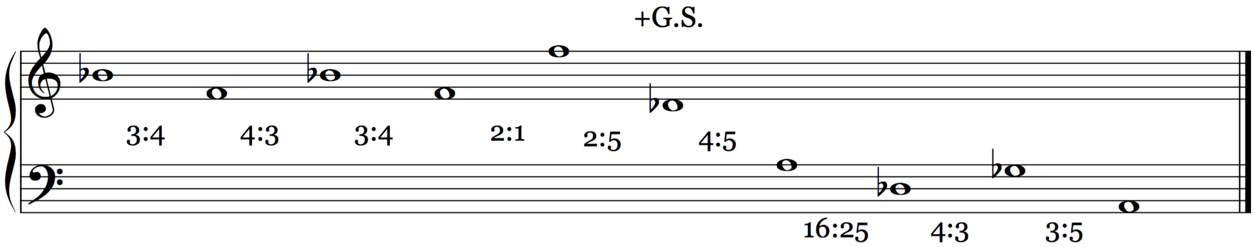


Figure 29: Tempo map showing proportional relationships as pitches.

3. Polyrhythm.

Polyrhythms by definition imply different tempi, so whilst the tempo series map illustrates different tempo markings on the score, other tempi are also often implied. In section A (bar 21), a 2:3 polyrhythm features in the new tempo of 90 b.p.m., creating not one but two relationships following the preceding tempo of 120 b.p.m., being 3:4 and also 2:3.

Sometimes these polyrhythms are used to preempt changes of tempo, thus setting up the change for the performers (e.g. bass guitar bar 131). At other times they are used as a layer for textural richness and ambiguity (e.g. bass guitar bars 133–134 and 161).

When the chronology of tempi is disregarded, and only one iteration of each tempo used is mapped, a simple lattice is formed, again revealing the relationships of 2:3:4:5 inherent in the composition.

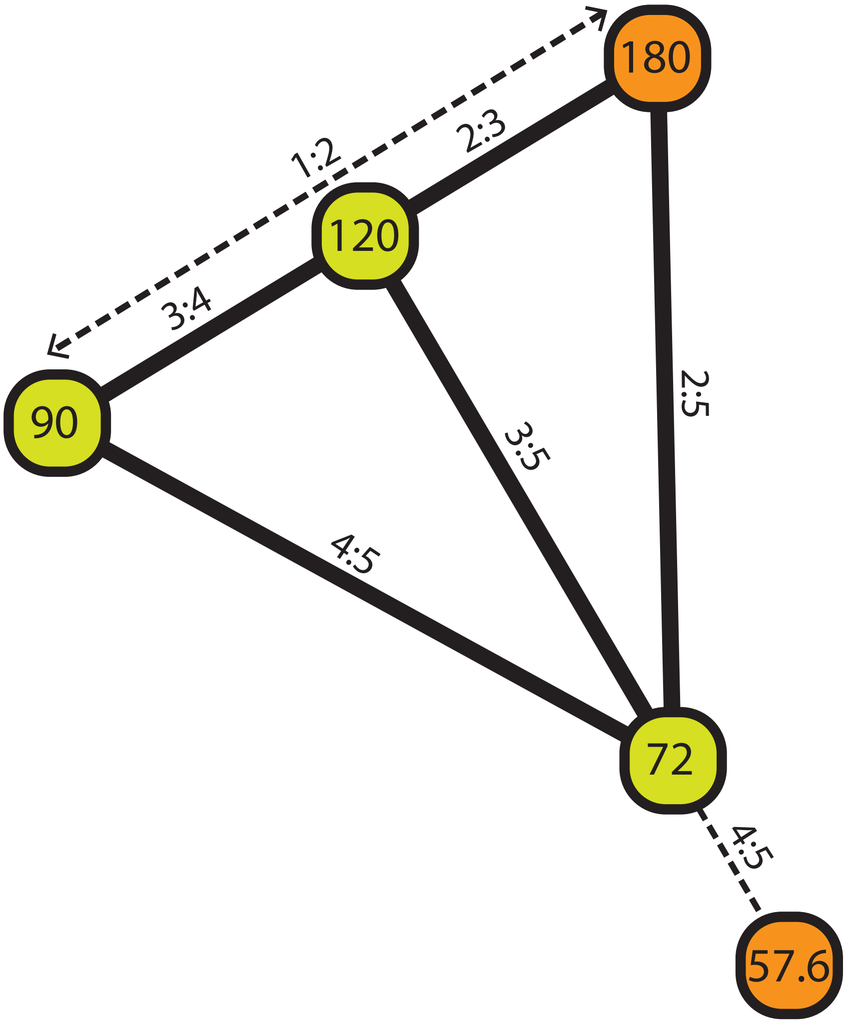


Figure 30: Tempo lattice showing relationships of tempi and inherent 2:3:4:5 proportion.

The Pythagorean triple ratios of 3:4, 4:5 and 3:5 are all present, and the arithmetic progression is again extended to also include ratios of 1:2, 2:3, 2:5 and 3:5.

After initially planning the broad structural concepts, the tempi in *Mod Times* were chosen intuitively—only the desire to maintain 3:4:5 relationships was predetermined. These diagrams demonstrate the integrative effect of simple underlying concepts upon intuitive compositional decisions concerning tempo, polyrhythm and structure.

6.8 Self-similarity in phrase lengths and sections

The meta-structure is comprised of smaller self-similar patterns, manifested in the lengths of phrases and sections. It was my intention to create phrase lengths of 3, 4, and 5 bars. I also sought to compose sections 9, 16, and 25 bars long. These proportions are again drawn from the ratio of the Pythagorean triple 3:4:5 and the applicable formula $3^2 + 4^2 = 5^2$.

Some examples of the application of these proportions in the lengths of phrases and sections are as follows:

- Bars 25–33: 3-bar phrasing, 9-bar section (3^2)
- Bars 34–49: 4-bar phrasing, 16-bar section (4^2)
- Bars 50–58: 9-bar interlude/recapitulation (3^2)
- Bars 59–71: 25-bar section (inclusive of repeats) (5^2)
- Bars 114–129: 4-bar phrasing, 16-bar section (4^2)
- Bars 144–152: 9-bar section (3^2)
- Bars 162–177: 4-bar phrasing, 16-bar section (4^2)

The trombone melody in bars 135 to 143 contains phrases of 3, 4, then 5 attacks sequentially, followed by 5, 4, and 3 attacks. Again, this undulation of phrase length reinforces the Pythagorean triple 3:4:5.

6.9 Self-similarity in pitch and rhythm

The role of the Pythagorean 3:4:5 triangle is shown in the opening theme of the piece. In musical harmony and tuning systems, the 4:3 ratio creates the consonant perfect fourth, the *diatessaron*, illustrated in Figure 2 (Doczi 8). This interval is the span of the incessant motive featured in the bass trombone and bass guitar for the first 20 bars of the piece, and also returns in bars 50–58.

In *Mod Times* I sought to render clear, simple self-similar sequences to give the listener an insight into the compositional logic. I consequently chose to employ counting sequences that did not exceed the number 5, nor count on more than three levels.

Whilst the pitch material in *Mod Times* is usually isomorphically related to the rhythmic structures (as explained below), in section B (from bar 72) the pitches in the bass trombone and bass guitar follow a self-similar counting sequence[8] as follows:

1
1221
122333221
1223334444333221
1223334444555554444333221
1223334444333221
122333221
1221

Figure 31: Self-similar pitch sequence in section B, from bar 72.

Some initial observations about this sequence:

- There are eight stages, omitting the ninth (1) in order to render a smoother transition when the sequence is repeated.
- This is a palindromic repetitive-accumulative counting sequence with an apex of 5, reached in its fifth stage.
- Each stage advances with a total number of events[9] increasing to the apex as follows:

number of events = number of events of prior stage + current stage number + prior stage number.

The pattern then decreases in the same exponential manner; i.e. the event count of 1, 4, 9, 16, 25, 16, 9, 4, 1 advances by 3, 5, 7, 9 and back.

- There are a total of 84 events per sequence.
- The Pythagorean triple proportions of 3^2, 4^2, and 5^2 appear symmetrically around the sequence's apex (9, 16, 25, 16, 9).
- The shape of the sequence (from small to large and back to small again in mirror formation) resembles a South Indian *mridanga yati*.

8 Refer to Johnson *Self-Similar Melodies*.

9 I use the generic term *events* here because the sequence is being isomorphically applied to both pitch and rhythm.

Figure 32 illustrates the same palindromic repetitive-accumulative counting sequence in two perspectives. On the right-hand panel, the Pythagorean triple 3:4:5 is illustrated in geometric form (in three dimensions), with the 5x5 square forming the hypotenuse of the 3:4:5 right-angled triangle (in red).

On the left-hand panel, the sequence is listed in its nine progressive stages from top to bottom, illustrating the progressive expansion and contraction of the *mridanga yati* shape. Colour coding correlates the right panel to the left.

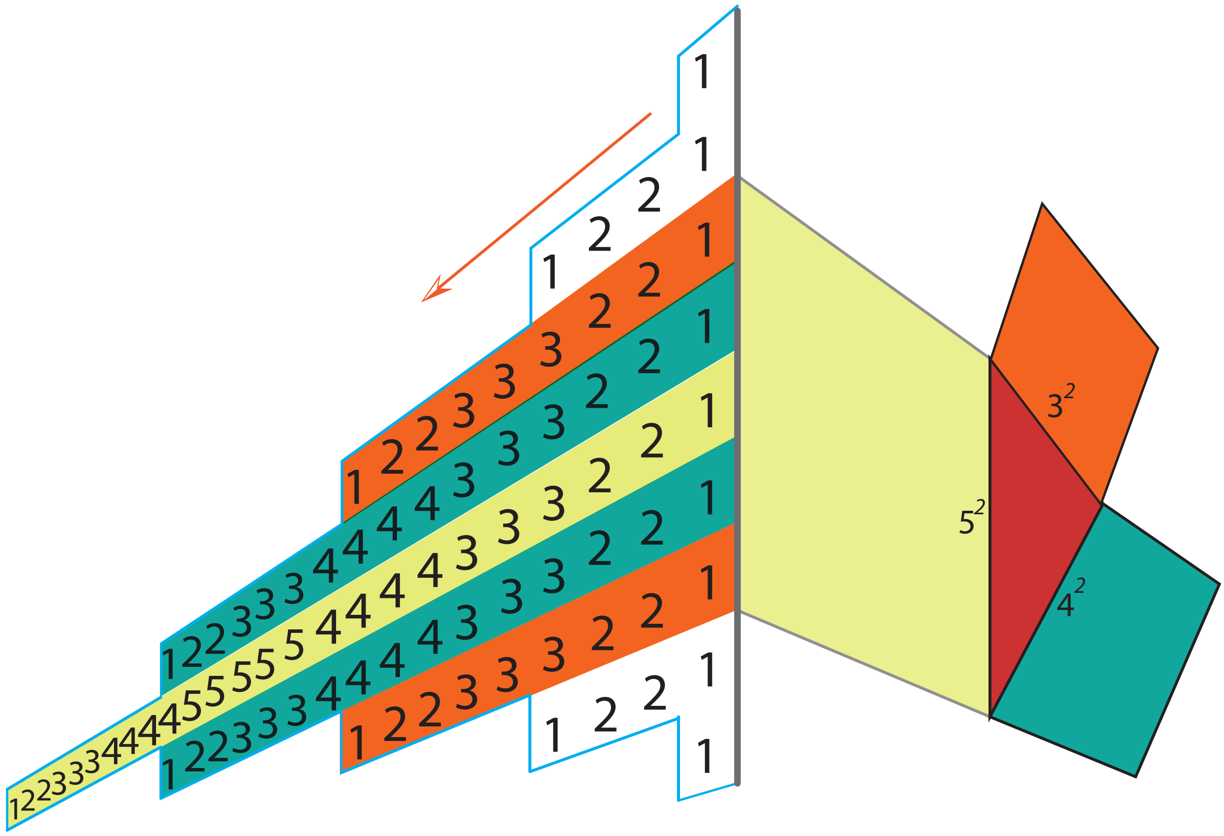


Figure 32: Palindromic repetitive-accumulative *mridanga yati* correlating the Pythagorean triple with the self-similar pitch sequence of Figure 31.

The following figure illustrates this palindromic repetitive-accumulative counting sequence as a matrix to further illuminate its properties.

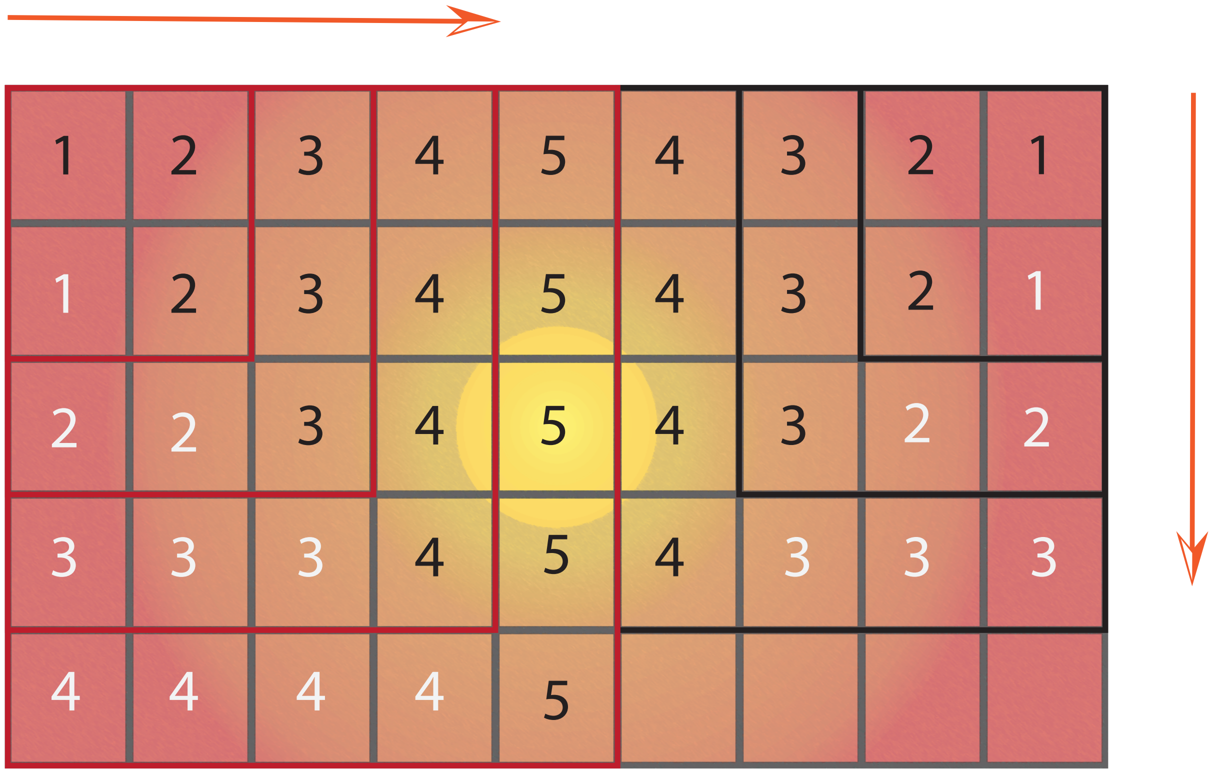


Figure 33: Palindromic repetitive-accumulative *mridanga yati* as a matrix.

This matrix is read as nested squares for each stage of the sequence, reading from the top left corner, following the cells in an anti-clockwise direction as the arrows indicate. The numbers in white represent the returning sequence in each palindrome. For example in stage 4, after reading off the descending columns for 1, 2, 3, and 4, the rows of white 3s, 2s, and 1 are read right-to-left to complete the 4x4 square and its concentric squares in the red borders. After the sequences' apex, the concentric squares with the black borders on the right side of the diagram are followed, again in an anti-clockwise direction. This nonlinear representation of the sequence illuminates the symmetry and self-similarity of the sequence, and bears musical relationship to the note choices made, as will be discussed. It is also interesting how the exact mid-point of the apex (the highlighted central 5) falls in the centre of the rectangle—something that is not revealed by the previous numerical representation.

This palindromic repetitive-accumulative counting sequence is rendered 5 times from bars 72–113, with progressive variations in orchestration. The palindromic nature of this sequence relates to the palindromic nature of all cumulative rhythms inherent in polyrhythms, and also their contoured variations employed in *Mod Times*.

In order to render this sequence as pitch, I made the following pitch-integer relationships in the bass guitar part:

1=G, 2=Bb, 3=B, 4=Db, 5=E

Figure 34 tracks the unfolding of this sequence.

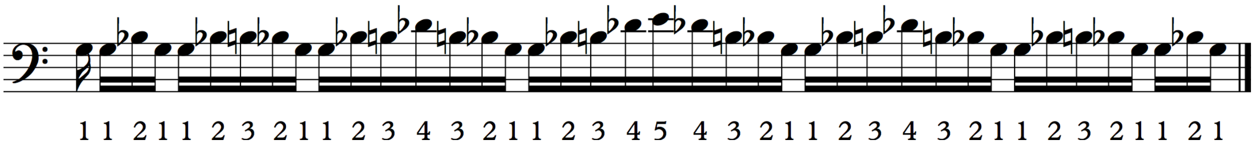


Figure 34: Palindromic repetitive-accumulative counting sequence as pitch, bass guitar bars 72–129.

These pitches are not arbitrary, and the intervals relate to the prevailing polyrhythm, as will be explained below in my discussion of isomorphic relationships.

Each pitch is repeated by its ascribed number in the counting sequence, so in the first stage there is 1 G, stage 2 is 1 G followed by 2 B-flats and 1 G, and so forth. In this way, the pitch and attacks unite to illuminate the pattern of the palindromic repetitive-accumulative counting sequence.

The durations are not determined by the sequence itself, but rather maintain the cumulative rhythm for the 4:3 polyrhythm that features in this section. The cumulative rhythm for 4:3 is <3-1-2-2-1-3> (as seen in the bass guitar from bar 72). What I managed to achieve was a unification of the 6 attacks inherent in the cumulative rhythm per bar, with the 84 events required to pass through each of the 14-bar palindromic repetitive-accumulative counting sequences.

By examining the bass guitar part in one rendering of this sequence (e.g. bars 72–85), it can be seen that the effect is that the idea expands in bounds to the apex, and then retreats progressively in an arch form. The shape of each mini-bound is self-similar to the overall arch form of the entire sequence.

From bar 86 the pitches of the trombone and bass parts are in mirror relationship, so that whilst the bass plays the sequence with 1=G, 2=Bb, 3=B, 4=Db, 5=E, the trombone is playing in contrary motion (descending): 1=Bb, 2=G, 3=E, 4=Eb, 5=Bb. The axis of symmetry arising from this mirror relationship is around pitch G3. During bars 86–99 (the fourth 14-bar section) the trombone's durations are locked into the 'four side' of the 4:3 polyrhythm, so whilst the pitches are in mirror inversion to the bass, they are quantized to the four possible attacks rather than the full six attacks of the cumulative rhythm in the bass. This relationship becomes rhythmically in lock-step in the fifth and final sequence from bar 100–113.

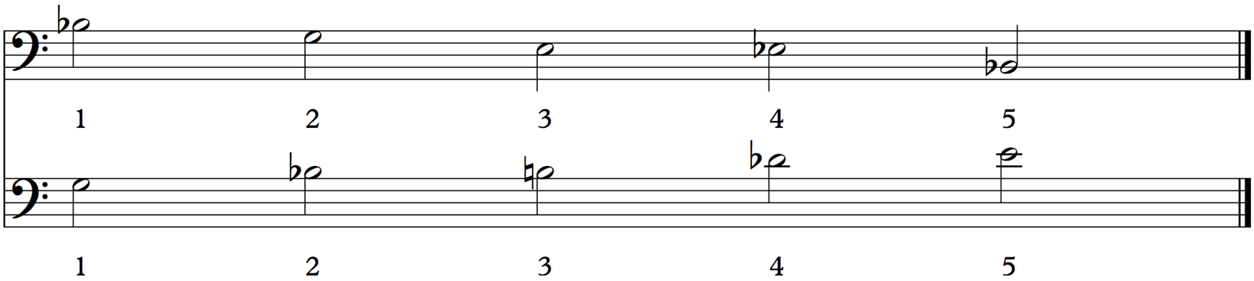


Figure 35: Palindromic repetitive-accumulative counting sequence as pitch with contrary motion between parts. Bass trombone (top) and bass guitar (bottom).

Prior to section B, the previous A section features a palindromic-accumulative counting sequence, in bars 59–71. This is illustrated in Figure 36.

1
121
12321
1234321
12321
121
1

Figure 36: Self-similar pitch sequence in section A, bars 59–71.

Some initial observations about this sequence:

- There are seven stages.
- This is a palindromic accumulative counting sequence with an apex of 4, reached in its fourth stage.
- Each stage advances with a total number of events totalling (current stage number + prior stage number) to its apex, and then decreases in the same manner. I.e. the event count of 1, 3, 5, 7,5, 3, 1 advances by 2 to the apex and back.
- There are 25 events in total, which corresponds to the 5^2 hypotenuse from the 3:4:5 triangle.

- 25 is partitioned into 9 + 16 (per the Pythagorean triple) when considering the sequence as either pitch events or as rhythmic groupings. Considering each number as an event, the first three stages total 9 (1+3+5 = 9) and the first four stages total 16 (9+7=16). Considering each number as a rhythmic grouping, stages 3 and 5 total 9 (1+2+3+2+1=9), and stage 4 totals 16 (1+2+3+4+3+2+1=16).
- Considered as rhythmic groupings, the sequence of 7 stages demonstrates the geometric progression 1^2 , 2^2 , 3^2 , 4^2 , 3^2 , 2^2 ,1^2 .(totalling 1+4+9+16+9+4+1).
- The expanding-contracting geometric shape of the sequence again resembles a South Indian *mridanga yati*.

Each stage of the sequence is realised as a discrete bar in this section of *Mod Times*, and is realised as rhythmic duration in 3/4 metre. This differs from the approach in section B, where the sequence was overlaid upon the 4:3 cumulative rhythm, and is the reason for the requisite change of subdivision for each bar (see bars 59–71).

In stage one, a 1:1 ratio of duration applies, hence the dotted minim in bar 59. In stage two, a 4:1 ratio of duration applies, hence the dotted quavers grouped in the 1+2+1 sequence in bar 60. In stage three, 9:1 applies, which requires quaver triplet subdivisions grouped 1+2+3+2+1, etc. In this manner, another self-similar effect arises from the mini-bounds of each bar echoing the arch form of each 6-bar sequence, further reflecting the arch form of the entire 25-bar (5^2) section.

I realised this palindromic-accumulative counting sequence as a 25-bar section by writing the seven-stage sequence effectively four times, but omitted the first stage after the initial presentation. The first stage is the same as the seventh, and I found the section had more momentum by removing the redundant 1 and making the sequences overlap, as illustrated in Figure 37.

1
121
12321
1234321
12321
121
1
121
12321
1234321
12321
121
1
121
12321
1234321
12321
121
1
121
12321
1234321
12321
121
1

Figure 37: Self-similar pitch sequence in section A, bars 59–71, showing dove-tailed repetition.

The following diagram illustrates the same palindromic-accumulative counting sequence in two perspectives. On the right-hand panel, the sequence expands out from the top left pink square to the green strand and contracts during the outer purple strand. All numbers are read from right-to-left on this right-hand panel, as indicated by the arrows. It is notable that the entire sequence is contained by the 25-grid (5x5) square. On the left-hand panel, the sequence is listed in its seven progressive stages from top to bottom, illustrating the progressive expansion and contraction of the *mridanga yati* shape.

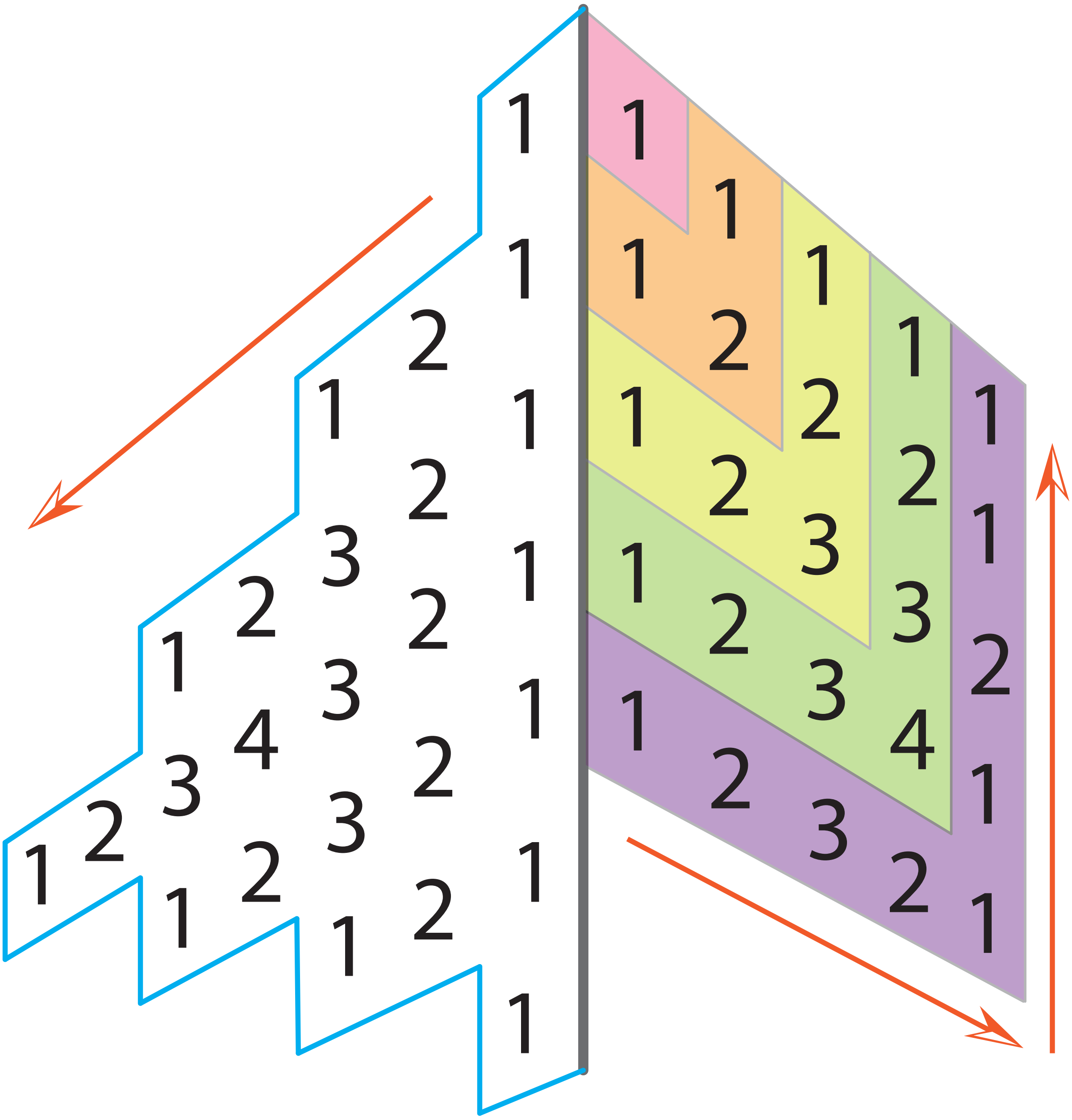


Figure 38: Palindromic-accumulative *mridanga yati* correlating the Pythagorean triple with the self-similar pitch sequence of Figure 36, bars 59–71.

In the Coda of *Mod Times* (bars 181–187) the same palindromic-accumulative counting sequence from section A (bars 59–71) is used (see Figure 36). The difference between this version and the version introduced in section A (bar 59) is that the composition's essential 3:4:5 relationship is embedded in a three-part canon, where each part is a scaled version of the other.[10]

Essentially, the 7-stage palindromic-accumulative counting sequence is rendered in 5/4, 4/4, and 3/4 simultaneously by the bass guitar, trombone, and drums as follows:

- Bass follows 5/4 (35 beats) sequence, with the palindromic-accumulative counting sequence as 1:5, 4:5, 9:5, 16:5, 9:5, 4:5, 1:5
- Trombone follows 4/4 (28 beats) sequence, with the palindromic-accumulative counting sequence as 1:4, 4:4, 9:4, 16:4, 9:4, 4:4, 1:4
- Drums follows 3/4 (21 beats) sequence, with the palindromic-accumulative counting sequence as 1:3, 4:3, 9:3, 16:3, 9:3, 4:3, 1:3
- The beat ratios of 21:28:35 is a multiple of 3:4:5, and each of the renderings of the counting sequence naturally contains the 3^2, 4^2, 5^2 sequence.

The bass trombone and bass guitar sound the pitches in Figure 39 as they progress through the counting sequence. The isomorphic relationship between the pitches and polyrhythm is discussed in the following section.

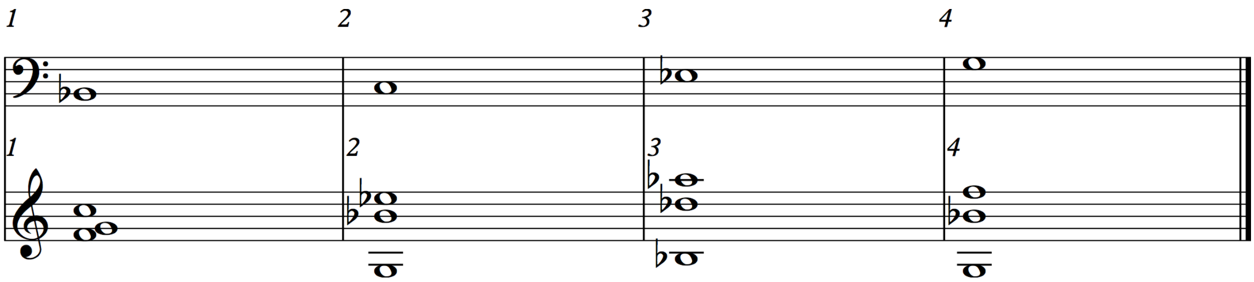


Figure 39: Pitch material in coda showing association between palindromic-accumulative counting sequence, bars 181–187. Bass trombone is the top stave.

The entrance of the three canonic layers is timed so that each voice's final onset finishes simultaneously exactly at the end of the piece, creating an effect resembling the fractal Chinese nested boxes. The staggered entries and scaled layerings of the underlying arch-form counting sequence yields a complex and satisfying introspective climax in bar 185.

10 This approach to self-similarity has been employed by Nancarrow in his player piano *proportion canons* (Scrivener 188).

6.10 Isomorphic relationships

The inter-onset intervals of the polyrhythms featured in *Mod Times* are frequently transformed into pitch intervals. This type of inter-parametrical self-similarity shows further commitment to further developing the formative ideas of the composition, and further integration of the parts to the whole. The following figures identify the scales in service of the pitch material at various stages of the composition, and the isomorphic source being the polyrhythms used in this composition. Many of these polyrhythms' cumulative rhythms are subjected to my *contour modulation* technique. Notice the systematic progression of polyrhythmic relationships that progress along the 2:3:4:5 ratio that scaffolds the entire composition.

Introduction

Bars 1 to 20

This section clearly features the 3:2 polyrhythm.

The 3:2 polyrhythm contour modulated +1 yields <3-2-2-3>. This onset interval series is used as pitches in this section, built from C.

Figure 40: 3:2 +1 as <3-2-2-3>, isomorphic scale bars 1–20.

In the transitional section (bars 17 to 20) this scale is partitioned into an upper and lower part, with the bass trombone harmonising above the bass guitar.

Section A

Bars 21 to 24

The 3:2 polyrhythm contour modulated +2 mod 3 yields <1-3-3-1>. This onset interval series is used as pitches built from C in first four bars of this section by the bass guitar, whilst playing the corresponding polyrhythm.

Figure 41: 3:2 +2 mod 3 as <1-3-3-1>, isomorphic scale bars 21–24.

Bars 25 to 33

From the bass trombone’s entrance with the theme in the pickup to bar 25, the <2-1-1-2> interval series is used as pitches from C. This series is derived from the 3:2 polyrhythm’s cumulative rhythm without modulation.

Figure 42: 3:2 +0 as <2-1-1-2>, isomorphic scale bars 25–33.

Bars 34 to 49

Each of the four 4-bar phrases in this section feature isomorphic rhythms and scales that progress through the three contour modulations for the 3:2 polyrhythm mod 3, as well as 3:2 +2. See Figures 42 to 45.

Bars 34 to 37

3:2 +0 as <2-1-1-2>

See Figure 42.

Bars 38 to 41

3:2 +2 mod 3 reordered as <3-1-1-3>

Figure 43: 3:2 +2 mod 3 reordered as <3-1-1-3>, isomorphic scale bars 38–41.

Bars 42 to 45

3:2 +1 as <3-2-2-3>

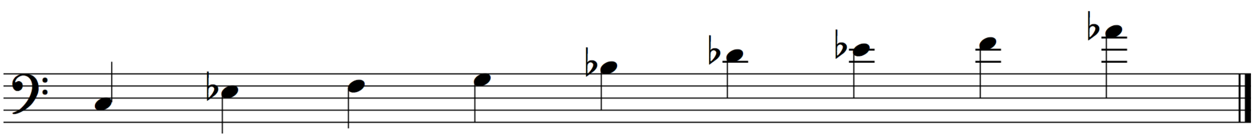


Figure 44: 3:2 +1 as <3-2-2-3>, isomorphic scale bars 42–45.

Bars 46 to 49

3:2 +2 as <4-3-3-4>

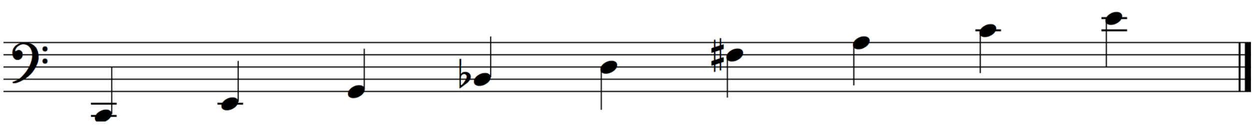


Figure 45: 3:2 +2 as <4-3-3-4>, isomorphic scale bars 46–49.

After a recapitulation of introductory material in reduced form from bars 50–58, the first of the *mridanga yati* sections occurs. This section uses the same pitch material as the introduction, being the pitches based upon the 3:2 polyrhythm contour modulated +1 being <3-2-2-3>, built from C (Figure 40). This scale is partitioned into two sections and used in contrary motion by the trombone when it enters and joins the bass in rhythmic unison from bar 65–71.

Section B

Bars 72 to 132

This section clearly features the 4:3 polyrhythm. The onset interval series is used as pitches in this section, built from G.

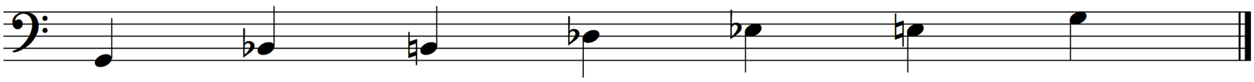


Figure 46: 4:3 +0 as <3-1-2-2-1-3>, isomorphic scale bars 72–132.

This section commences with a bass trombone improvisation that draws freely from this mode, whilst the bass guitar progresses through the palindromic repetitive-accumulative counting sequence examined previously. See Figures 31 to 35.

In bars 86–113 the trombone and bass are in mirror formation on this counting sequence, so that whilst the bass sounds the sequence as 1=G, 2=Bb, 3=B, 4=Db, 5=E, the trombone is playing in contrary motion (descending): 1=Bb, 2=G, 3=E, 4=Eb, 5=Bb. The self-similarity of this section was discussed previously. (See Figure 35.)

Bars 114–129 feature a trading section between the pitched instruments and the drums. Such trading sections typically occur at this stage of a performance of North Indian classical music, and are called *Jawal Sawab*. The bass guitar and trombone show the rhythm isomorphically in two dimensions, being their melodic intervals (horizontally) and harmonic intervals (vertically), notated in the figure below. Compound symmetry arises from the combination of the palindromic motive with its retrograde-inversion to create bilateral symmetries in rhythm and harmony between the two parts. The <3-1-2-2-3> motif now saturates the texture as a fitting climax to section B.

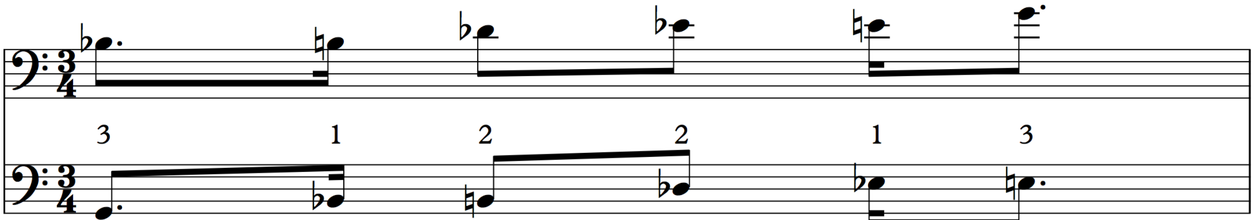


Figure 47: Isomorphic application of 4:3 polyrhythm as <3-1-2-2-1-3> as melodic intervals and harmonic intervals displaying compound symmetry, bars 114–129.

Whilst the <3-1-2-2-3> interval series is maintained in pitch in the trading section to bar 129, the rhythm is distorted following the contour modulation process as previously discussed. I chose to maintain the mode and interval relationship to provide a reminder of the source rhythm from which the contour modulated rhythmic relationships are derived.

During the transitional section (bars 130–132), the bass guitar chords are derived isomorphically from the 3:2 polyrhythm contour modulated +1 being <3-2-2-3>, built from C. This mode has been used frequently in the composition (see Figure 40), but is introduced now as a chordal idea with a transitional purpose, and is used similarly with this transitional function from here until the end of the composition. See bars 130–132, 169, 173, and the coda.

Section C

Bars 133 to 180

From bar 133 the drums initially play a groove based on a modulation of the 4:5 polyrhythm, and the pitch material in the bass and trombone is derived from 4:5 polyrhythm's cumulative rhythm <4-1-3-2-2-3-1-4> from G. This mode remains in place through to bar 152.

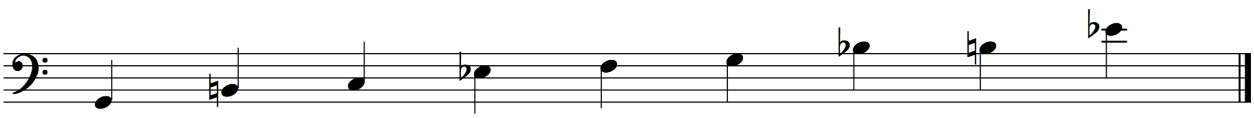


Figure 48: 4:5 +0 as <4-1-3-2-2-3-1-4>, isomorphic scale bars 133–152.

The contrasting idea explored in bars 153 to 161 draws from the 4:5 polyrhythm contoured +1 mod 4, being <1-2-4-3-3-4-2-1> from E.

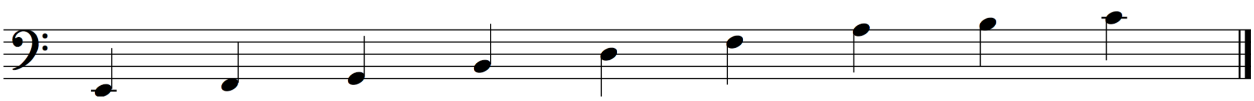


Figure 49: 4:5 +1 mod 4 as <1-2-4-3-3-4-2-1>, isomorphic scale bars 153–161.

A *D.S.* to bar 133 effectively repeats the material of this section.

In bars 162–176 there are a series of four *tihai*. These feature the contour modulations of 4:5, along with isomorphic pitch relationships.

Bars 162 to 164

Tihai one

4:5 +0 as <4-1-3-2-2-3-1-4> from G.

See Figure 48.

The bass guitar chords in bar 165 are again derived isomorphically from the 3:2 polyrhythm contour modulated +1 being <3-2-2-3>, built from C. This association and the role of these chords as a transitional device was discussed in Section B, bars 130–132. The chords appear in the transitional ‘gap’ bars of this *tihai* section, being bars 165, 169, and 173.

Bars 166 to 168

Tihai two

4:5+1 mod 4 as <1-2-4-3-3-4-2-1> from E

See Figure 49.

Bars 170 to 172

Tihai three

4:5 +2 mod 4 as <2,3,1,4,4,1,3,2> from E

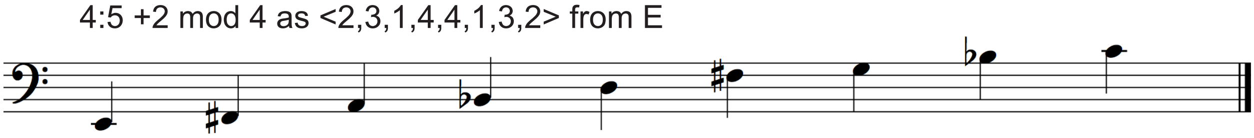


Figure 50: 4:5 +2 mod 4 as <2-3-1-4-4-1-3-2>, isomorphic scale bars 170–172.

Bars 175 to 176

Tihai four

4:5 +3 mod 4 as <3-4-2-1-1-2-4-3> from E

Figure 51: 4:5 +3 mod 4 as <3-4-2-1-1-2-4-3>, isomorphic scale bars 175–176.

The transitional section that follows this series of *tihai* (bars 178–180) is a 5-bar section that recapitulates the material of the transitional section from bars 159–160. The difference is that now the trombone and bass guitar swap roles.

Coda

Bars 181 to 187

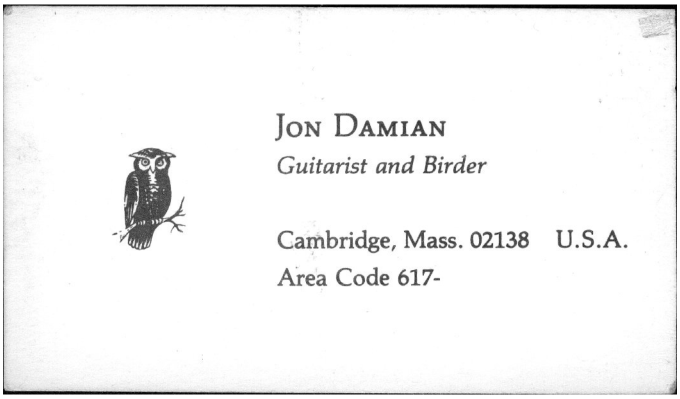
The coda features the 3:2 +1 as <3-2-2-3> scale that opened the piece, built again from C (see Figure 40). The return of this harmonic material based upon the simpler 3:2 proportion gives the piece a sense of homecoming after the climax and density of the previous section. New development is still taking place on a deeper level, however. The chordal idea, only first introduced in the bass guitar in bars 130–132, is now expanded upon and subjected to a new kind of stratification across the ensemble, as discussed in Section 9. (See Figures 36, 38 and 39.)

6.11 Real-time geometric analysis

I have created a real-time analysis featuring geometric diagrams that accompany a recorded performance of the work. This video reveals many of the polyrhythmic, self-similar, structural and symmetrical properties of *Mod Times* discussed in this chapter. Please visit:

https://youtu.be/bGA8dOO1FNI

Chapter 7
Analysis of work 2—*Birder*



7.1 Background

Dedicated to the American guitarist, composer and educator Jon Damian, this composition gains its title from Damian's hobby of bird-watching. This composition offers my respect to Damian, his teaching and music, in that it grants the guitarist a featured position in the ensemble, with musical inspiration in the composition derived from the calls of a selection of Australian birds.

Figure 1 shows a sketch of the form of the work, providing a macroscopic overview of the 800-bar composition. It tracks the undulating intensity over the 21 minutes projected duration of the work, and structural markers including changes in metre, tempo and harmony.

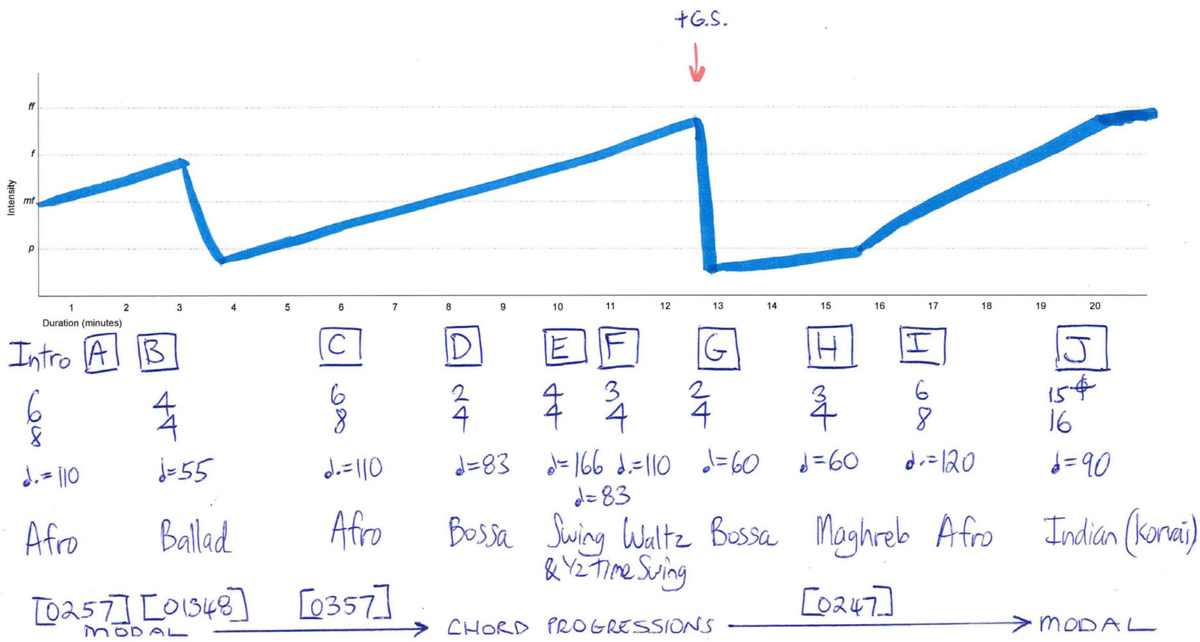


Figure 1: Sketch of form.

7.2 Bird calls—thematic usage

I transcribed by ear Australian bird calls from recordings of an unknown source.[1] The themes in *Birder* were invariably derived from the following small selection of these Australian bird calls, chosen intuitively for their melodic virtue:

- *Eudynamys scolopacea* (Asian koel);
- *Vanellus miles* (Masked lapwing);
- *Pycnonotus jocosus* (Red-whiskered bulbul).

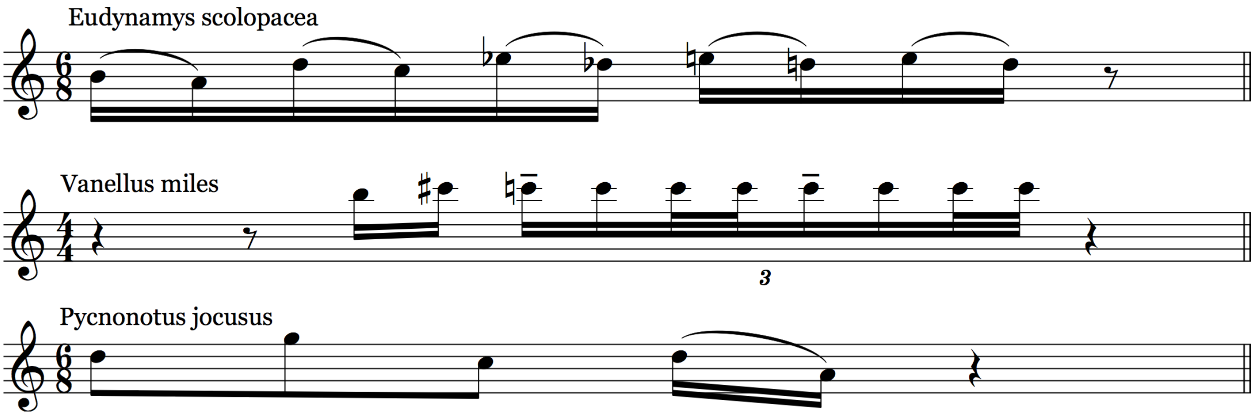


Figure 2: Transcription of selected Australian bird calls.

Some examples where these bird calls protrude into the foreground include:

- violin – bar 30
- electric guitar – bar 63
- piano – bar 383
- electric bass – bar 530
- soprano saxophone – bar 612
- viola – bar 651.

1 These recordings were made to cassette, collected around 2000–2005.

7.3 Bird calls—harmonic and melodic treatment

Using bird calls as superficial thematic material provides an obvious connection from the composition to the guitarist to whom the work is dedicated. However my primary intention in the composition was to constrain the amount of material introduced, and in-so-doing, unify the large-scale jazz composition through a myriad of perspectives on a limited number of ideas. My interest in self-similarity thus saw the original bird calls abstracted into other types of pitch material, and isomorphically into other parameters of music.

The first type of abstraction was to look at the harmonic potential of the bird calls, by abstracting the original melodic transcriptions into collections of pitch-class that disregarded order. The methodology I employed analysed the bird calls for their interval content using set theory.[2]

Pycnonotus jocosus
Set {0,2,7,9}
Prime form set type [0,2,5,7]
Interval vector <<0,2,1,0,3,0>>

This tetrachord is inherently pentatonic in sound, and this sound not only suited the style I had in mind for the composition, but also pandered to the natural capabilities of the guitar as the featured instrument. This characteristic is evidenced from the very first theme of the work (see guitar bar 18), which uses the original set as T_9 from E. The following notation shows the original bird call for *Pycnonotus jocosus* and some of its motivic transformations based on its pitch class set.

2 For a discussion of the related theories, refer to Rahn and Straus.

Figure 3: *Pycnonotus jocosus* and motivic transformations based on its pitch class set.

Motive variation 1A extends the motive rhythmically but reduces the pitch collection (and compass), and steers the focus from E to the dominant (B). Variation 1B is a fragment of the previous motive. Variation 1C captures the compass of the original motive, exposes more of the intervallic leaps, centres on the dominant (B), and also introduces a tripartite subsidiary phrase structure, with the three longer B's outlining the subsidiary resting points. Variation 1D returns to the original pitches of the bird call, but extends the motive with a different subsidiary phrase structure that uses pitch repetition and rhythmic augmentation to emphasise the cadence to ptich D5.

Figure 4 notates some of the pitch class set transformations applied to the original *Pycnonotus jocosus* bird call.

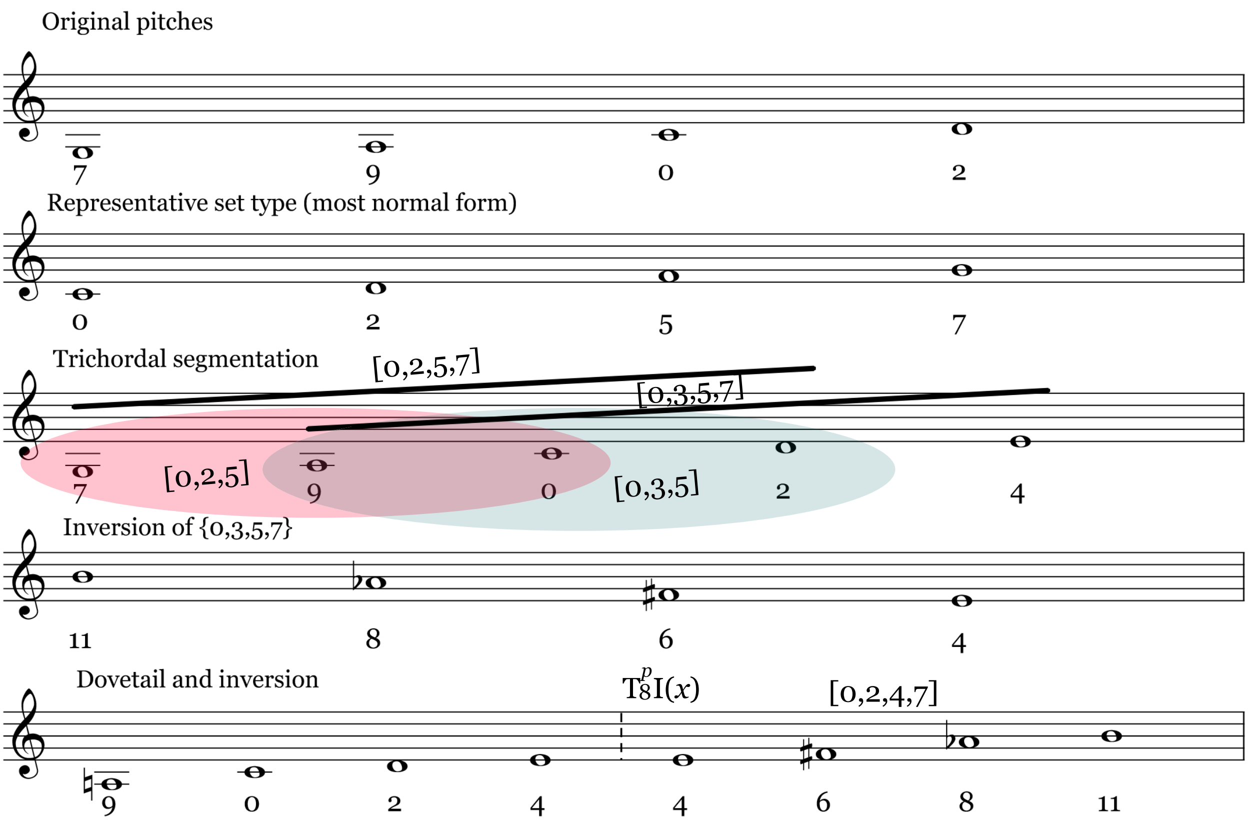


Figure 4: Pitch class set transformations applied to the original set for *Pycnonotus jocosus.*

The following circular diagrams map the *Pycnonotus jocosus* bird call into 12-tone pitch-class space in order to further illustrate the procedures used in my harmonic and melodic process.

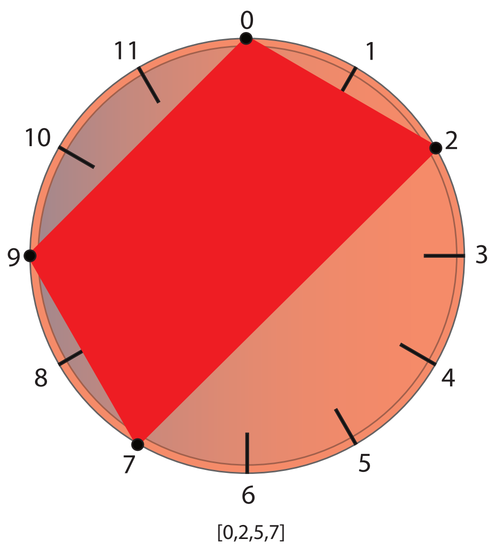


Figure 5: Pitch class set for the original *Pycnonotus jocosus* bird call.

From the original tetrachord {7,9,0,2} I segmented the first two trichordal subsets and found that {7,9,0} and {9,0,2} form the two inversionally-related trichords [0,2,5] and

(0,3,5). The bilateral symmetry created by this inversional relationship is emphasised by choosing a rotation that creates a vertical axis of symmetry.

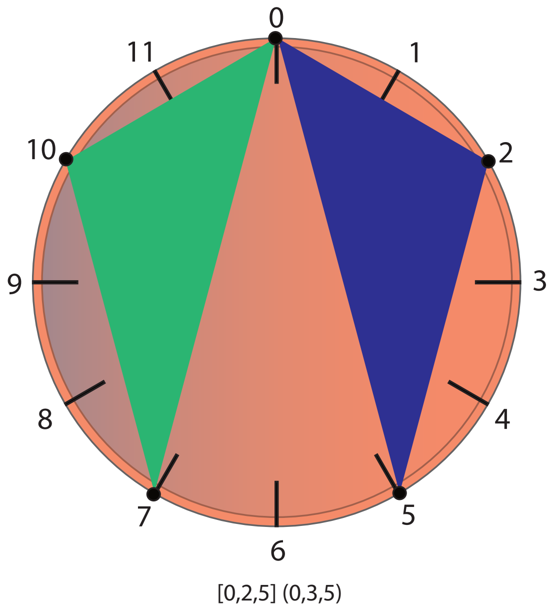


Figure 6: Two inversionally-related trichordal subsets of the *Pycnonotus jocosus* bird call.

Extrapolating this pattern to the tetrachord, I overlaid an overlapping inversionally-related tetrachord (0,3,5,7) from the second pitch-class in the original [0,2,5,7] tetrachord. I found this preserved the trichord segmentation and extended the pitch classes to a cardinality of five {7,9,0,2,4}. The preservation of the component trichords in the previous figure is clearly illustrated in the illustration of the following pentachord.

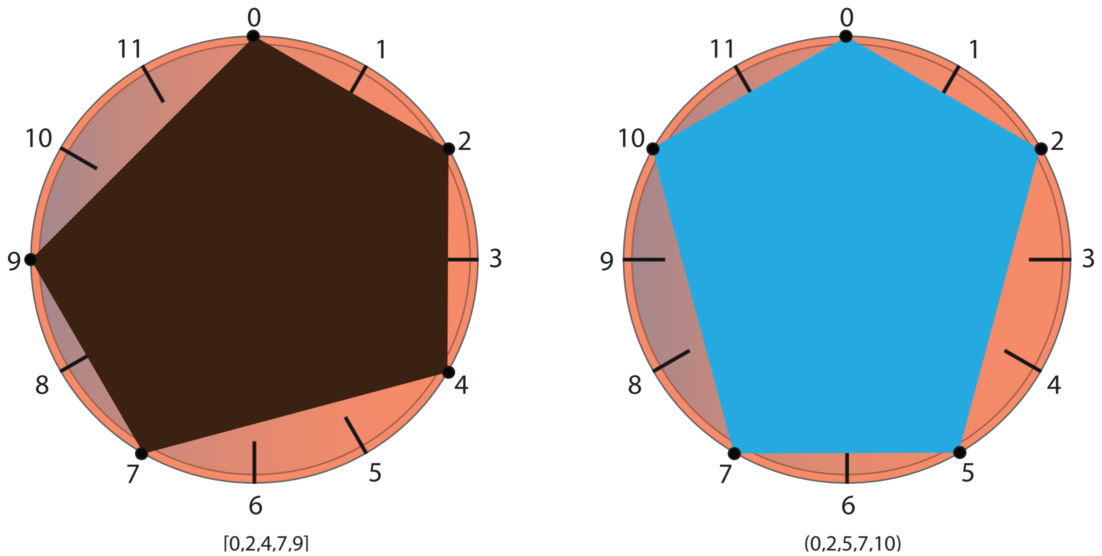


Figure 7: Pentachord comprised of [0,2,5,7] and overlaid (0,3,5,7) tetrachords. The left image shows the given transposition and the right image shows a rotation to reveal bilateral symmetry about a vertical axis and the component trichords of Figure 6.

The theme that appears in section E features a motive from this inversionally-related tetrachord (0,3,5,7) (see guitar bars 337–338). This theme appears also in the prior *bossa-nova* section D in a different harmonic setting (see saxophone bar 289) and in the contrasting section F in yet another key and rhythmic feel (bar 370).

Further experimentation saw the dovetailed (0,3,5,7) tetrachord inverted and transposed by 8 semitones to create the [0,2,4,7] tetrachord illustrated in Figure 8. Figure 9 is simply rotated to reveal its bilateral symmetry about a vertical axis.

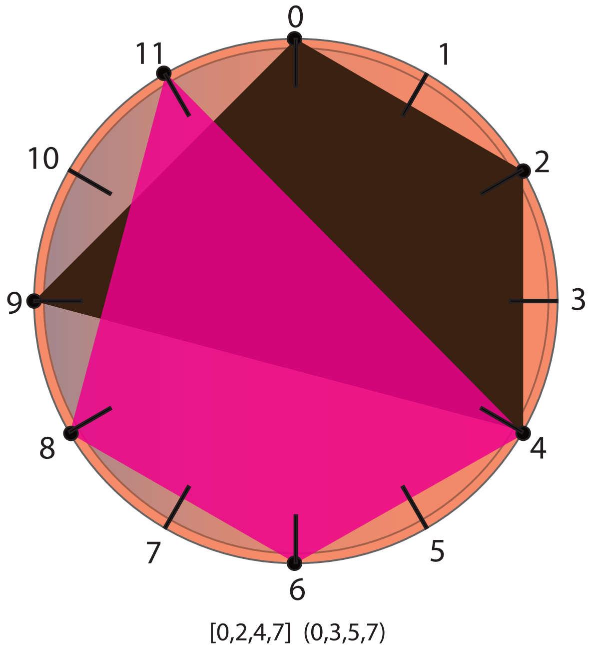


Figure 8: Tetrachord (0,3,5,7) (dark brown) with inversionally-related [0,2,4,7] tetrachord (pink).

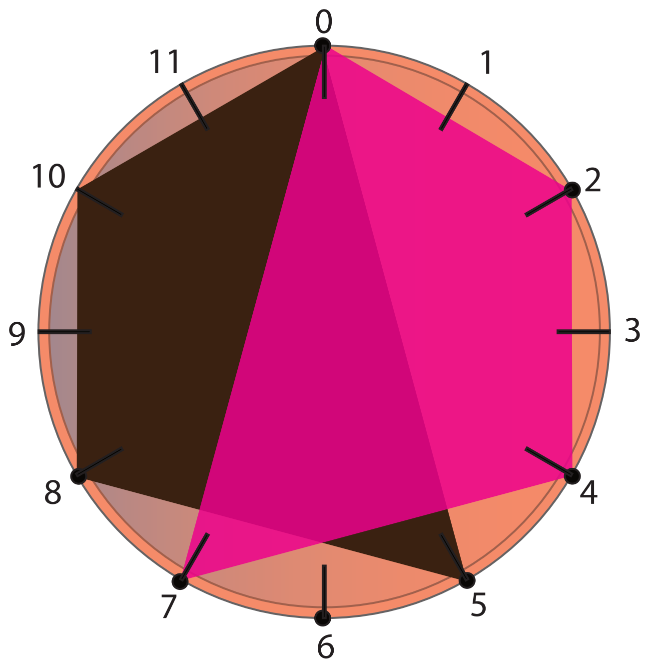


Figure 9: Tetrachord (0,3,5,7) (dark brown) with inversionally-related [0,2,4,7] tetrachord (pink), rotated to reveal bilateral symmetry about a vertical axis .

The combination of these dovetailed and inversionally-related layers is illustrated in the septachord of Figure 10, being the final heptatonic pitch collection.

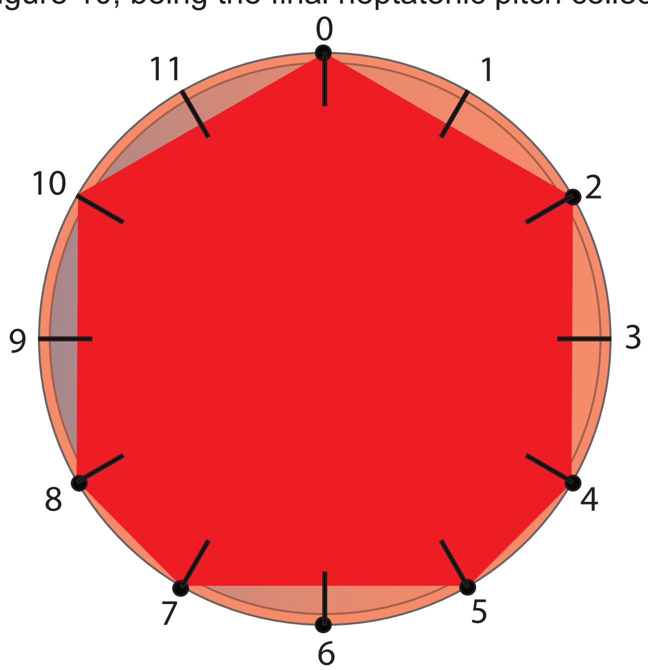


Figure 10: Septachord resulting from the combination of tetrachords of Figure 9.

This process of extrapolation demonstrates how I arrived at a seven-note pitch collection from the original *Pycnonotus jocosus* bird call via self-similar means. The final pitch-class set was used as guiding principle for the harmonic material as a macroscopic progression over the course of the work, progressing from [0,2,5,7] to (0,3,5,7) to [0,2,4,7] as indicated in the structural diagram (Figure 1).

Schenkerian analysis involves the identification of a fundamental background layer that underpins a composition, which Schenker called *Ursatz* ('fundamental structure') (Schenker 16). My approach in *Birder* is similar in respect that my goal is to unite inner and outer form. The difference is that the pitch classes in the final heptatonic pitch-class set are not a slow-moving sequence that is articulated note by note across the duration of the work. Rather, my approach began with the *Pycnonotus jocosus* bird call as surface material, and then wove this material back into the composition using modifications that are self-similar, both in foreground and background layers.

The dovetail and inversion of the *Pycnonotus jocosus* bird call notated in Figure 4 and illustrated in Figure 10 was also reconfigured as a heptatonic mode that united the (0,3,5,7) and [0,2,4,7] tetrachords into a South Indian *raga* known as *Charukesi*. Rearranged with a modal tonic of E, the essential scale of this *raga* is as follows:

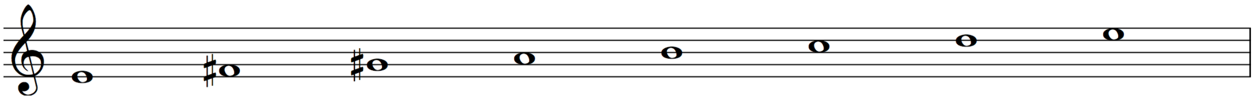


Figure 11: *Raga Charukesi* heptatonic mode derived from procedure leading to Figure 10.

These pitches are the basis of the harmonic material in the contrasting section C (bars 188–290). *Charukesi* in this key corresponds to the fifth mode of A melodic minor,[3] and harmonically to an E7 (b13) chord. This chord appears in the guitar in bars 266–289.

3 The jazz melodic minor scale is the source of such harmony, and corresponds to the ascending form of the classical melodic minor scale.

7.4 Other harmonic material

To contrast the anhemitonic pentatonic nature of the [0,2,5,7] tetrachord from the *Pycnonotus jocosus* bird call, and its derivations such as the [0,2,5] trichord, I chose pentachord [0,1,3,4,8]. Like [0,2,5,7] it is inversionally symmetrical, but it offers different intervallic content:

Prime form set type [0,1,3,4,8]

Interval vector <<2,1,2,3,2,0>>

Figure 12 shows [0,1,3,4,8] in a key chosen for its common pitch classes to those in the sets [0,2,4,7] and its inversion {0,3,5,7} notated in Figure 11, as well as an octatonic characteristic derived from the (0,1,3) subsets.

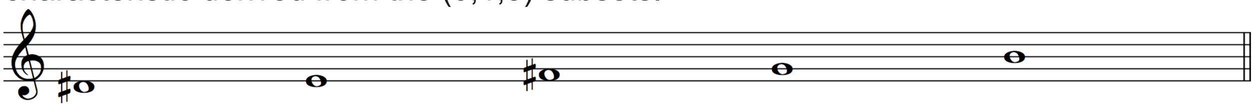


Figure 12: Pentachord [0,1,3,4,8].

This set features in the harmony of the distinct transitional interjections found in bars 118–124 and 328–332.

This set also provides the tonics of a slow-moving chord progression that underpins the exposition of the theme in section A—a kind of *cantus-firmus*. Figure 13 re-notates Figure 12 to match the order used in the electric bass part in bars 19, 35, 51, 67 and 86, being five sections with 16 bars per chord.

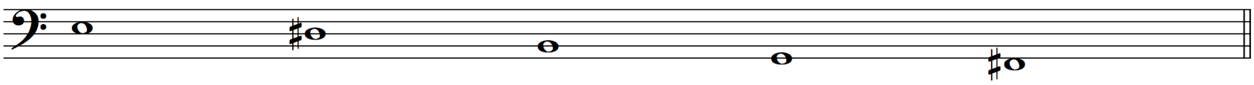


Figure 13: Bass chord progression tonic series derived from pentachord [0,1,3,4,8].

The chord progression was conceived as follows.

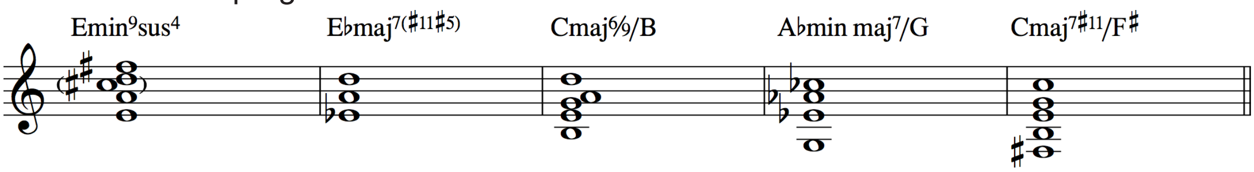


Figure 14: Chord progression with bass motion derived from pentachord [0,1,3,4,8].

It is apparent that each chord includes all or most of the pitches of Motive 1B (see Figure 3). This congruence allowed me to harmonically re-frame aspects of Motive 1 over the changing chords, leading to a feeling of progression and development whilst keeping a constant melodic thread. Further analysis reveals that various transpositions of the formative set [0,2,5,7] are also contained in these chords, which was a deliberate procedure I used to generate the chord qualities, as revealed in Figure 15.

Figure 15: [0,2,5,7] applied to *cantus-firmus* based on pentachord [0,1,3,4,8].

The pentachord [0,1,3,4,8] returns in section D to drive the chord progression in the *bossa-nova* section (see bar 290). Here the chord progression from the A section is reinterpreted and reordered to commence from the fourth chord and bass note, notated in Figure 16.

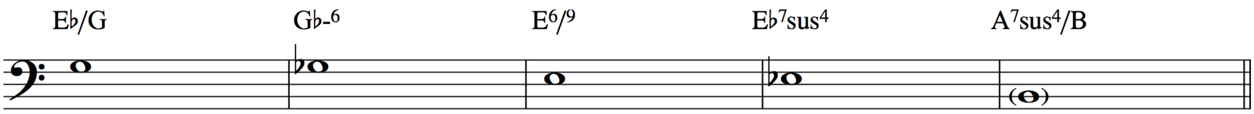


Figure 16: Alternative chord progression in section D based on pentachord [0,1,3,4,8].

Further development of strands of this sequence takes place in section D, where fragments of the bass movement or chord qualities can be seen in various transpositions. Examples include bars 303–305, and the entire saxophone solo from bar 315.

Following section A, the contrasting harmonic material in section B takes the same pentachord [0,1,3,4,8] and extrapolates the whole-half octatonic scale indicated by the first four pitch classes, built from pitch class B. For this section I created a chord progression consisting of 14 chords, each voice-lead in a canonic manner inspired by Schoenberg's *Farben* (Op.16, No.3). After studying Burkhart's analysis of this famous work which reveals its five-voice canonic 'organism', I devised my own approach[4] (Burkhart 143). I chose the canonic motive of an ascending major second, followed by a descending minor third for each of the four voices in a particular firing order as illustrated below. My four-voice 'organism' commences its canonic progression from a chord whose sound is not unlike the chord that opens Schoenberg's *Farben*. The texture of this contrasting section is also akin to Schoenberg's *klangfarbenmelodie*, where the melody of tone colour itself is featured. This aesthetic is maintained for the entirety of section B.

Figure 17: Four-voice 'organism' in section B, showing motive of +2, -3 in canonic firing order.

4 This insight is also attributed to Milton Babbit in *Words About Music*, ed. Stephen Dembski and Joseph Straus (London: The University of Wisonsin Press, 1987) 157.

Figure 17 indicates which voice type moves (Soprano, Alto, Tenor, or Bass), as well as the interval by which it moves (in semitones, following the +2, -3 motive). By the end of the 14-chord progression, the initial chord has reconstructed itself to rest two semitones lower than the key in which it began. Each voice in the canon has rendered the +2, -3 motive twice (because 2-3 = -1).

The firing order of the voices is devised in such a manner as to obfuscate the halfway point of the progression (when the initial chord is transposed down one semitone). This is achieved through avoidance of equidistance in the firing order of the four voices across the 13 voice-leading opportunities. In bar 9 of Figure 17 the bass, tenor and alto voices converge on the chord of the mid-point transposition (E-flat, B, G, C). The full realisation of this chord is foiled however by the soprano voice's preemptive movement up a tone two bars earlier, thus overlapping the two cycles of the overall +2, -3 motive. The 14-chord progression and this 4-voice canonic 'organism' is best observed during the piano solo, from bar 163.

This canonic section is not only symmetrical, resulting from the translation of the motive about the vertical and horizontal axes as a result of the transposition and temporal displacement of the generating +2, -3 motive. It is also arguably self-similar, due to the scaling present over time. Different voices move at different rates.

This 14-chord progression is reinterpreted in the guitar chord solo section from bar 149, and in an incomplete introductory form from bar 139.

The harmonic material that concludes *Birder* in the *korvai* (section J) draws from the intervals of the aforementioned *raga Charukesi*, but in a different manner to its harmonic application in section C. The pitch collection for the *korvai* is notated in Figure 18.

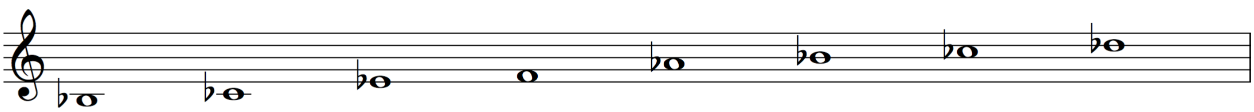


Figure 18: Pitch collection employed for section J *korvai*.

The fifth mode of A-flat melodic minor can still be viewed as the Western parent scale for this pitch collection. A modal tonic of B-flat aligns with the rhythmic starting point of the *korvai*, and the major third (G) of the parent scale is omitted. D-flat is also omitted from the lower octave. Through association of different pitches to the different stages of rhythmic expansion intrinsic to the *korvai*, motivic development is illuminated. The following figure illustrates the progressive unfolding of pitch in the *korvai* (section J).

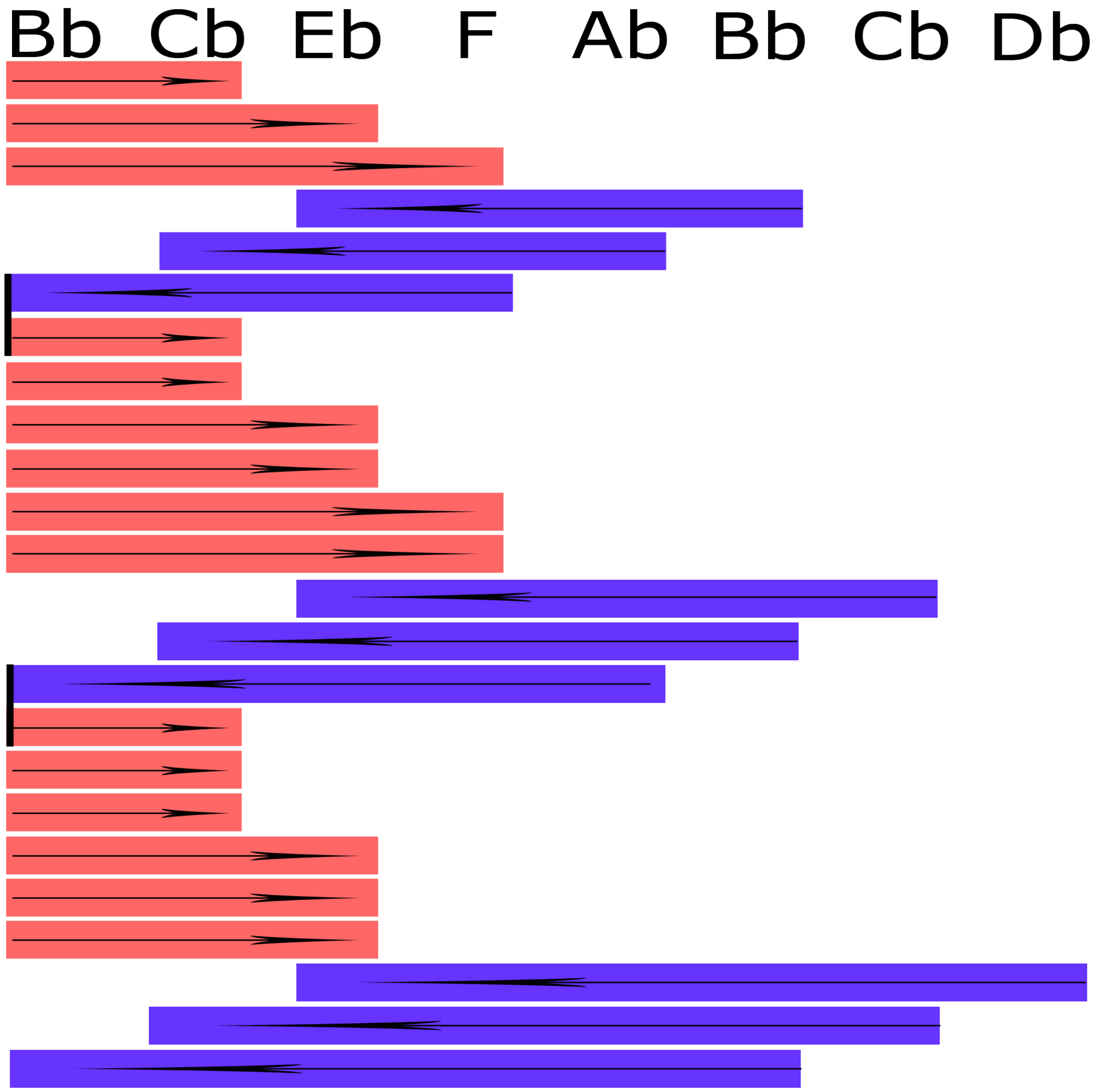


Figure 19: Pitch sequence employed for section J *korvai*.

This figure represents section J chronologically from top downwards, with the pitch headers showing the relevant pitches that fall in the (invisible) columns below. Each horizontal bar represents a phrase, and these are colour-coded red for ascending and purple for descending, as the arrows indicate. The structural repetition scheme will be discussed in the next section. This figure illuminates the progressive expansion of the melodic material in both the ascending and descending phrases. It also reveals the association of phrases of specific sizes and direction with specific pitches. For instance, two up is always B-flat, C-flat, whilst the groups of three descending phrases always target a descending cadence of E-flat, C-flat, B-flat regardless of phrase length. Comparison with the notation in the score at section J reveals that the last note in each set of descending phrases dovetails into the next ascending phrase. It shares its targeted pitch (B-flat) with the ascent that follows. These dovetailed joins are illustrated by black braces between sections in Figure 19. This approach fosters flow between phrases, and draws from South Indian rhythmic practice to articulate the first note in a subsequent group, including *karvai*.

7.5 Indian rhythmic structures

The first explicit occurrence of an Indian rhythmic procedure in *Birder* is revealed by the grouping designations in the guitar bars 42–48. The progressive reduction pattern 14 (2x7), 10 (2x5), 6 (2x3) is a *gopuccha yati* that occurs in the approach to the cadence at the end of this sub-section. This particular *yati* returns often, mostly climactically in the C theme group in section I. (This is discussed subsequently in Section 10.)

An expanding *srotogata yati* occurs in the approach to the next cadence in bars 75–80, comprised of 2 (2x1), 6 (3x2), 9 (3x3), 12 (3x4), 15 (3x5). A variation of this pattern occurs in bars 649–653. Here, expectation is aroused through the sounding of the first three stages of the pattern in the saxophone and guitar, but this attempt is foiled when the viola interrupts with the *Pycnonotus jocosus* bird call, causing a resumption of the pattern at bar 652. This then gathers increasing momentum as other instruments join the pattern to enforce the cadence to the next theme group from bar 654.

An example of a North Indian *tihai* occurs in the viola and double bass parts from bars 258–263, which consists of a 4 (10) 4 (10) 4 sequence to correspond with the final piano chord in that section.

Another example of an expanding structure appears from bar 86 in the violin part that accompanies the guitar solo and brings the ensemble to the final climactic cadence at bar 116. This structure consists of 5 (3) 7 (3) 9 (3), played three times from bars 86–101 and then another three times, gathering increasing instrumentation towards the unison final phrase in bars 115–116. It therefore combines the tripartite self-similar structure of *chakradar tihai*, with the phrase structure of *korvai*.

A *yati* figure features in the large pyramidal structure lead by the piano in bars 126–134 in the build-up to section B. Here, the expanding pattern 3 (2) 5 (2) 7 (2) 9 (2) 11 features a progressive ascent before the rapid descent into section B.

One original adaptation of the *yati* concept can be observed in the unfolding of the electric bass ostinato in bars 266–289. In this passage, a *srotogata yati* concept is loosely applied to the progressive and incremental expansion of the ostinato from one bar to four bars in duration.

The most explicit large-scale use of South Indian rhythmic structures occurs in the coda, section J. Here, the *korvai* indication in the score draws attention to form that unfolds, which has the following features:

- unison doublings of the melodic material
- expanding additive phrases
- pitches drawn from *raga Charukesi*
- a progressive orchestration from single-voice monophony, through polyphony and finally *tutti* monophony
- a large tripartite structure to create a climactic cadence and a sense of finality at the end.

The rhythmic procedure that underlies the *korvai* structure is revealed by the geometry of the following diagram.

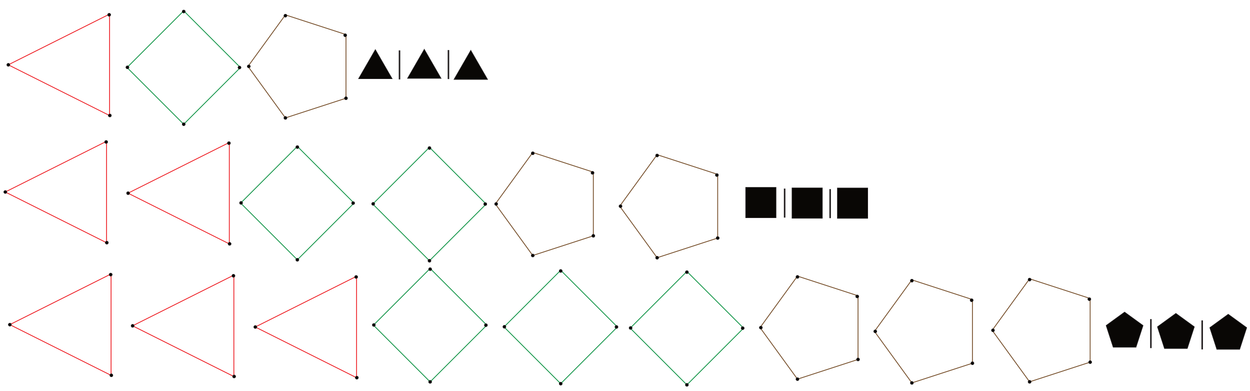


Figure 20: Geometrical representation of *korvai* structure, section J.

The self-similar role of the number 3 is clearly observed by this geometric illustration. On a macroscopic level, the entire structure illustrated in this figure is repeated three times. The other tripartite features of this *korvai* include:

- 3 lines to the structure
- 3 colours (polygons of different shapes) in each line
- Each line increases the number of polygons by 3 (3, 6, 9)
- The third line contains 3 of each polygon
- Each line of polygons is followed by a cadential figure (tihai) of 3 polygons (coloured solid black).

Figure 20 is read left-to-right and from top to bottom as a chronological sequence of phrases containing equidistant pulses. The three lines consist of the following design:

- <3-4-5> sub-grouped as (1+2), (2+2), (3+2), followed by a cadential figure (*tihai*) of <3-2-3-2-3>
- Doublings of each phrase of the korvai expansion (6, 8, 10), followed by a *tihai* of <4-2-4-2-4>
- Tripling of each phrase of the *korvai* expansion (9, 12, 15), followed by another incremental expansion of the *tihai* to <5-2-5-2-5>.

The total value of this *korvai* is 120 and is rendered three times for a grand total of 360 pulses.

The self-similarity of this *korvai* is further revealed by the following formulaic diagram that represents the mathematical structure of the groupings of each phrase (denoted *p*). Each of the three lines of Figure 20 are denoted in Figure 21 as a *stage* (denoted *s*). There are three stages, from top to bottom, each with three phrases, from left to right. At the end of each set of three phrases is a *tihai*, itself comprising a tripartite phrase structure. Note that the *karvai* (rests) are indicated by the bracketed single digits (2), the onset of which are articulated per South Indian practice. Note also that the third (being the last) resting gap in every *tihai* is omitted, as is conventional North Indian practice.

Stage (*s*) \ Phrase (*p*)	1	2	3	tihai
1	*s* x *p* (2)	*s* x *p* (2)	*s* x *p* (2)	(*s*+2) (2) x 3
2	*s* x *p* (2)	*s* x *p* (2)	*s* x *p* (2)	(*s*+2) (2) x 3
3	*s* x *p* (2)	*s* x *p* (2)	*s* x *p* (2)	(*s*+2) (2) x 3

Figure 21: Formulaic representation of *korvai* structure, section J.

What is immediately apparent from this formulaic diagram is that the *stage* and *phrase* values are the drivers of the changing structure, and that the counting procedure is self-similar throughout.

The *korvai* in section J is not the final structure of the composition, but this 360-pulse cadence is capped off with a short *tihai*, played *tutti*. The ensemble is consequently unified for the final climax, as the bass and drums were rendering a contrasting 15/16 layer during much of the *korvai*. The placement of the *tihai* was chosen to dovetail out of the final phrase of the *korvai*, making the last 5 (2) 5 (2) 5 sequence of the *korvai* the first sequence of the *tihai* that follows (see bars 780–782). Inclusive of this shared phrase, the *tihai* structure is as follows:

5 (2) 5 (2) 5 (8)

5 (3) 5 (3) 5 (8)

5 (4) 5 (4) 5 (8)

The incremental expansion of the internal gaps (*karvai*) in each line, and the lengthy partitions of 8 pulses of rest after each stage, combine to build up anticipation of the final chord in the work.

7.6 Brazilian rhythmic structures

The Brazilian forms of *bossa-nova* and *samba* are commonly found in jazz repertoire and performance, and so I was confident that Jon Damian would respond well to material showing this influence in my composition. The themes I used throughout *Birder* translated naturally in the most obvious Brazilian settings, including section D and section G.

Brazilian rhythmic influence is also found in less obvious settings, such as section C from bar 210. Here, the binary Brazilian *bell* pattern has undergone *ternarization.*[5]

The Brazilian *bell* pattern (*clavé*) is <2-2-1-2-2-2-2-1-2>, with nine onsets in a cycle of 16 pulses. In Figure 22 it appears in its native metre of 2/4, and therefore is considered a binary timeline. This form of the *clavé* appears in the drum kit from bar 290.

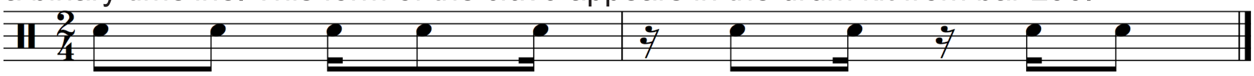


Figure 22: Conventional binary form of the Brazilian *bell* pattern.

In Figure 23 the Brazilian *bell* pattern is transformed into 6/8, with the same series of onsets <2-2-1-2-2-2-2-1-2>, but in quavers. The resulting transformation fills 8 bars of 6/8, as it requires 3 phases of the rhythm to complete a periodic cycle. The start of each phase is indicated with an accent for clarity.

Figure 23: Ternary form of the Brazilian *bell* pattern.

The *specific rhythmic contour* of Figures 22 and 23 are invariant, yet there is a scaling operation in effect. *Ternarization* is therefore a self-similar technique. This ternary transformation is used in its full three-phase form in the drum kit part from bar 218, the guitar in bars 226–289, the saxophone from bar 250, and again in the drums from bar 670 during the recapitulation. The metric modulation at section D hinges on the transformation of the Brazilian *bell* pattern from its ternary to binary form.

Figure 24 illustrates the geometric effect of overlaying the timelines of the Brazilian

5 Binary timelines can be divided by 2 but not 3, such as timelines with 2, 4, 8, 16, or 32 pulses. Ternary rhythms have timelines with a value of 3, 6, 12, and 24 pulses, which can be divided by 3. Refer to Chapter 3.2 and the Glossary for an explanation of *Binarization* and *Ternarization*.

bell clavé, native in a 16-pulse timeline, onto four cycles of a 12-pulse timeline. The outside concentric circle is tiled by the clavé.

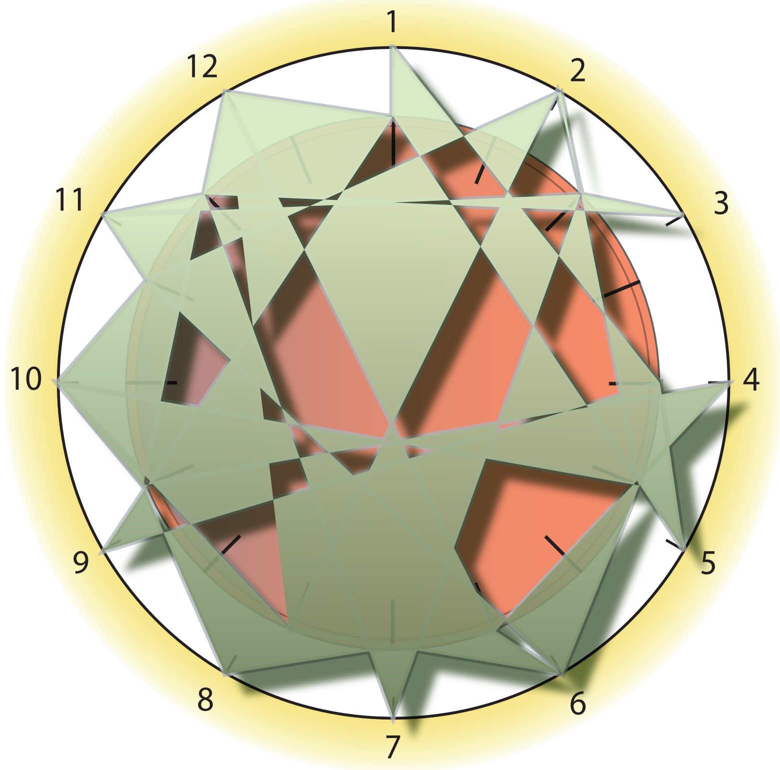


Figure 24: Pattern resulting from the ternarization of the Brazilian bell clavé.

One of the most common rhythmic figures used in Brazilian music is the *fork* rhythm (Neto 3).

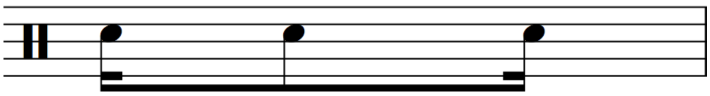


Figure 25: Brazilian fork rhythm.

This simple rhythmic cell can be found in various settings in *Birder*, including background figures (e.g. double bass bars 553–557 and viola 307–310), foreground melodies (e.g. guitar bars 549–552), and *tutti* figures (e.g. bars 328–331).

7.7 Afro-Cuban rhythmic structures

The Afro-Cuban *bell* pattern is a pervasive feature of the 6/8 sections. It is present in some form in all the 6/8 Afro-Cuban sections of the composition.

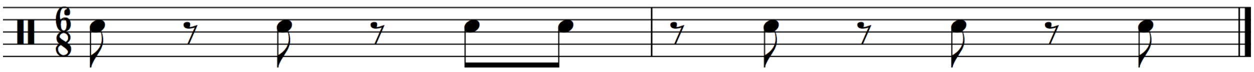


Figure 26: Conventional ternary form of the Afro-Cuban *bell* pattern.

This pattern commences the work, as played on the cymbal of the drum kit. The timeline <2-2-1-2-2-2-1> functions as the *clavé* that organises other rhythms around it, such as the electric bass guitar groove from section A.[6]

The Afro-Cuban *bell* pattern appears in background accompaniment, such as in the guitar harmonics from bar 528, the violin and double bass figures from bar 553, and the saxophone and viola figures bars 559–562.

6 This 12-pulse timeline is considered a *robust rhythm*, as it and its necklaces (phases) are ubiquitous to so many genres of music around the world (Toussaint, “The Geometry of Musical Rhythm” 75).

7.8 Hybrid rhythmic structures

Some rhythmic structures are hybrids derived from different genres. In section A, the first portion of the ternary form of the Brazilian *bell* pattern (Figure 23) is subjected to an Indian *korvai* expansion in tripartite form. Figure 27 notates the rhythm of the 5-bar sequence.

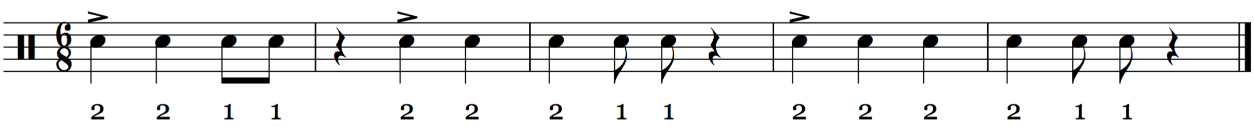


Figure 27: Brazilian bell pattern in ternary form with Indian *korvai* expansion.

The additive rhythmic design of this pattern is:

5 (3)

7 (3)

9 (3)

where the sub-groupings are 2+3=5, 2+2+3=7, and 2+2+2+3=9.

The explicit appearance of this pattern in the violin in bars 107–116 is preceded by its subtle orchestration under the guitar solo from bar 86.

Another rhythmic hybrid is found in section H in bars 520–535. Brazilian *bossa-nova* is merged with the Afro-Cuban *bell* pattern before succumbing to a North African *Maghreb* rhythm from bar 536. The tapestry that commences the section from bar 520 is a kind of musical mobile.[7] Ostinati of different durations are overlaid, including 11 (violin), 4 (saxophone), 3 (double bass), and 8 semiquavers (guitar), with various pitch cycles within those groupings also.

The *Maghreb* region of North Africa encompasses the territory of Morocco, Algeria, and Tunisia, and has spawned its own unique intercultural musical styles, including *rai* (Samba 12). The *Maghreb* groove functions as a *clavé*, and features a characteristic dialogue between the low and high percussion sounds, as notated below.

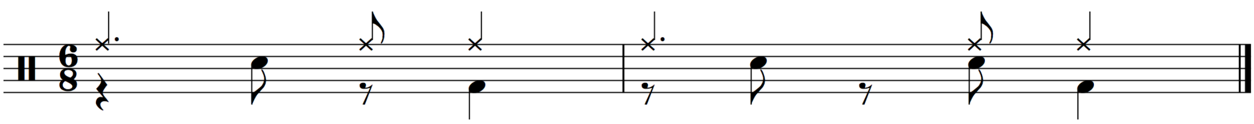


Figure 28: *Maghreb* groove notated for drum kit.

7 Musical mobile can be found in Stravinsky's *Three Pieces for String Quartet* (1914), and the ballet *Pulcinella* (1919). Ostinati of different lengths are superimposed to create phasing relationships.

In *Birder*, I have taken this groove and reinterpreted it in a slow 3/4 metre in section H, from bar 536. This complies with the same metre and tempo as the preceding hybrid Brazilian/Afro-Cuban groove, albeit with an increase in intensity. The aforementioned essential dialogue between the bass drum and snare drum in the 6/8 groove is maintained in this hybrid version of the *Maghreb* groove, as the following notation illustrates.

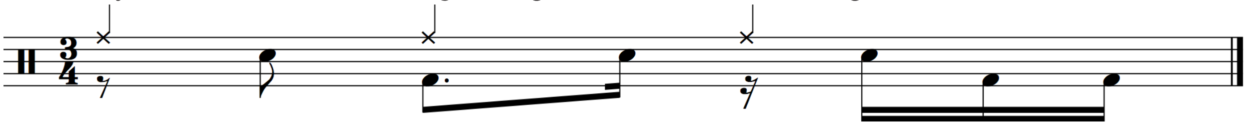


Figure 29: 3/4 version of the *Maghreb* groove notated for drum kit.

The resultant groove was a perfect choice for a powerful, earthy foundation for the distorted guitar and saxophone soli, and the drum and bass fills that give way to this groove in the preceding bar 535 are some of the most energetically-charged expressive moments of the work.

There is an isomorphic correspondence between the *Maghreb* groove and the ubiquitous *Pycnonotus jocosus* bird call.

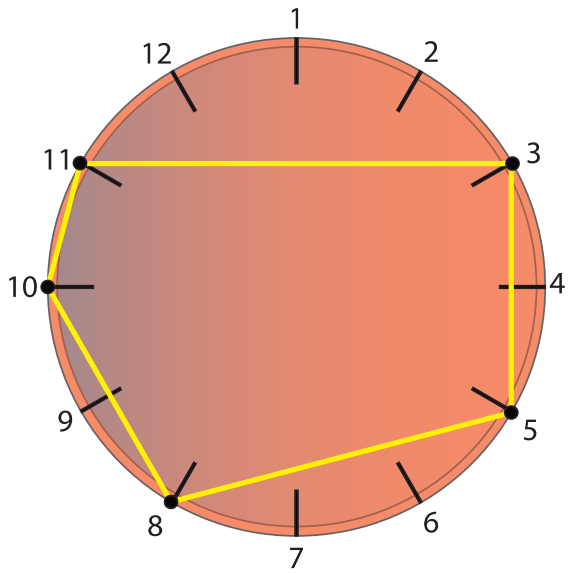


Figure 30: *Maghreb* groove timeline.

Compare this timeline with the [0 2 5 7] pitch class set from the *Pycnonotus jocosus* bird call in Figure 31. I have rotated (transposed) the set to (3 5 8 10) and placed it on a rhythmic timeline (with 1 at twelve o'clock) for ease of structural comparison.

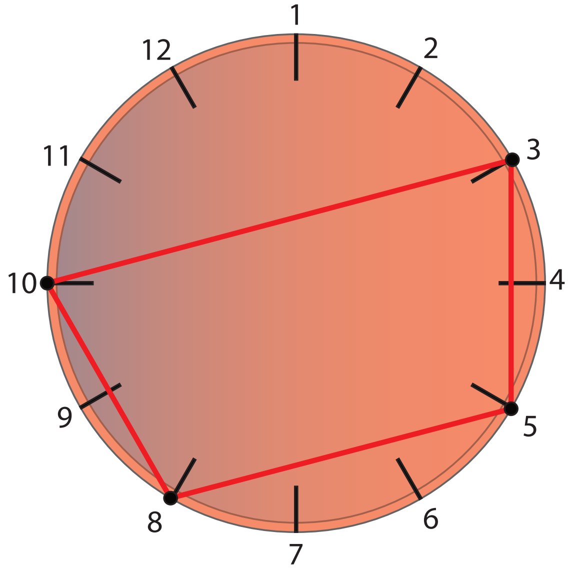


Figure 31: *Pycnonotus jocosus* bird call as a timeline showing isomorphic relationship to the *Maghreb* groove timeline.

The likeness of the groove and the bird call is uncanny, with the latter simply omitting the final onset (11). These diagrams illustrate how my composition embeds self-similarity through isomorphic relationships.

7.9 Use of quotation

The use of quotation to draw relationships between different tunes was something I enjoyed as a young fledgling jazz improviser. Indeed, the application of 'licks' in jazz draws from the same approach of interrelating given chord symbols or progressions with premeditated melodic patterns. Even master jazz artists draw upon licks and quotations. Legendary bassist Oscar Pettiford opens his improvised solo on *Tricotism* with the theme from the *Looney Tunes* cartoon,[8] and David Baker warrants both quotation and melodic pattern observations sufficiently important to include them as categories of observations on his analysis templates for jazz solos.[9] When I think of Jon Damian's musical aesthetic, I am reminded of the legendary jazz guitarist Jim Hall. Hall quotes the theme from the Jerome Kern classic *Can't Help Lovin' That Man* at 3:07 within his improvised solo on the track *The Way You Look Tonight,*[10] as follows:

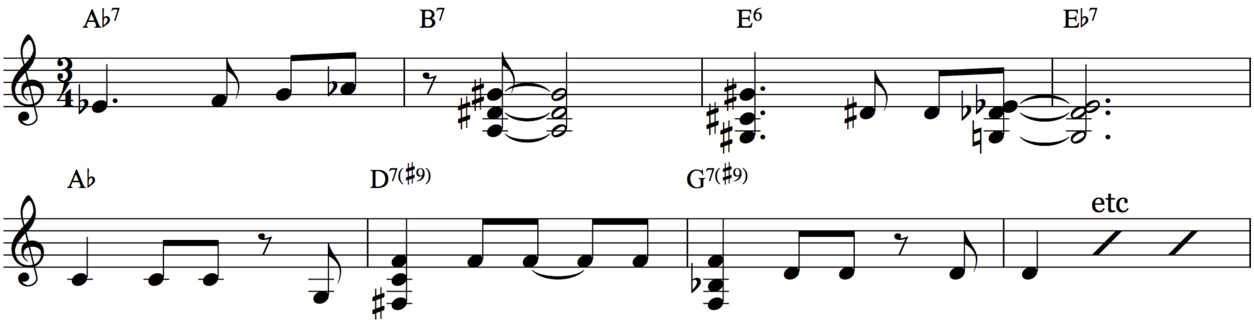


Figure 32: Jim Hall quotation of the theme from *Can't Help Lovin' That Man*, during *The Way You Look Tonight*.

I make the connection back to Jim Hall via Jon Damian through the subtle use of this same quotation in *Birder* bars 503–505. The saxophone renders the same theme behind the guitar solo, albeit in a slow 2/4 metre, *bossa-nova* feel and different key. This same quotation is pre-empted by the violin in bars 479–481.

In the guitar chord solo section from bar 149 I made a correlation and personal reference to a composition with a similarly introspective mood and harmonic colour that I composed 17 years prior, *Spool.*[11] This self-quotation is evidenced by the displaced quaver pattern in the guitar, the role of the lydian augmented chord, and the nature of the

8 Oscar Pettiford *The Bass* (1956).

9 David Baker's series of *Jazz Monographs* feature these templates, such as the one on legendary trombonist J.J. Johnson.

10 Jim Hall *Jim Hall Live!* (1975).

11 Loops *Ek!* (2001).

voice-leading throughout the progression.

Figure 33 is an excerpt from the first two bars of *Spool*. Compare this excerpt with bars 178–179 of *Birder*.

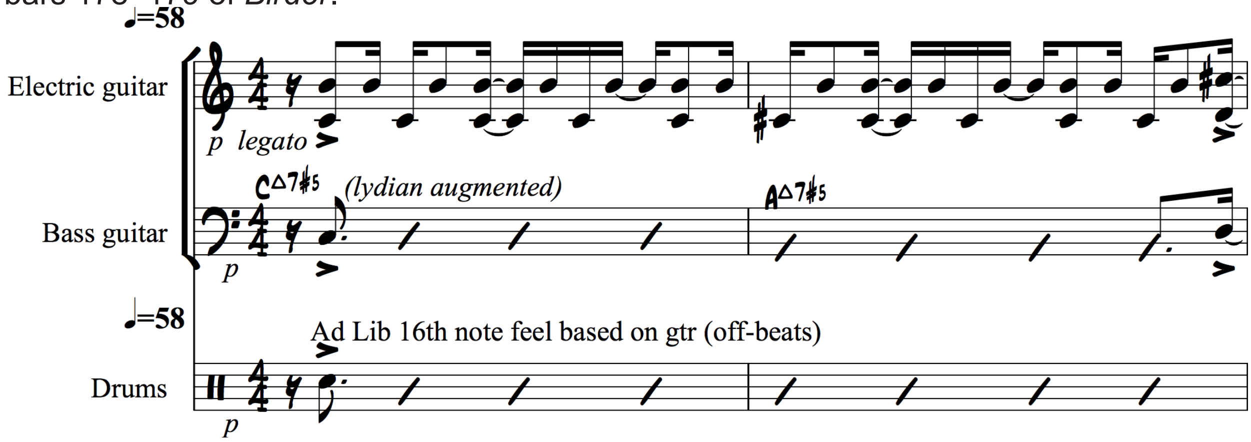


Figure 33: Excerpt from the first two bars of *Spool*.

In the penultimate section of the composition I sought to build tension for subsequent release in the final *korvai* section. A sequence of 5-note chord voicings were written that matched theme group D of melodic material (discussed in the next section). The climax of this sequence occurs in bars 697–700, through dynamic, rhythmic and harmonic means. The so-called 'Tristan chord' is quoted in bars 697–698, in the same key as Wagner's Prelude to Act 1 from *Tristan and Isolde*. This highly emotionally-charged sound suited my purposes at this moment. Figure 34 is a score reduction of *Birder* and shows how I used this appoggiatura, followed by a score reduction of the opening three bars from *Tristan and Isolde* as a point of comparison.

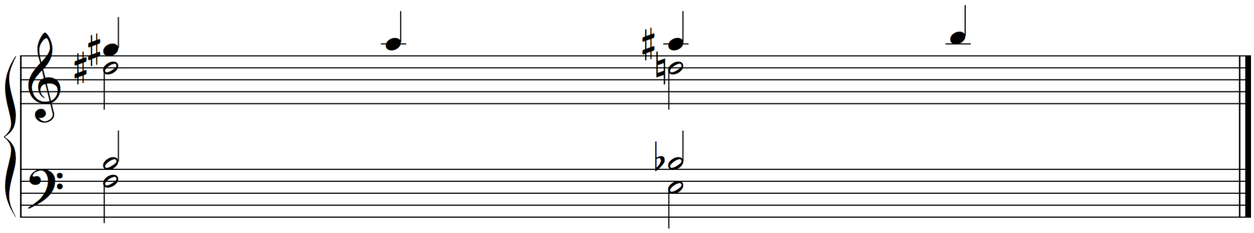


Figure 34: Score reduction of bars 697–698, showing use of *Tristan*-like appoggiatura.

Figure 35: Score reduction of the opening three bars from Wagner's Prelude to Act 1 of *Tristan and Isolde*.

7.10 Other structural observations

The positive Golden Section point is denoted '+G.S.' in Figure 1. This point represented an opportunity for a dramatic reduction of intensity in order to build to the final climax. Beyond this intention, I did not use mathematics to realise the position of this point, but rather used the sketch in a more intuitive manner to guide my writing in the large-scale form.

Post-composition analysis of the score and its performance placed the positive Golden Section point in bars 451–452, or 13'53" in the recording.[12] This point falls in the final phrases that precede section G (bar 470), a section of dramatic contrast, and so matching my original structural intention. This G.S. point results from applying the 1:1.618 ratio to both the real-time and bar-number approaches to calculation. In such cases, I claim the structure has *multi-mensural Golden Section alignment*.

The structural plan and overview of Figure 1 shows that changes of metre become more rapid in the approach to the Golden Section point, with an average of one change per minute in the lead up to that point, as opposed to a sparser distribution prior and thereafter.

There are a range of tempi employed in *Birder*, but it was my intention to not overly stress my performers with tempo modulations. The modulations are mostly intuitive,

12 The *korvai* section letter J was initially written entirely in 15/16 metre, as this suited the macroscopic cycle length, and was the metre of the bass/drums ostinato upon which the korvai pattern was composed. After the composition's initial rehearsal I decided to re-bar this section to favour the groupings of the microscopic phrases that were overlaid by the other instruments, facilitating their realisation at the expense of the bass and drums. The original barring of this section meant *Birder* was 731 bars long. 731 bars and 1348 seconds are the metric and real-time figures used in my Golden Section calculations, the results of which are within 2% of the 61.8% mathematical Golden Section point.

featuring doubles and halves, and in some cases changes between simple duple and compound triple metres. The modulations are also largely setup by rhythmic figures that cue the new tempo. Figure 36 shows an example of this approach, where the electric bass sets up the modulation that occurs at bar 135.

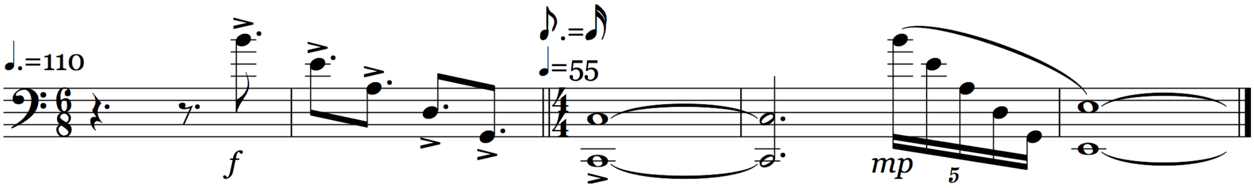


Figure 36: Setup of metric modulation at section B.

The exception to this intention is the modulation at the major Golden Section point (section G). Not only is the modulation a complex 4:11 ratio, but the new tempo is not prepared, but rather "crashed into" surprisingly. This was a deliberate compositional choice and effectively draws attention to the guitar in its new introspective mood and soloistic role from bar 470. The solo guitar emerges from the "crashed" chord by the ensemble with a gentle *bossa-nova* pattern.

The penultimate section I (bar 566) features recapitulation of past themes, contrasting the final section—which features Indian rhythmic processes as a coda. The recapitulations are not verbatim repetitions, but rather show some element of transformation, be it in modified motives, harmony or rhythm.[13] For example, the guitar theme that commences the composition (from bar 18) returns in bar 565 at a faster tempo, in a more strident tessitura of the guitar, over a chord progression with a faster harmonic motion, and in a different key. It is through these types of modifications that the recapitulating themes create a sense of journey, climax and completion.

13 The model of intense thematic growth can be found in Schoenberg's *developing variation*, where basic compositional units undergo ongoing modification (Schoenberg, "Style and Idea" 397).

The following figure identifies the theme groups in this penultimate section:

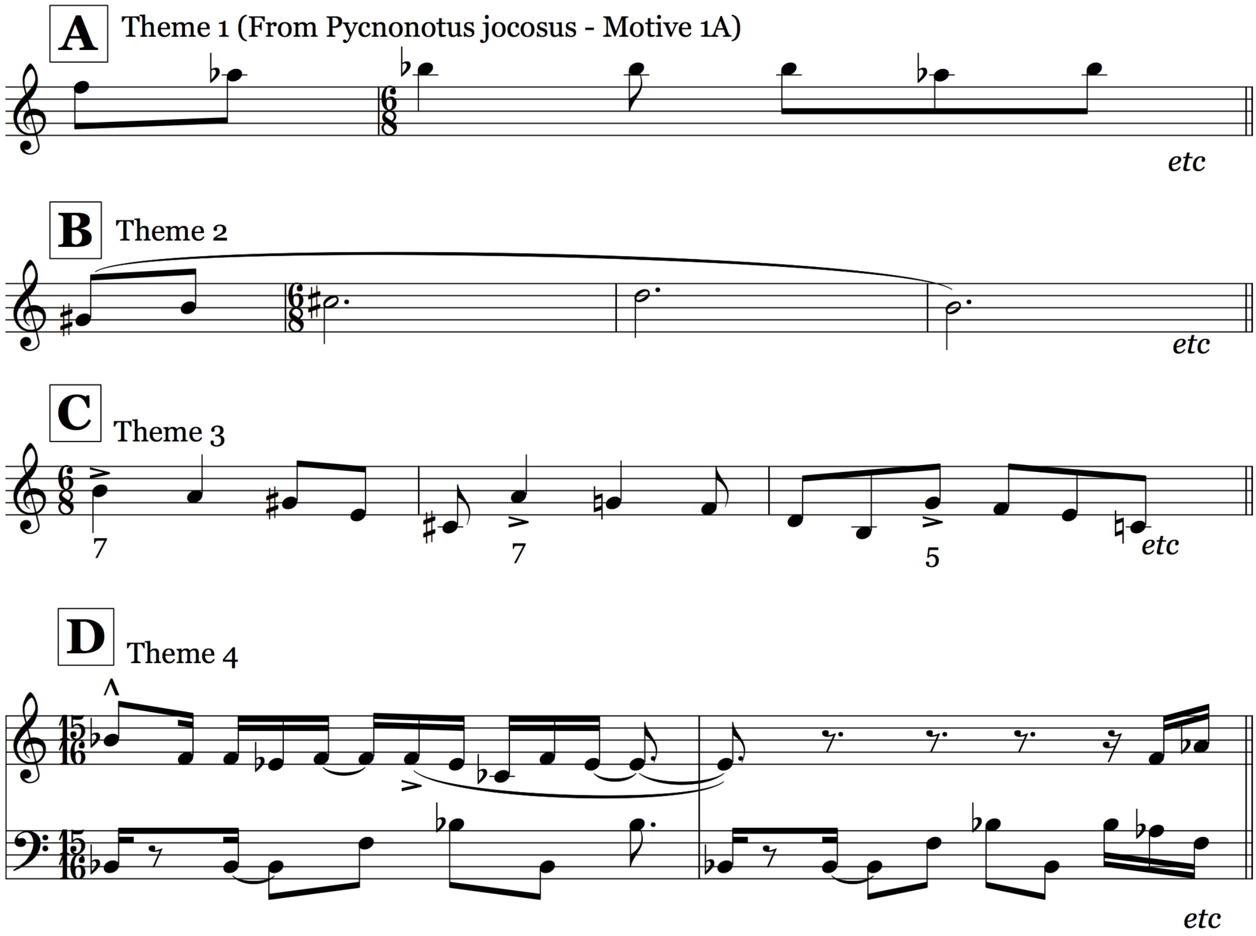


Figure 37: Themes employed in section I, from bar 566.

The sequencing of these four theme groups is as follows:

Bar 566–605: A B C D (39 bars)

Bar 606–653: A B C D D (47 bars)

Bar 654–701: A B C D D (47 bars).

This section is structured in a tripartite expanding fashion. The identity of these four theme groups is maintained through distinct rhythmic, harmonic and orchestration characteristics.

Theme group C features a *yati* reduction pattern of 14 (2x7) 10 (2x5) 6 (2x3), introduced at the start of *Birder* in the electric guitar (bars 42–48). The pattern is now used in a cadential manner to set up the 15/16 metred theme group D, and ultimately the final section J (see bars 590–596).

Figure 38 shows the chord progression drafted in the planning of this section, with four cycles that not only match the theme groups that were ultimately used, but function cyclically individually as well as a collective whole.

Figure 38: Chord progression for section I showing correspondence with four theme groups (Figure 37).

There are two interruptive silent bars in Birder, one at bar 117 and the other at 327.[14] Both of these silent bars are followed by *tutti* homophonic material that is distinctive and unique to these occurrences in the composition.

The first occurrence from bar 118–124 is in 6/8 and the ensemble renders a voicing of the *Pycnonotus jocosus* bird call as a static chord. The pitches G-A-C-D are drawn directly from the original *Pycnonotus jocosus* bird call (refer to Section 3). The addition of the pitch B to the voicing adds a desirable dissonance that further enhances the

14 Bar 117 is functionally silent even though it contains a pickup by the guitar at the end. The bar still feels like a silent break from the sections that precede and follow.

interruptive nature of this passage, and expands the pitch class set from [0,2,5,7] to [0,2,3,5,7]. This pentachord combines the original tetrachord with its inversion (0,3,5,7) to create the added pitch. The voicing is open and aims to feature the strident nature of the chord, further enhancing the interruptive sensation.

This chord is set to a peculiar rhythm that is syncopated and does not appear to align with a *clavé* or any predictable pattern. The static nature of the harmonic material, coupled with the long/short duration opposition makes the rhythm resemble Morse code.[15] The articulations specified indeed fall into binary combinations of long and short durations, indicated by the long/short accent markings across the ensemble. I attempted to decode the rhythms from Morse code. The message I deciphered was:

A F U F R E T

There is no record in my composition notes that reveals the idea behind this coding, but there may have been other symbolic procedures employed.[16]

The second occurrence of Morse code material follows the silent bar at 327. The chord in the passage from bars 328–332 is sounded now in 2/4 instead of 6/8—a *binarization* of the previous occurrence. Perhaps it is sufficient to identify this passage rhythmically as a *vishama yati*, employed to contribute surprise element at these structural positions. The strident nature of the chord is increased by the nature of the [0,1,3,7,8] pitch classes and the open voicing utilised.

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15 As an Amateur Radio operator since my youth, there is a precedent for using Morse code in my composition that dates back to the early 1980s, such as *Charlie Foxtrot*.

16 Without my original calculations I have been unable to find further evidence of the symbolism originally intended.

Chapter 8
Conclusion

This dissertation aimed to identify some of the organising forces that inform creative ideas in my compositions. Through practice-based research I have revealed the pervasive nature of certain attributes that have contributed to my particular compositional voice. These attributes—namely self-similarity, symmetry, polyrhythm and intercultural music—though at times consciously engaged and applied, have also arisen intuitively, and as such may be contemplated as much from an aesthetic perspective as the mathematical, geometric and constructuralist perspective adopted here. The former perspective may well be an avenue of future research.

The folio of compositions demonstrate some of the potential of a range of contemporary compositional techniques that fuse intercultural approaches with jazz and Western art music, resulting in a hybridized contemporary style. Particularly significant is the contribution of the rich musical heritage of the Indian subcontinent to the field of rhythm and time. The assimilation of these contributions into a contemporary jazz language offers seemingly unlimited creative potential for Western practitioners.

If geometry is ‘number in space’, then music could be likened to ‘number in time’, and it is with this scientific lens that this dissertation has employed simple geometric tools to offer insight into my music’s organising principles—albeit frozen in time (Lundy et al. 84). By employing a variety of diagrammatic representations, I have facilitated various perspectives on my music that have focussed upon musical intervals, compositional structure, and rhythmic patterns. This has illuminated inter-parametric relationships and formative patterns that would otherwise have been subsumed by the complexity and changeability of music’s temporal and multifaceted reality. The circular geometric representations of the temporal features have revealed that timelines have a deep and consistent role in the organisation of my composition. As a result, my music shares a sense of *chronos*[1] with the principal genres with which it has hybridized, being North and

1 *Chronos* time is defined as the time of clocks, and of temporal frameworks which form the basis of tension, release, and momentum. This contrasts with the philosophy of *Christos* time, being the spiritual, “timeless” time (Leake, “Relating Sound and Time” 108).

South Indian music.[2]

This dissertation and folio of compositions have uncovered certain theoretical areas of research that offer potential for future creative composition and analysis, which may be taken up by myself, as well as other composers and music theorists.

1. The role of self-similarity in music has been addressed by Madden and Johnson. Further research into its occurrence and application to composition—including intercultural composition—promises to be fruitful.
2. The *kotekan* patterns of Balinese *gamelan gong kebyar* orchestra contain rich symmetries, revealed by a geometric approach to their representation. A study of the metallophones, *kendang* drums and their repertoire with this perspective would be a rewarding investigation.[3]
3. South Indian *yati* rhythmic patterns could benefit from further geometric analysis, considering the topics of rhythmic contour and figurate numbers. Deduction of generative formulae for creation of customised patterns and variations (through exchange of grouped onsets and *karvai* gaps) would be a useful resource for composers and performers.[4]
4. A survey of intercultural timelines that represents not only the constituent onsets but also their relative *metrical weight* would be a rewarding endeavour.[5] A geometric approach using polygons in not two but three dimensions (polyhedrons) could be used to represent timelines in such a way as to co-illustrate their metrical weight, and thus objectively communicate a timeline's pattern of onsets as well as their metrical complexity.

2 The music of sub-Saharan Africa shares the same *chronos* (Anku 1; Leake, "Master Drummers of West Africa" 187).

3 Research by Tenzer and Vitale paves the way for a focus on symmetry in this genre.

4 This would amalgamate and extend the work of Sharma, Krishnamurthy, Iyer, and Reina.

5 *Metrical weight* is a measure of syncopation proposed by Lerdahl & Jackendoff (termed *metrical complexity* by Toussaint). This would extend the work of Flanagan to include timelines beyond the *clavé* rhythms of Brazil, Cuba and Sub-saharan Africa.

5. Experimental determination of the limit of perception of nonconsecutive rhythmic events would be of interest to composers and theorists working with complex rhythmic strata.[6] Whilst it is suspected that the brain subconsciously recognises distant relationships, there is no experimental evidence yet to prove or disprove the extent to which these relationships play a role in the perception of rhythmic similarity.[7]
6. Experimental comparison of the aural versus visual perception of symmetry in timelines with various orientations of reflection lines. An outcome of such experiments may be the development of a method for improving the cognition of such symmetry, including the more challenging symmetries with diagonal reflection lines.[8] Such a method could assist musicians to render various necklaces of timelines, reducing disorientation arising from the skewing effect of the rhythm in relation to the underlying beat.
7. The investigation of intercultural genres whose rhythmic language exhibits the mathematical property of tiling would be of interest. This could include the *palmas* of Flamenco, *kotekan* of Balinese gamelan, and *garagab* instruments of the Maghreb region of Africa.[9]

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6 This could build on the research of Phillips and Sethares.

7 Experiments have been conducted in this way for nonadjacent pitches, including Quinn's research that employs Polansky & Bassein's *combinatorial model of contour*.

8 The nine classes of symmetric rhythm are categorised in Toussaint *The Geometry of Musical Rhythm*, chapter 30.

9 The mathematical properties of tiling have been investigated with a musical perspective by Tangian, Johnson, Moreno, Toussaint, Andreatta, and Hall & Klingsberg. A study that brings together the range of musical variations of the tiling principle with an intercultural survey would contribute to a creative musical understanding of the topic.

The role of collaboration deserves mention, as it acts as an essential ingredient in the creation of intercultural music. Though this topic has been broached in the context of Indian music and jazz by other researchers (including Evans; Wren), it remains a large and changing field with more potential for specialised research. The intercultural collaboration required for the creation of my body of work recognised the risks of intercultural hybridity identified in Chapter 1. I conclude that the following countermeasures could mitigate these risks during the process of collaboration:[10]

- A balance of *emic* and *etic* approaches by all ensemble members and the composer;
- Humility and an acceptance of 'not-knowing' in all situations;
- Respect for traditional musics by honouring their people, cultural values and rituals;
- Acting in the moment, rather than enforcing a preconceived agenda for its own sake;
- Acceptance that the journey towards creation of successful intercultural music can be a lengthy one, and that the emphasis should be turned patiently away from any goal and rather toward the process;
- Continual refinement of ones aural skills in order to facilitate the conceptual understanding of unfamiliar musical languages.

These countermeasures align with the factors determined to be integral to intercultural music by the First International Symposium and Festival on Intercultural Music, held in London in 1990 (Kimberlin & Euba 3). Two of these four factors include:

- A highly intimate knowledge and understanding of creative and performance processes of other cultures;
- Expansion of modes of expression in the creation and performance of new music through the utilization of musical elements, processes and techniques of other cultures, whilst honouring the integrity of indigenous value systems.

10 This list of countermeasures arises from my own research, and is intended as a guide rather than a prescriptive list, just as in the list Evans produced from her research (201).

In my compositions, I recognise the semiotic nature of interculturally-derived musical objects, and treat the inter-semiotic translation of such objects to their consciously-juxtaposed environment with reverence and care.[11] Intercultural composition and collaboration thus presents both the risk and the opportunity that fuels my activity, and represents potential future growth in technical understanding and aesthetic empathy alike. I agree with Evans in identifying bi- or poly-musicality as a most desirable trait of a contemporary musician (198). It represents an artist attuned to today's global village—one with cultural competency that does not segregate, but rather mediates tradition, identity and context in a dynamic fashion (Wren, "Improvising Culture" 87).[12]

The practice-based research that this work embodies is part of a long-term process of learning and development—one which in reality has no end.

11 Wren refers to the 'indexical' nature of such musical objects similarly (Wren, "Improvising Culture" 22). In musical language, synecdochical signs and shortcuts are used to represent and communicate multiple parameters as generalised and easily transportable concepts (Pareyón 110). Appiah makes the point that cultural appropriation isn't as much an issue of intellectual property, but rather one of respect—as long as respect is shown for authorship and ownership, that cross-pollination of cultural works (including "borrowing, and if necessary stealing from other cultures") is a desirable and necessary part of human expression and development (Appiah 9).

12 Refer to Slimbach for a discussion of cultural competencies.

Glossary

Additive rhythm

Additive rhythm builds durations from concatenating small, atomic units of time. Additive rhythmic procedures are suited to the realisation of music that contains frequent changes of metre, prime metres with no option for perfectly even beats (as in 7/8), polyrhythm, melody based on speech, passages that are phrase-centric and those that incorporate numerical procedures.

Angsel

Term for the structural cues performed by the Balinese *kendang* drums that dictate where cadences and contrasting sections occur in the *gamelan gong kebyar* orchestra.

Apsis

The point where the constituent layers of a polyrhythm are most distant from each other on their periodic trajectory. (See also *Coincident hit point*.)

Arudi

A rhythmic cadence that belongs to South Indian music. It features a phrase sounded three times, as in AAA. (See also *Mora, Tihai*.)

Axis of reflection

In bilateral symmetry, the formula for determining the location of the axis of reflection (a) in any sequence of members (m) is:

$a = (m + 1) / 2$

Axis of symmetry

In bilateral symmetry, the line about which a mirror image occurs is the axis of symmetry. In music, symmetries about the vertical (y-axis) are temporal, whilst symmetries about the horizontal (x-axis) concern pitch.

Beat

A perfectly regular (uniform) grouping of pulse that gives rise to the sensation of rhythmic periodicity. Beats form regular polygons on isochronous timelines.

Bilateral symmetry

Also known as *mirror symmetry* and *reflectional symmetry*, bilateral symmetry in two dimensions shows invariance about a particular axis. In musical terms, symmetries about the vertical (y-axis) in music are temporal, whilst symmetries about the horizontal (x-axis) tend to concern pitch.

Binarization

The modulation of a ternary timeline into a binary one. The *specific rhythmic contour* is invariant during the procedure, yet there is a scaling operation in effect. *Binarization* is thus a self-similar technique. Those timelines with both 2 and 3 as factors (such as length 6 and 12) are readily binarized, as compound duple and compound quadruple metres are easily converted to simple triple metres. Various methods can be used to binarize other timelines. Interesting grooves and musically-rewarding effects can be produced by reinterpreting a binary rhythm in a ternary setting, and *vice versa*. (Refer to Anku, and Toussaint "The Geometry of Musical Rhythm" 57). (See also *Ternarization*.)

Binary timeline

Binary timelines have a beat that is simple—it can be divided by 2 but not 3. Timelines with lengths of 2, 4, 8, 16, or 32 pulses are common examples. (See also *Ternary timeline*.)

Bol

A grouping of rhythmic syllables that form a syntactic word in North Indian music, such as *tirakita*. (See *Solkattu*.)

Bracelet

If two rhythms are mirror image reflections of each other, they are considered of the same bracelet. (See also *Necklace*.)

Chakradar

A North Indian melodic and rhythmic device that features three occurrences of the same phrase within a larger over-arching tripartite structure. (See also *Tihai*.)

Clavé

A rhythmic pattern of central importance in the organization of other rhythmic layers. Usually sounded as a persistent *ostinato*, its presence is associated with the music of sub-Saharan African music, Afro-Cuban music, and Brazilian music. (See also *Timeline*.)

Coincident hit point

The pulse in a polyrhythm where the constituent layers share an onset. (See also *Apsis*.)

Complementary rhythm

A rhythm that completely fills in the gaps or silences of another rhythm is its complement. Together, they sound a continuous stream of onsets. (See also *Tiling*.)

Contour modulation

A technique of rhythmic augmentation and diminution created and featured in *Mod Times*, whereby constituent inter-onset intervals of a particular motive are varied whilst the generic rhythmic contour and any inherent symmetry is maintained. This technique constrains the subdivision to a minimal duration.

Cumulative rhythm

The resultant rhythmic phrase which represents the combination of two or more layers of a polyrhythm. It is palindromic.

Duration

The length or rhythmic value that spans two onsets.

Eduppu

In South Indian classical music, *eduppu* is the displaced starting point of a composition or line, occurring such as positions as 1 or 1.5 beats away from beat 1 (*samam*).

Fibonacci Series

A type of *summation series* that is self-similar and consequently can be defined geometrically and recursively. Its proportions relate to *Golden Section*.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

Figurate Numbers

Numbers that can be represented geometrically through the plotting of equally spaced points. Two-dimensional plane figurate numbers include triangular numbers, square numbers, etc. My research demonstrates an intimate connection between trapezoidal numbers and polyrhythm.

Fractal

A term coined by Mandelbrot in 1975 that refers to a branch of geometry that is more complex than standard Euclidean geometry, in that it deals with the patterns and systems exhibited by the natural world complete with their seeming chaos, statistical irregularity, similarity across multiple scales, and dimensional constructs that elude 1-, 2- and 3- dimensional measurement—indeed possessing a fractional dimension. (See also *Self-similarity*.)

Gati Bedam

A South Indian rhythmic technique that gives rise to the illusion of tempo modulation in the same way that polyrhythm can. In *gati bedam*, phrases are represented in changing subdivisions (e.g. a phrase in semiquavers performed subsequently in quaver triplets). Any tempo modulation effect is framed in the context of a consistent and unchanging *tala* (cycle) and tempo (*laya*).

Generic rhythmic contour

A measurement of the relative chronological change in duration between a series of inter-onset intervals. The arrows ↑ ↓ → respectively represent a relative lengthening, shortening and repetition of inter-onset intervals. (See also *Specific rhythmic contour*.)

Golden Section proportion

Golden section, symbolised Ø (*phi*) can be expressed algebraically as:

a:b = (a+b):a

It represents a geometric proportion where the ratio of one part to the larger is the same as that of the larger to the whole. It therefore has the property of *self-similarity*. (See also *Fibonacci Series*.)

Grouping

The addition of some quantity of pulses into a unit of duration, as in additive rhythmic procedures.

Hit point

A targeted pulse in a measure or timeline. (See also *Coincident hit point*.)

Inter-onset interval

The distance between two onsets, measured in pulses, and denoted by the sides of a polygon plotted on a circular timeline.

Interval generator

Messiaen's modes of limited transposition demonstrate the application of interval generators to create symmetrical scales from equal partitions of the octave. For example, 2+1 can be used as the interval generator for a symmetrical partition of four 3s (i.e. a diminished seventh chord) in an octave.

Interval of reflection

The interval of reflection applies to both horizontal and vertical axial types of bilateral symmetry, and refers to the distance between the reflected forms. (See also *Interval of translation*.)

Interval of translation

The interval of translation (or translation length) applies to both horizontal and vertical axial forms of translational symmetry, and refers to the distance between the translated forms. (See also *Interval of reflection*.)

Interval vector

In this dissertation, interval vectors are used as an approach to representing the characteristic interval content of both pitch and rhythmic entities. Rhythmically, adjacent inter-onset interval vectors summarise the inter-onset interval multiplicity for all intervals sounded in a timeline. Full interval vectors, by contrast, show all of the smallest possible (geodesic) distances between all onsets (including those that are non-contiguous), being analogous to the interval-class vectors of post-tonal theory. Double triangular brackets <<g,h,i>> are used to distinguish interval vectors from an ordered series of inter-onset intervals.

Isomorphic

Derived from the Greek words meaning ‘equal form or shape’. Isomorphic relationships are considered an aspect of self-similarity, whereby ideas are reapplied trans-parametrically. Isomorphism of rhythm and scale has been identified in the compositions of the *ars nova* of the fourteenth and fifteenth centuries.

Isorhythm

A rotationally symmetrical procedure employed by fourteenth century Flemish composers (such as Machaut and Dufay). The composer draws from two groups with different membership size, one group being the pitch content (*colour*) and the other rhythmic duration (*talea*).

Karvai

A term that refers to the structural gaps or spaces in South Indian music. In practice, *karvai* act functionally as separations and are not necessarily silent (being often filled with a sustained sound or subordinate articulations). This function relates directly to the concept of *interval of translation,* an appropriate analogy given the predominance of situations where *karvai* are applied to symmetrical structures. Such structural gaps many be conceptualized differently in North Indian music.

Kendang

Balinese drums incorporated into the *gamelan gong kebyar* orchestra and also featured in their own solo tradition called *kendang tunggal.*

Khyal

The prominent style of North Indian classical music performed today. A vocal style, it is inherently improvisational, as the translation of *khyal*'s Arabic root suggests—'imagination'.

Konnakkol

The South Indian term for the vocalization of its rhythmic language. The term is sometimes used synonymously with *Solkattu.* (See *Solkattu*.)

Korvai

Meaning 'strung together', *korvai* are South Indian compositions that typically consist of a sequence of phrases that are not identical, but that follow a logical pattern of development. *Korvai* possess two sections, the first often creating rhythmic opposition to the metre (*tala*), and the second containing a repeated *mora* cadence.

Kotekan

Interlocking patterns featured in the Balinese *gamelan gong kebyar* orchestra.

Laya

The term for tempo used in North and South Indian music. More broadly, *laya* represents all the interrelationships that contribute to an inward orderly flow to the music, even on a philosophical level.

Lehara

A type of "melodic metronome" featured in North Indian *tabla* solos, involving the repetition of a suitable theme. Traditionally performed by the *sarangi*.

Matra

A beat in North Indian classical music.

Mirror Point

See *Axis of reflection*.

Mora

A rhythmic cadence that belongs to South Indian music. It features a phrase sounded three times, as in AAA. (See also *Arudi*, *Tihai*.)

Mukhra

Hindi for ‘face’, or ‘feature’. North Indian classical music term for the important catch phrase that functions as the melodic “hook” of the tune. As such it acts as the recurring refrain that bookends improvisation.

Multi-mensural Golden Section alignment

The property that results when the outcome of multiple *Golden Section* measurement methods leads to congruent results for specific material.

Necklace

If two rhythms are rotations of each other, they are considered of the same necklace. (See also *Bracelet*.)

Ombak

Meaning ‘waves’, refers to the dynamic and tempo fluctuations featured in Balinese music.

Onset

The initial articulated pulse in a grouping. On timeline diagrams, onsets are represented by black dots upon the circumference of the circle. Sub-groupings may have subsidiary attacks of additional onsets (e.g. 7 = 3+2+2).

Polyrhythm

The superimposition of two or more layers of regular groupings of pulse. All polyrhythms have bilateral symmetry and palindromic cumulative rhythms.

Pulse

An atomic unit of time, which are grouped to form rhythms and beats. Simile for *Subdivision*.

Reflection rhythm

A special kind of *complementary rhythm* whereby the mirror-symmetric image of a rhythm serves as the original rhythm's complement. The combination of the original and the reflection fills all the pulses in the timeline as a *tiling*. The single paradiddle drum rudiment LRLLRLRR is one example.

Rhythmic canon

Rhythmic canons are created when two or more rotations (necklaces) of a rhythm are sounded at the same time where no simultaneous onsets occur. Gaps can occur in the timeline, but Toussaint suggests that the different voices require different timbres. Rhythmic canons thus exhibit translational symmetry.

Rhythmic tiling canon

A subcategory of rhythmic canon where all pulses of the timeline are sounded with no simultaneous onsets between the voices. *Nyog cag kotekan* is a simple form of rhythmic tiling canon. (See also *Tiling*.)

Rhythmic contour

The relative change in duration of the attacks in a particular motif. (See also *Generic rhythmic contour*.)

Self-similarity

The presence of self-similar relationships may be indicated by scaling, recursive patterns and geometric sequences. Awareness of the principles of self-similarity encourages the composer/analyst to detect features and relationships that traverse multiple parameters, over different scales of resolution, that may work together to create a web of inter-relationships. (See also *Fractal*.)

Solkattu

A grouping of rhythmic syllables that form a syntactic word in South Indian music, such as “ta di ki na dom”. (See *Bol*.)

Specific rhythmic contour

A measurement of the chronological change in duration between a series of inter-onset intervals. (See also *Generic rhythmic contour*.)

Subdivision

A portion of a beat. Simile for *Pulse*.

Sam

Beat 1 in a North Indian rhythmic cycle. Also referred to as *Samam* in a South Indian rhythmic cycle.

Stages

The stages of a counting sequence are the intelligent partitions that reveal the pattern and symmetry of the lines that comprise the sequence. Most applicable to Indian counting sequences. (See also *Yati*.)

Tabla

North Indian hand drums, used in a pair consisting of the *bayan* (left bass drum) and *dayan* (right treble drum, tuned to the drone).

Tala

The metre or time cycle of Indian music. Literally meaning “clap”, the *tala* is represented by a series of claps/taps and waves in both North and South Indian classical music. *Tala* of various lengths are denoted using prefixes, e.g. *tintal* (North Indian 16 beats) and *aditalam* (South Indian 8 beats).

Tani Avartanam

A percussion solo portion of a South Indian classical performance, combining improvisation with compositional norms.

Tanpura

Drone instrument used in Indian classical music, consisting of a long fretless neck upon a gourd, and four to six strings, typically tuned to the tonic and dominant.

Ternarization

The modulation of a binary timeline into a ternary one. The *specific rhythmic contour* is invariant during the procedure, yet there is a scaling operation in effect. *Ternarization* is thus a self-similar technique. (See also *Binarization*.)

Ternary timeline

Ternary timelines have a beat that is compound—it can be divided by 3. Timelines with lengths of 3, 6, 9, 12, or 24 pulses are common examples. (See also *Binary Timeline*.)

Tihai

A rhythmic cadence that belongs to North Indian music. It features a phrase sounded three times, as in AAA. (See also *Arudi*, *Mora*.)

Tiling

Tiling in music adopts the one-dimensional tiling principles of mathematics so that two or more voices combine in such a way that each and every pulse in a cycle has one and only one onset. In its pure form, tiling is designed so that displaced repetitions of a single motif are responsible for covering all the onsets without overlaps. The result is a homogeneous pulse train that is shared by the voices—a tessellation. (See also *Rhythmic tiling canon*.)

Timeline

In this dissertation, a timeline is both a specific periodic strand of time, and also the diagrammatic representation of such a strand of time. (See also *Clavé.*)

Translational symmetry

In music *translation* means to repeat, and this can occur about a horizontal axis as transposition (movement to a new key) or about a vertical axis as repetition (temporal displacement). (See also *Interval of translation*.)

Urlinie

A term from Schenkerian analysis that has relevance to self-similarity. Schenker worked to show that a reduction of the upper voice called the *urlinie* ('fundamental line') often creates a stepwise descent to the tonic over the course of a piece.

Vibhag

A metric subset of a *tala* (rhythmic cycle) in North Indian classical music. Similar to a bar or measure.

Yati

South Indian rhythmic patterns that can be represented geometrically to show their symmetry. *Yati* exhibit bilateral symmetry, shear symmetry, translation, or a compound of symmetries.

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